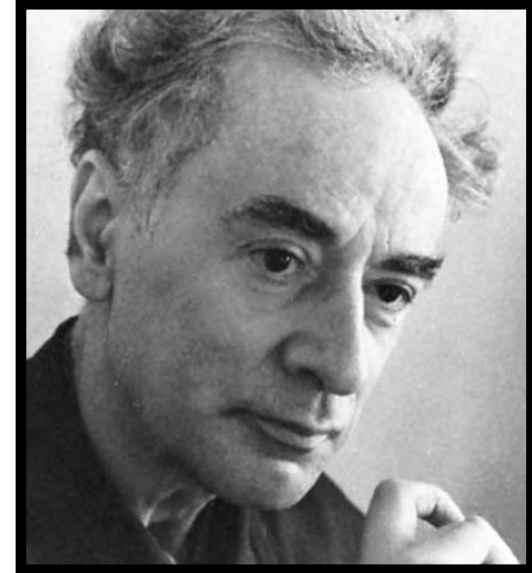


**Landau Congress**  
100 years of Lev Landau  
**Memorial meeting**  
Moscow, June 19-20, 2008



Some Mysteries  
about  
High-Temperature Superconductors

Elihu Abrahams  
Rutgers University

**RUTGERS**  
CENTER FOR MATERIALS  
THEORY

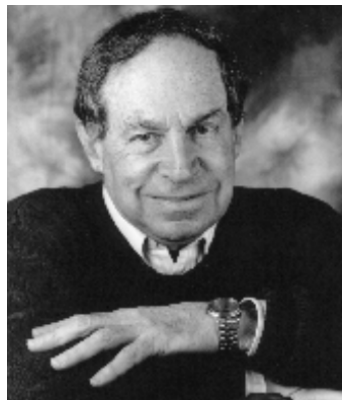
# WHY I AM HERE

Elihu Abrahams  
Rutgers University

# 10<sup>TH</sup> LOW TEMPERATURE PHYSICS CONFERENCE, MOSCOW 1966

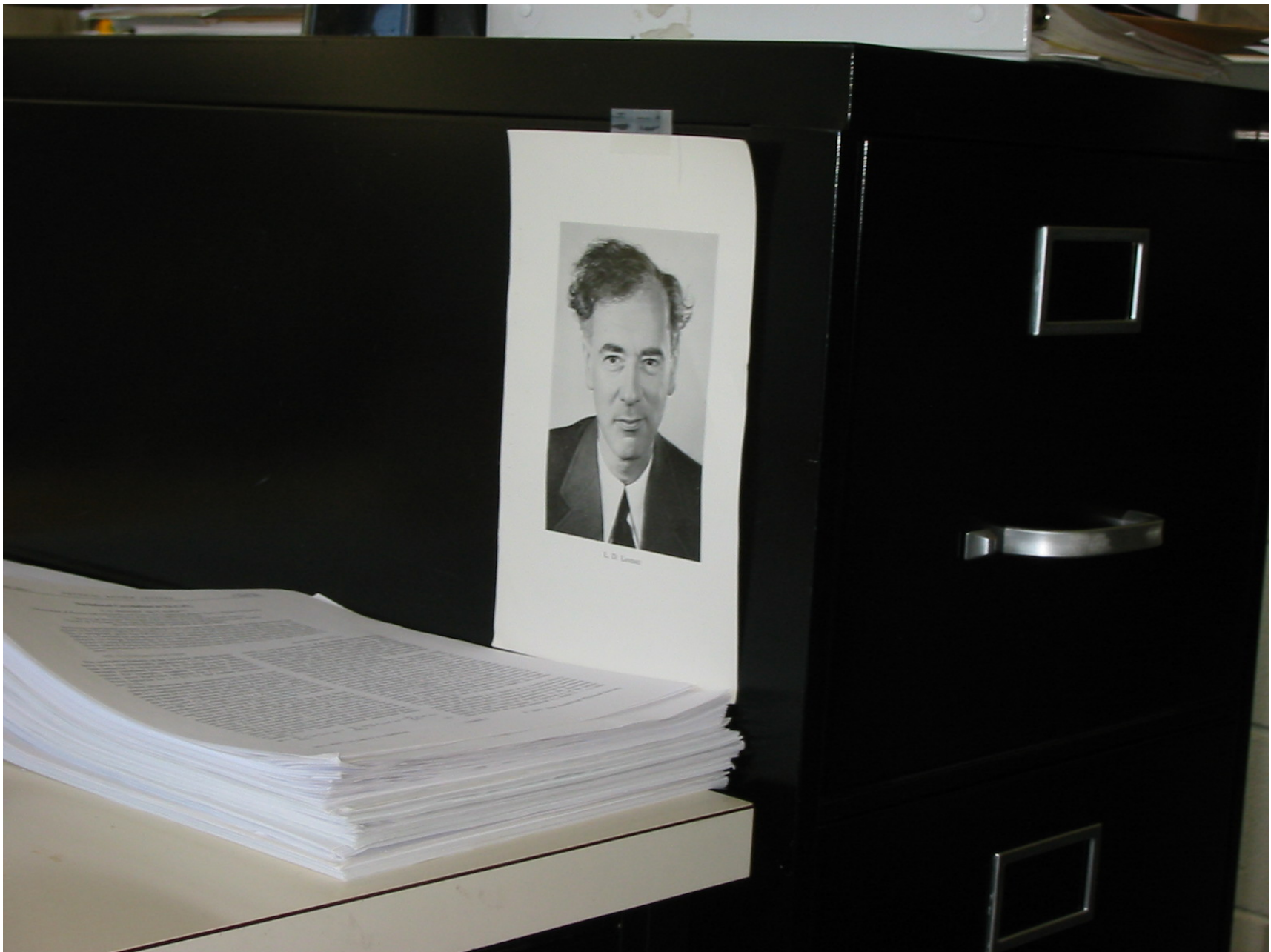
## US/USSR Workshops in Condensed Matter Theory 1968, 1970, 1974, 1978, 1988

D. PINES



I.M. KHALATNIKOV





Back in 1986



Z. Phys. B – Condensed Matter **64**, 189-193 (1986)

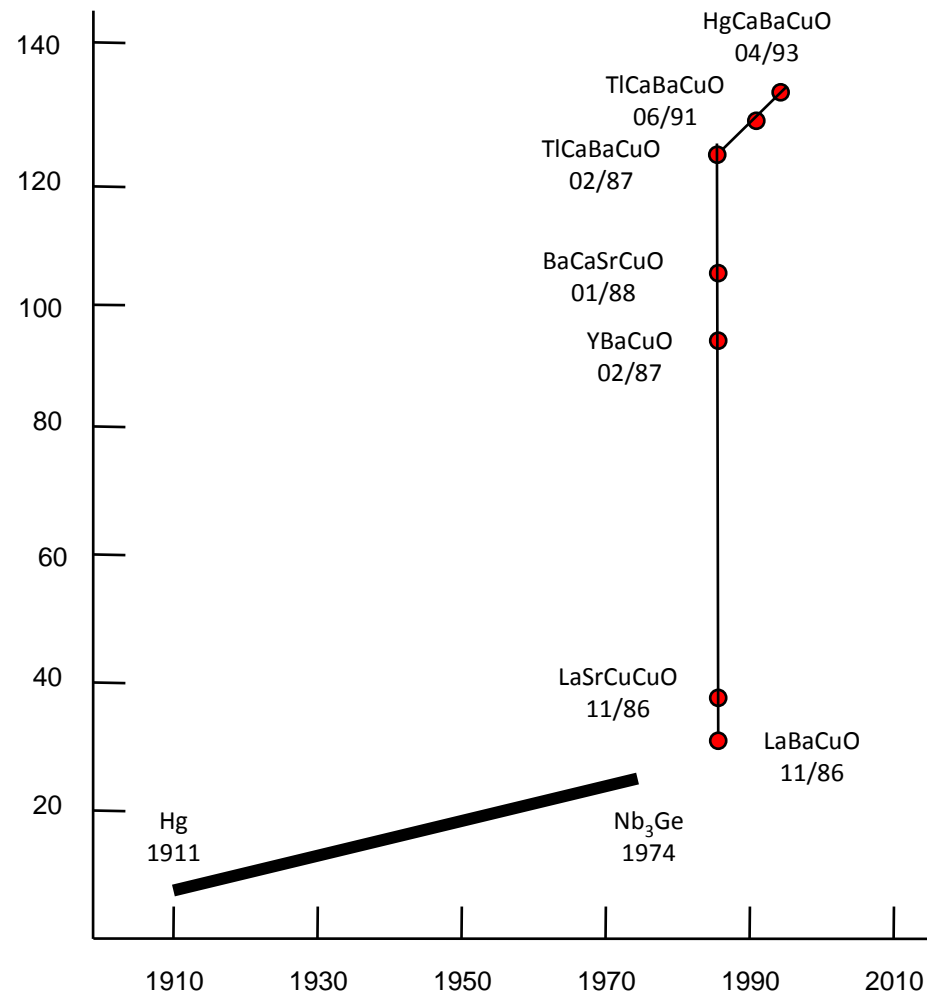
## **Possible High $T_c$ Superconductivity in the Ba – La – Cu – O System**

**J.G. Bednorz and K.A. Müller**

IBM Zürich Research Laboratory, Rüschlikon, Switzerland

Received April 17, 1986

# $T_c$ vs $t$

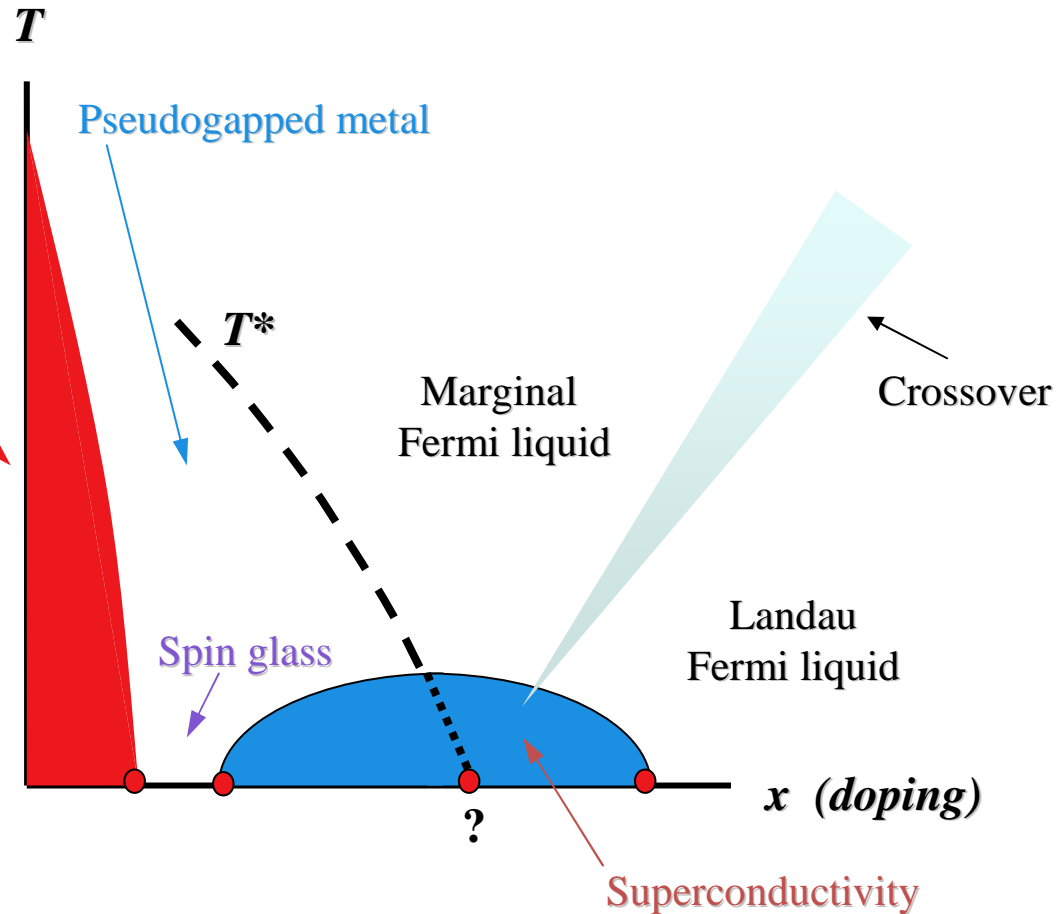


# Schematic phase diagram of a high- $T_c$ superconductor

For example:

$\text{La}_{2-x}\text{Sr}_x\text{CuO}_4$   
x holes in  $\text{CuO}_2$   
plane

Antiferromagnetism



● Quantum critical points

Competition: Almost everything is a mystery

# Theories connected with high- $T_c$ superconductivity

Spin fluctuations

Anisotropic phonons

Excitons

Charge fluctuations

Plasmons

Circulating currents

Bipolarons

Resonating valence bonds

Stripes

Interlayer tunneling/Kinetic  
energy lowering

Spin bags

Spin liquids

Flux phases

BCS/BEC crossover

Marginal Fermi liquid

Van Hove singularities

Quantum criticality

Anyon superconductivity

Time reversal violation

Dynamical mean field theory

Slave bosons

Gauge theory

d density waves

Gossamer superconductivity

SO(5)



# Reflections on Theory

Some important quotes:

“It’s so hard that the only way to win is to be ‘right’.”

“It is dark law.”

**PESSIMISTS**

“It could be that focusing on the mechanism is the reason that the mechanism hasn’t been found.”

“We are close to a theoretical consensus on the magnetic origin ---”

“The superconductivity is understood in detail.”

“The cause of high- $T_c$  superconductivity is no mystery” -  
and - “If it is accepted, why no one wants to accept it.”

**OPTIMISTS**

“Eighty percent of the field is against anything – especially anything that might solve the problem.”

“As a result I can state that the so called ‘mystery’ of high- $T_c$  superconductivity does not exist.”

# The theory question

Has the “problem” of the theory of high- $T_c$  been solved?

If so, what is it?

The model solution:

## Theory of Superconductivity\*

J. BARDEEN, L.N. COOPER,<sup>†</sup> AND J.R. SCHRIEFFER<sup>‡</sup>  
*Department of Physics, University of Illinois, Urbana, Illinois*  
(Dated: July 8, 1957)

A theory of superconductivity is presented . . . . The ground state of a superconductor . . . is lower in energy than the normal state by an amount proportional to an average  $(\hbar\omega)^2$ , consistent with the isotope effect. . . . The theory yields a second order phase transition and Meissner effect in the form suggested by Pippard. Calculated values of specific heats and penetration depths and their temperature variation are in good agreement with experiment. There is an energy gap for individual-particle excitations which decreases from about  $3.5kT_c$  at  $T = 0^\circ\text{K}$  to zero at  $T_c$ . Tables of matrix elements of single-particle operators between the excited-state superconducting wave functions, useful for perturbation expansions and calculations of transition probabilities, are given.

# Comprehensive BCS Theory!

## I. INTRODUCTION

## II. THE GROUND STATE

A. Ground-State Energy

B. Energy gap at  $T = 0^\circ\text{K}$

## III. EXCITED STATES

A. Minimization of the Free Energy

B. Critical Field and Specific Heat

## IV. CALCULATION OF MATRIX ELEMENTS

## V. ELECTRODYNAMIC PROPERTIES

A. The Meissner Effect

B. Current Density

C. Penetration Depths

## VI. CONCLUSION

APPENDIX A. CORRECTIONS TO GROUND STATE ENERGY  
APPENDIX B. CHANGE IN ZERO-POINT ENERGY OF LATTICE VIBRATIONS  
APPENDIX C. EVALUATION OF THE KERNEL IN THE PIPPARD INTEGRAL  
APPENDIX D. CORRELATION OF ELECTRONS OF OPPOSITE SPIN

Ultrasonic attenuation  
NMR  $1/T_1$   
Thermal conductivity  
Microwave conductivity  
Infrared absorption

# Теория сверхпроводимости в купратах

XXXX, YYYY и ZZZZ

*Институт сильно взаимодействующей физики Рога и копыта, где-то*

Абстракт: Предлагается теория сверхпроводимости в купратах ...

## I. EFFECTIVE HAMILTONIANS

A. Ground State

B. Normal State

## II. THE PHASE DIAGRAM

A. Undoped State

## III. NORMAL STATE PROPERTIES

A. Transport

B. Thermodynamics

## IV. UNDERDOPED REGION

A. Origin and behavior of the Pseudogap

B. Transport and Thermodynamics

C. Nernst Effect

D. Evolution of the Fermi Surface

E. Magnetic Oscillations

## V. SUPERCONDUCTING STATE

A. Energy gap and Tunneling

B. Transport and Thermodynamics

Comprehensive  
high- $T_c$  theory

# Schematic phase diagram of a high- $T_c$ superconductor

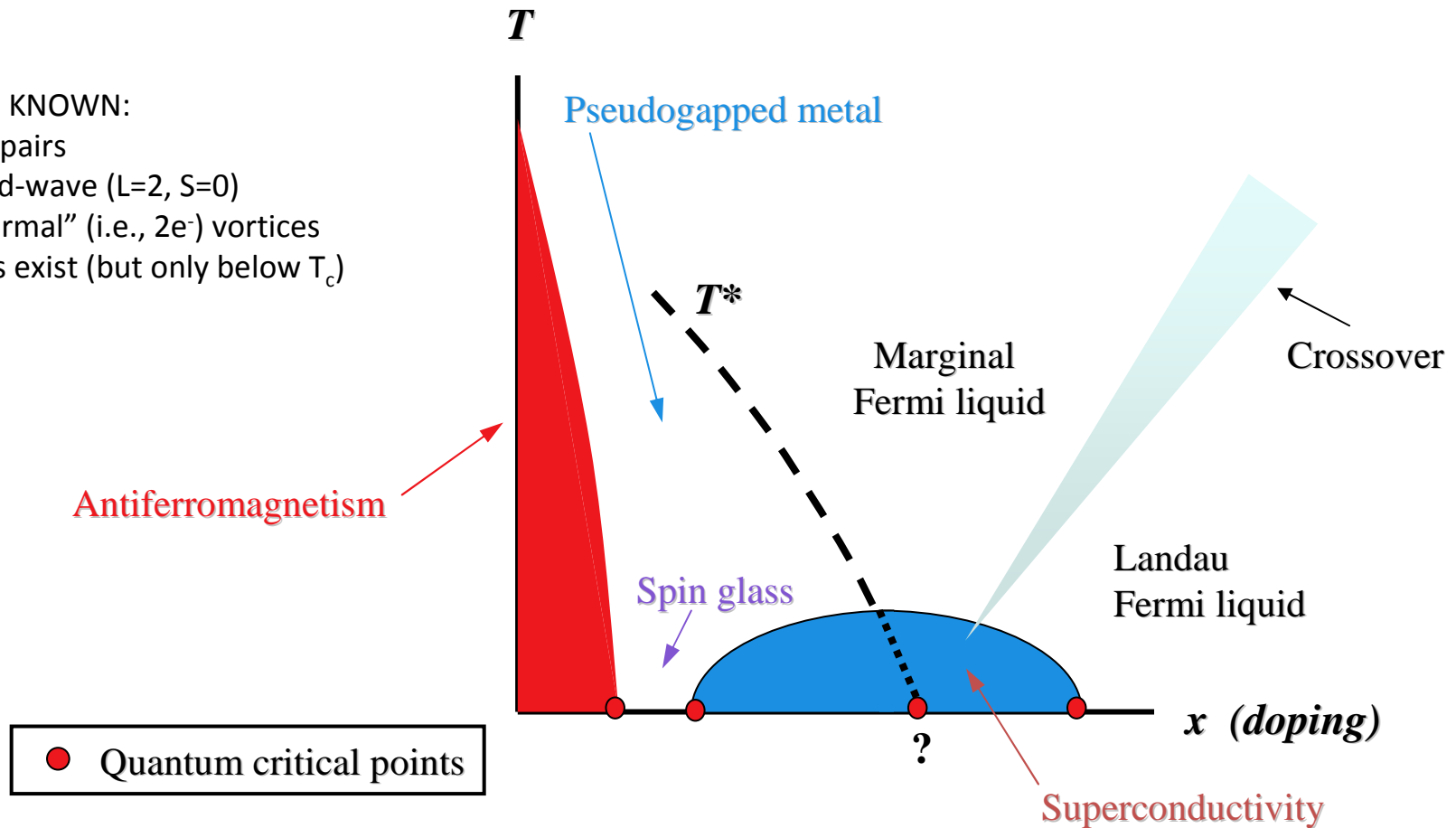
WHAT ELSE IS KNOWN:

There are  $2e^-$  pairs

The pairs are d-wave ( $L=2, S=0$ )

There are "normal" (i.e.,  $2e^-$ ) vortices

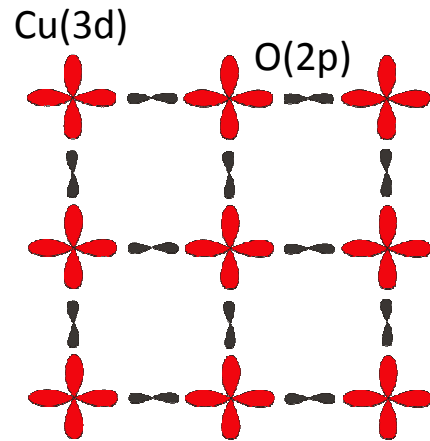
Quasiparticles exist (but only below  $T_c$ )



All these behaviors need to be explained!

# Effective hamiltonians

The maximal minimal model: 3-band model

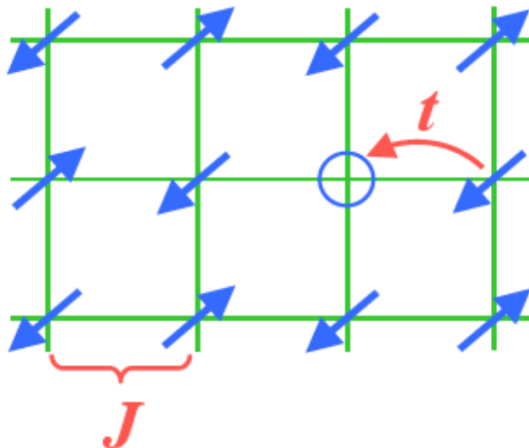


$$\begin{aligned} \mathcal{H} = & \sum \epsilon_d n_d + \sum \epsilon_p n_p \\ & + t \sum (c_{d\sigma}^+ c_{p\sigma} + h.c.) + t' \sum (c_{p\sigma}^+ c_{p\sigma} + h.c.) \\ & + U_d \sum n_{d\uparrow} n_{d\downarrow} + V \sum n_d n_p \end{aligned}$$

The minimal minimal model: Hubbard model

$$\mathcal{H} = -t \sum_{\langle ij \rangle \sigma} (c_{i\sigma}^+ c_{j\sigma} + h.c.) + U \sum_i n_{i\uparrow} n_{i\downarrow}$$

Zhang Rice

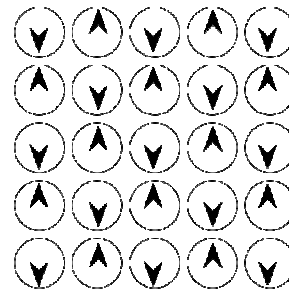


The optimistic minimal model: t-J model

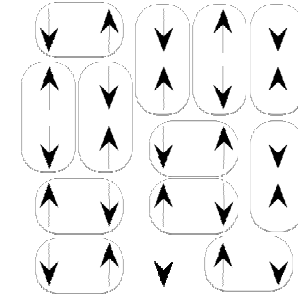
$$\mathcal{H} = -t P \sum_{\langle ij \rangle \sigma} (c_{i\sigma}^+ c_{j\sigma} + h.c.) P + J \sum_{\langle ij \rangle} S_i \cdot S_j$$

# RVB phase diagram

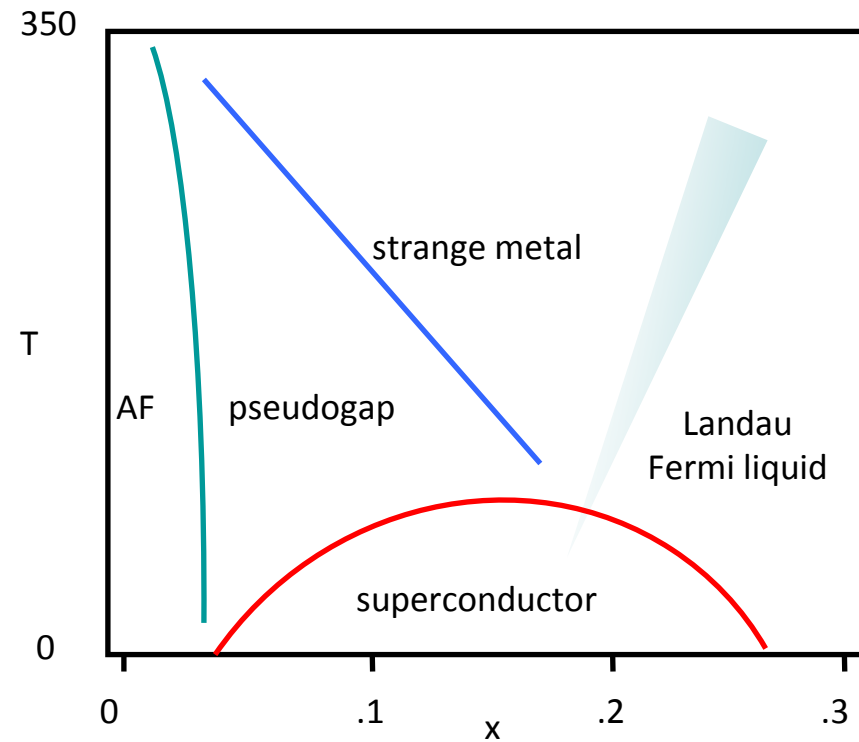
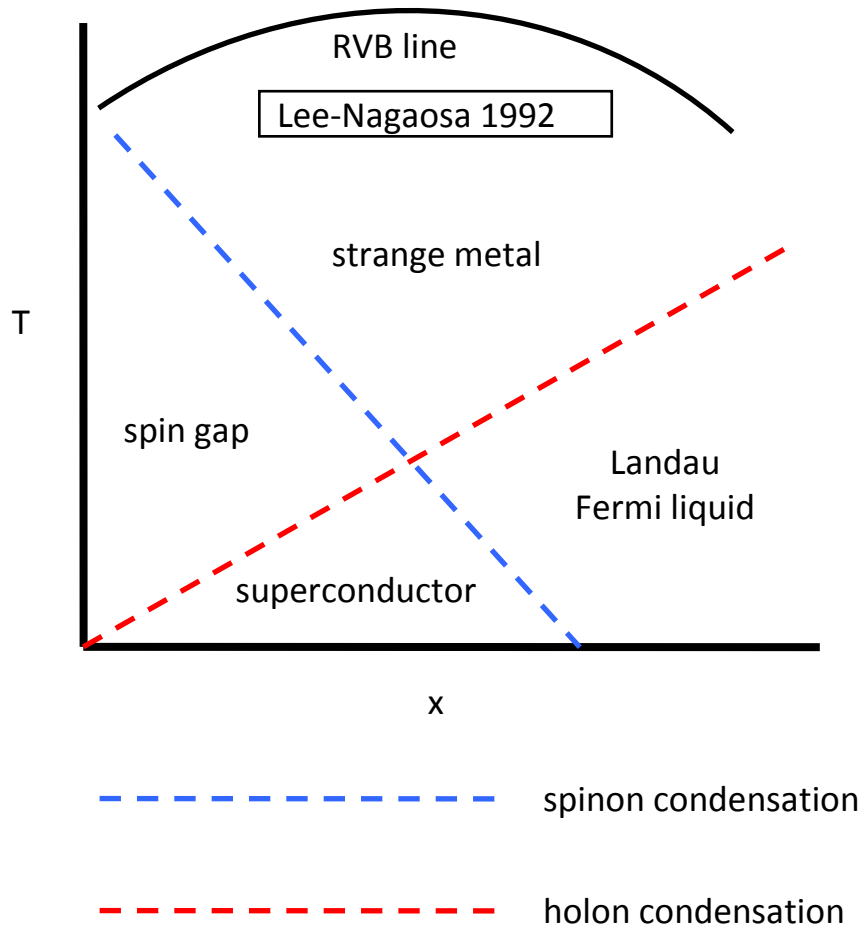
(the t-J model is enough)



Neel state



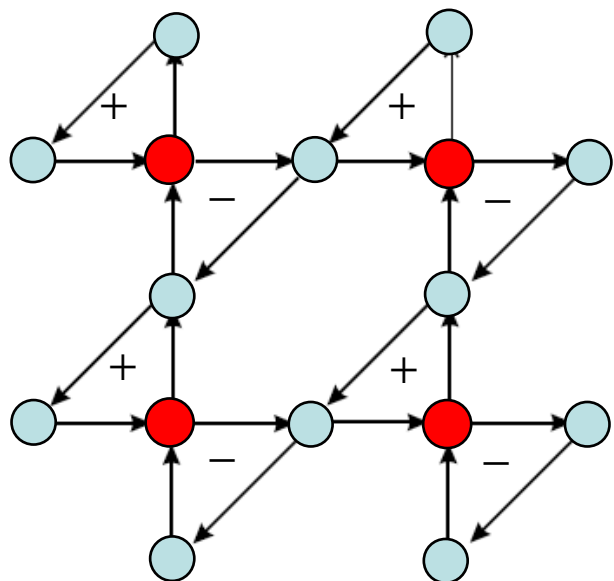
RVB spin singlet liquid



# Circulating currents

T breaking, translation symmetry preserved, or not?

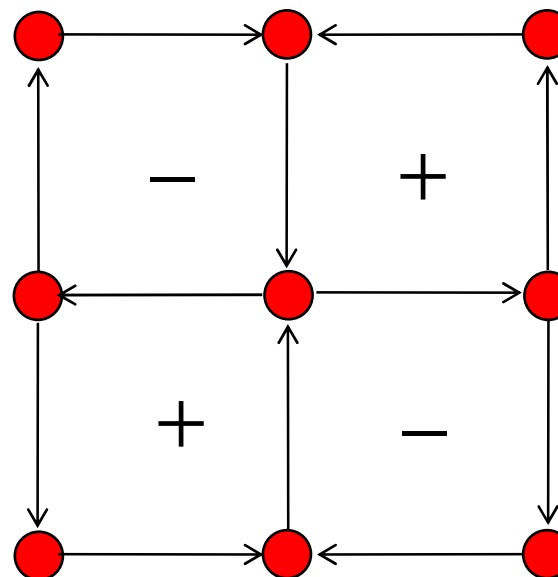
(You need the 3-band model!)



Simon and Varma 2002

$t$  (Cu-O)                      ● Cu  
 $t'$  (O-O)  
 $U_{Cu}$  (mean field)            ● O

(Hubbard model works!)



Chakravarty, et al 2001

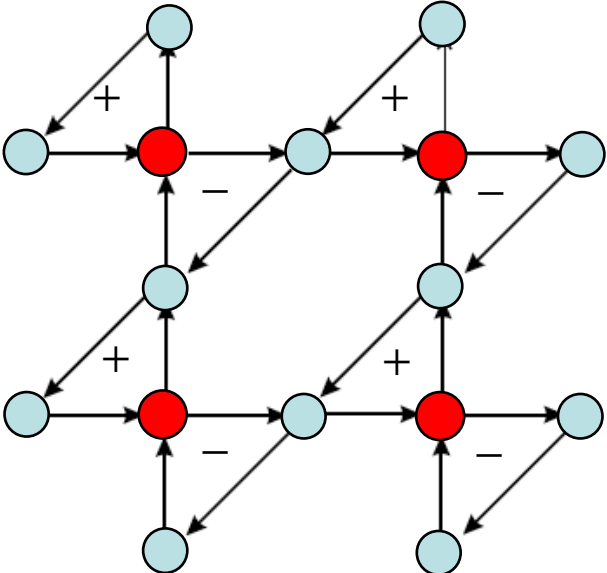
"d-density wave"  
 Unit cell is doubled



# Circulating currents

T breaking, translation symmetry preserved

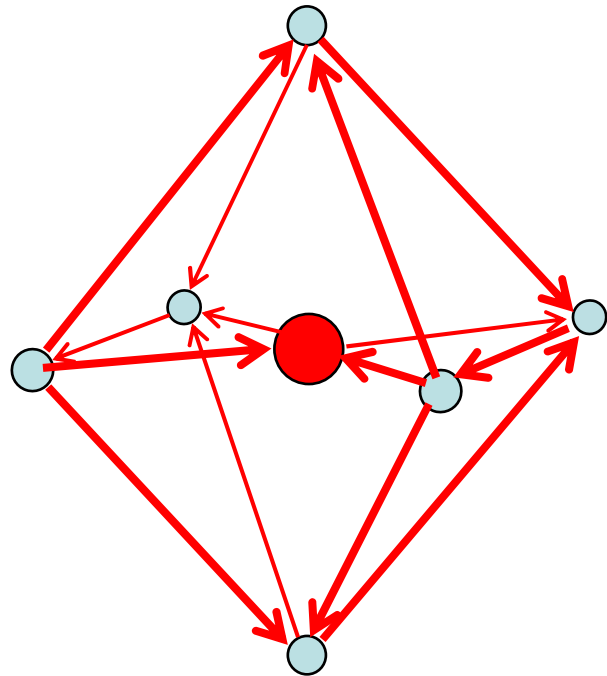
(You need the 3-band model!)



Simon and Varma 2002

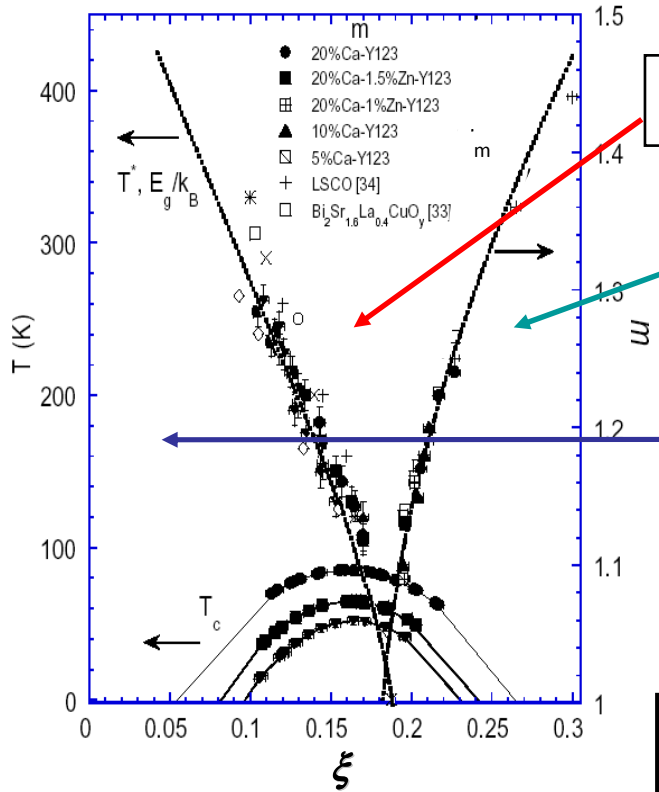
- $t$  (Cu-O)                      ● Cu
- $t'$  (O-O)
- $U_{Cu}$  (mean field)            ● O

(And: you need apical oxygens!)



C. Weber et al 2008  
Li, Bourges et al 2008

# Quantum critical point ?

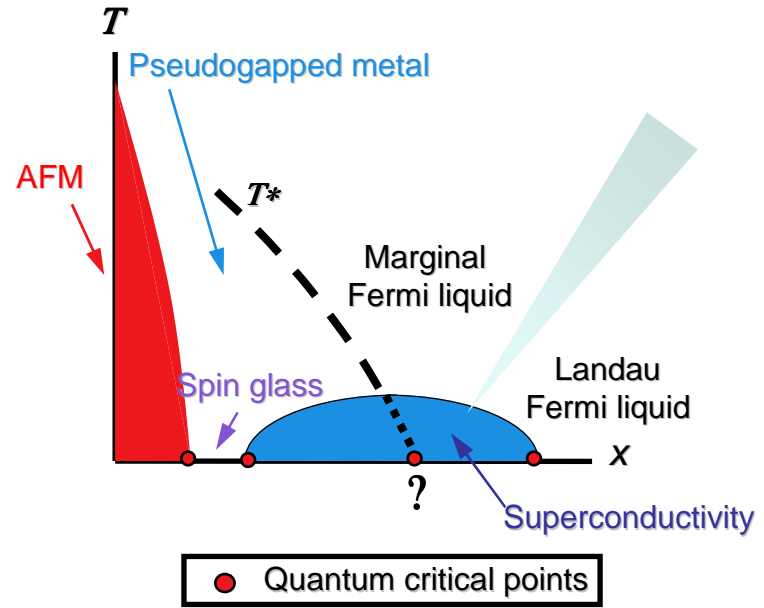


Naqib et al 2002

The marginal Fermi liquid region,  $\rho \propto T$

Overdoped region,  $\rho = \rho_0 + \alpha T^\mu$

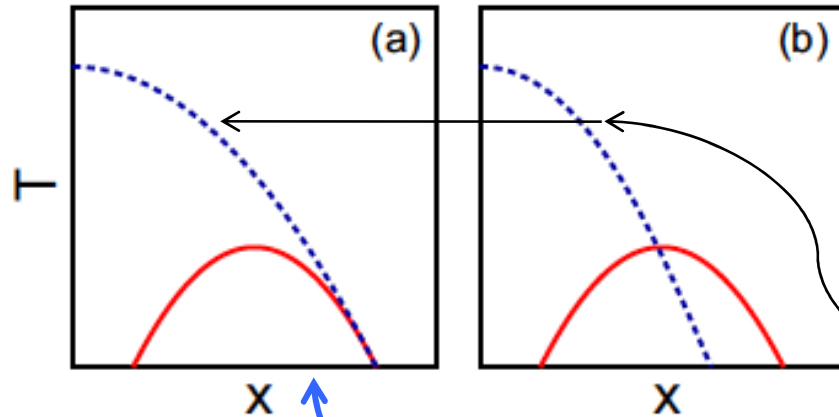
Pseudogap region,  $E_\gamma$  from specific heat,  $T^*$  from various



Marginal Fermi liquid phenomenology: Varma, Littlewood, Schmitt-Rink, Abrahams, Ruckenstein 1989

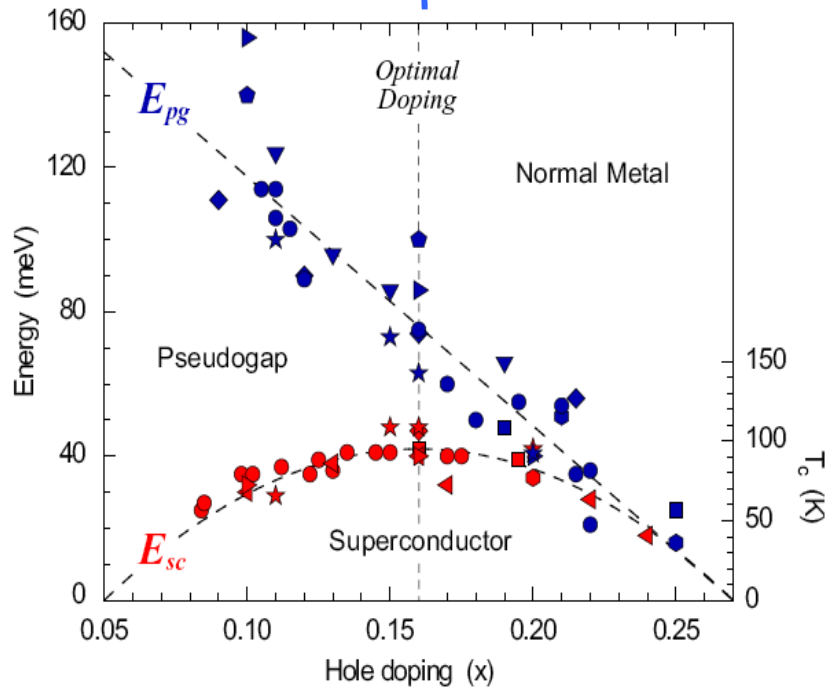
- Raman scattering
- Spectral function (ARPES)
- Linear-T resistivity

Assumes  $\omega/T$  scaling of correlator.



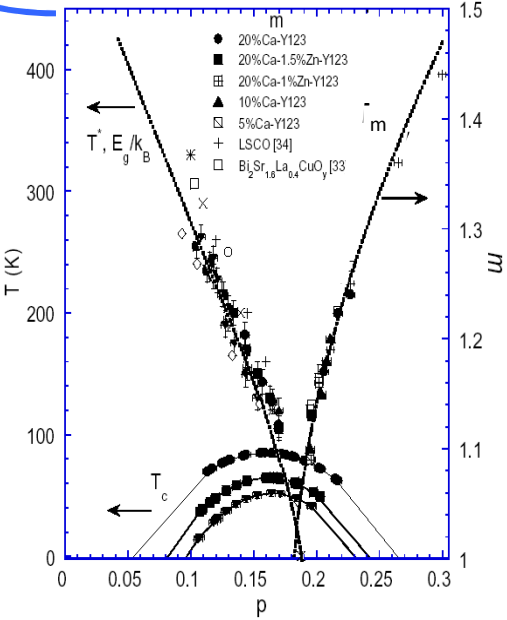
**However**  
 (do not be blinded by guides to the eye)

Is this a phase transition?  
 Circulating currents?  
 d-density wave?, SDW?

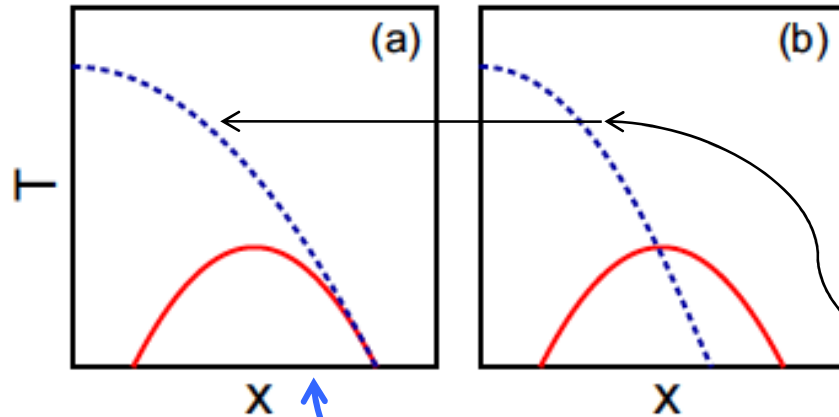


Hüfner et al 2007

- ◆ Bi ARPES<sup>48</sup>
- ★ Bi RS-B<sub>1g</sub><sup>45,46</sup>
- Bi SIS<sup>34,35</sup>
- ▼ Bi STM<sup>38</sup>
- ▶ Bi SIN<sup>34</sup>
- TI ARPES<sup>56</sup>
- TI HC<sup>68</sup>
- ◆ Y HC<sup>67</sup>
- Bi ARPES<sup>49</sup>
- Bi INS<sup>64</sup>
- ★ Bi RS-B<sub>2g</sub><sup>45,46</sup>
- ▲ Bi SIS<sup>35</sup>
- ◆ TI INS<sup>66</sup>
- ▶ Y AR<sup>69</sup>
- Y INS<sup>64</sup>

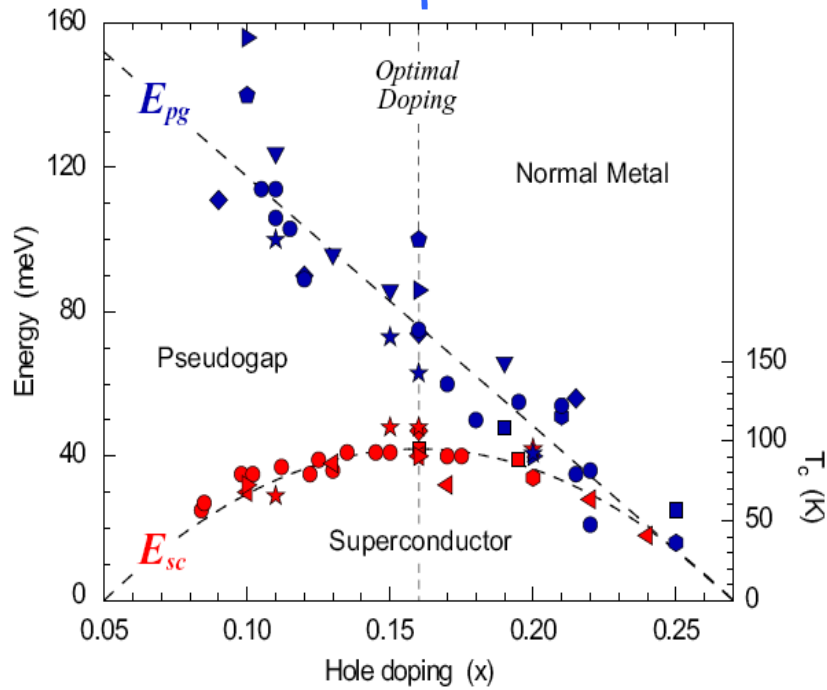


Naqib et al 2002



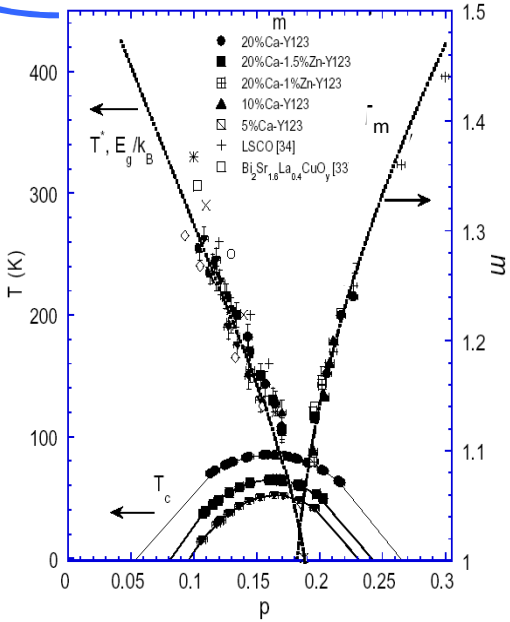
**However**  
 (do not be blinded by guides to the eye)

Is this the onset of pairing,  
 without long range order.  
 Phase fluctuations?



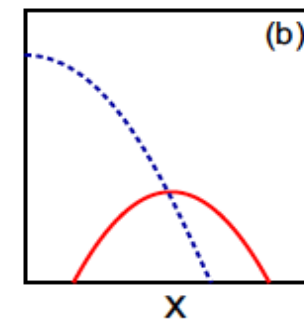
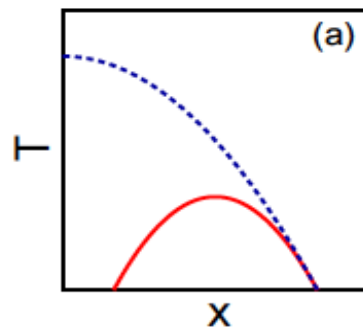
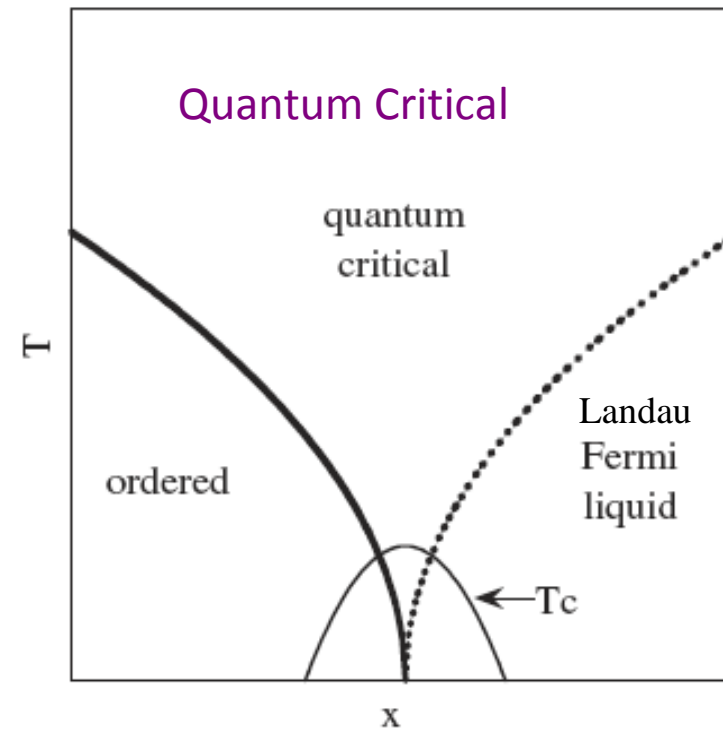
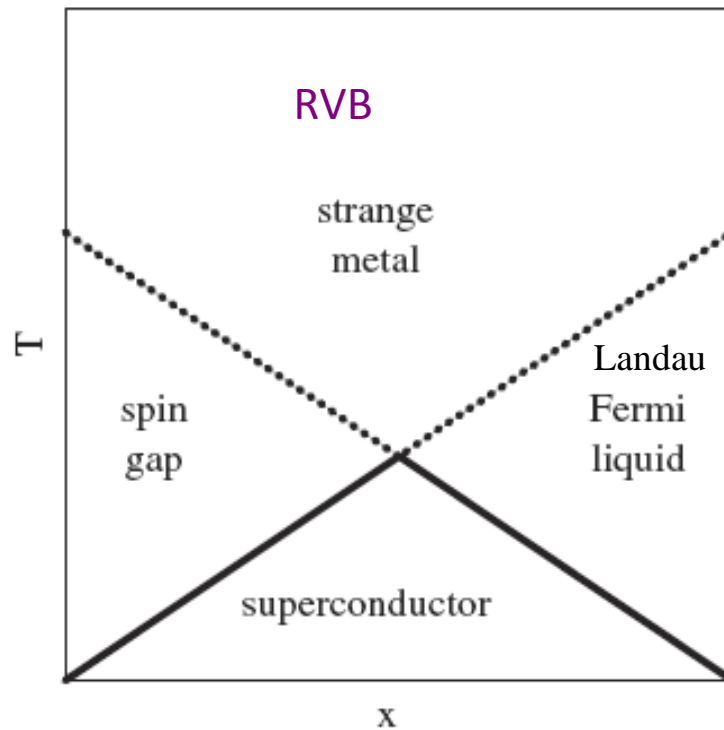
Hüfner et al 2007

- ◆ Bi ARPES<sup>48</sup>
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- ▲ Bi SIS<sup>35</sup>
- ◆ TI INS<sup>66</sup>
- ▶ Y AR<sup>69</sup>
- Y INS<sup>64</sup>



Naqib et al 2002

# Two Theories of the Phase Diagram

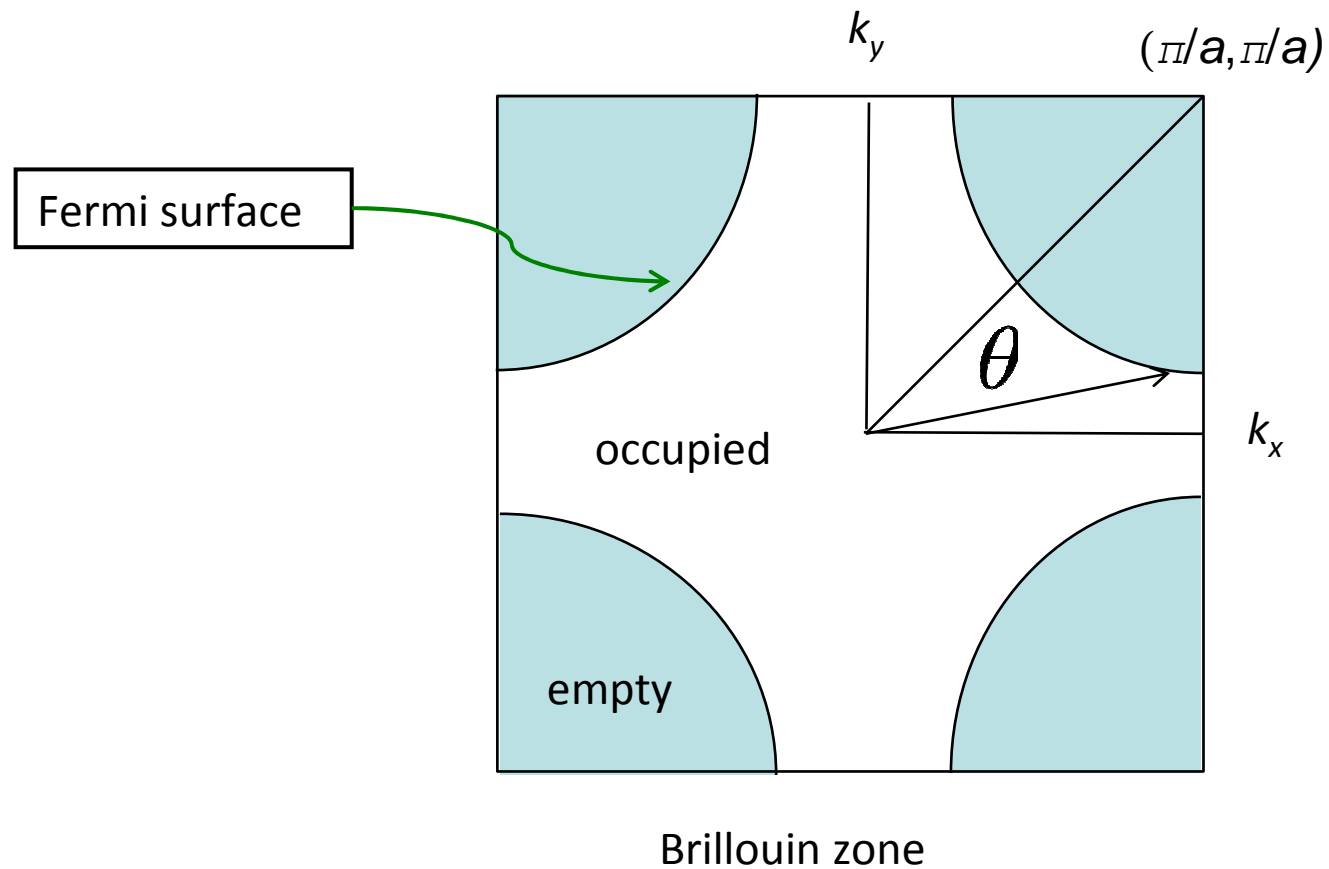


## Some mysteries of the pseudogap:

1. What causes it?- preformed pairs, sdw, ddw, circulating currents, stripes, cdw, resonating valence bonds?
2. Does the pseudogap phase have an order parameter?
  - Is  $T^*$  a phase transition line to a competing phase?
3. Do the pseudogap and the superconducting gap coexist below  $T_c$ ? One gap or two?
4. What is the nature of electronic states below  $T^*$ ?
5. What is the correct position of the  $T^*$  line?
  - Is there a hidden quantum critical point?
6. What is the origin of “Fermi arcs?”

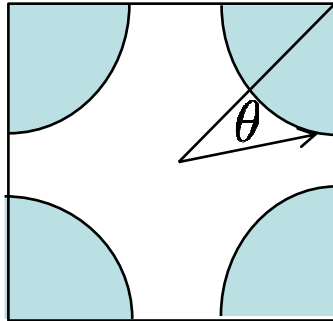
# Fermi surface of optimally-doped cuprate

Angle-Resolved Photoemission Spectroscopy (ARPES)



# Marginal Fermi Liquid fits to ARPES

$$\mathcal{A}(\mathbf{k}, \omega) = -\frac{1}{\pi} \frac{\Sigma_2(\mathbf{k}, \omega)}{[\omega - \varepsilon_k - \Sigma_1(\mathbf{k}, \omega)]^2 + [\Sigma_2(\mathbf{k}, \omega)]^2}$$

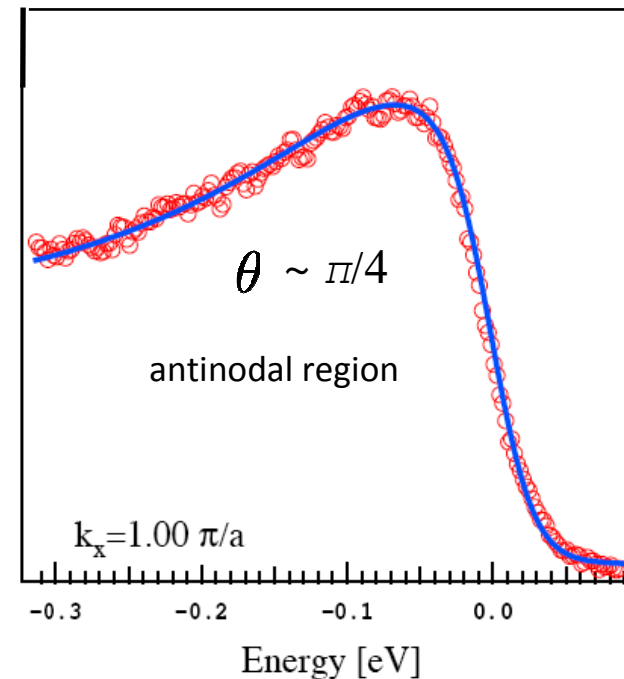
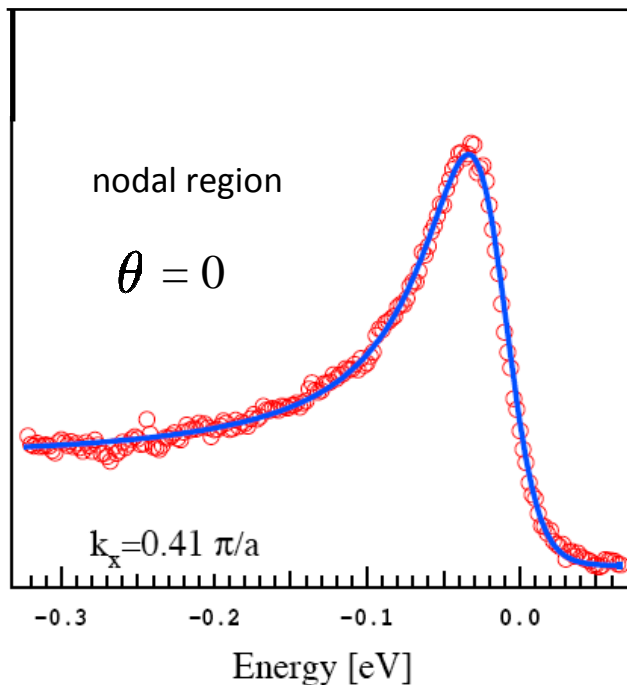


Kaminski et al 2005

MFL scenario gives

$$\Sigma_2(\omega) = a(\theta) + b\omega$$

Where does this come from?

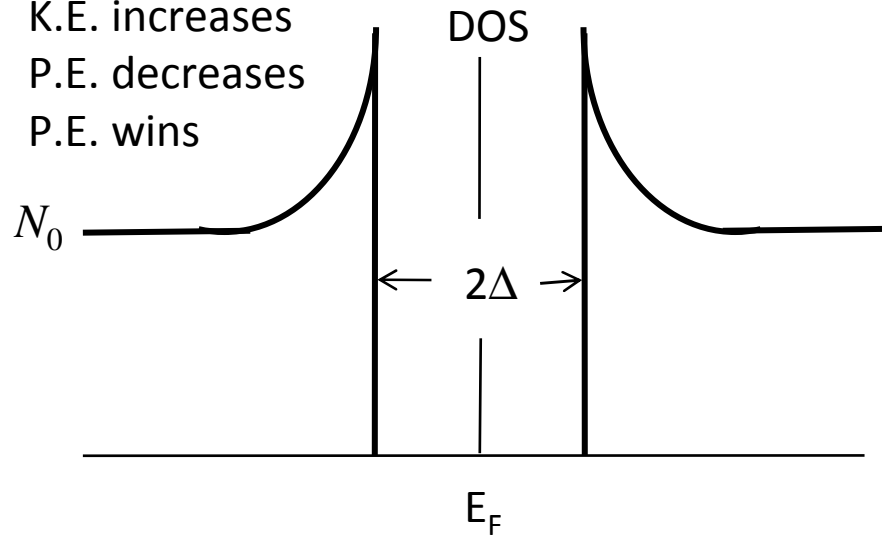




# Condensation energy

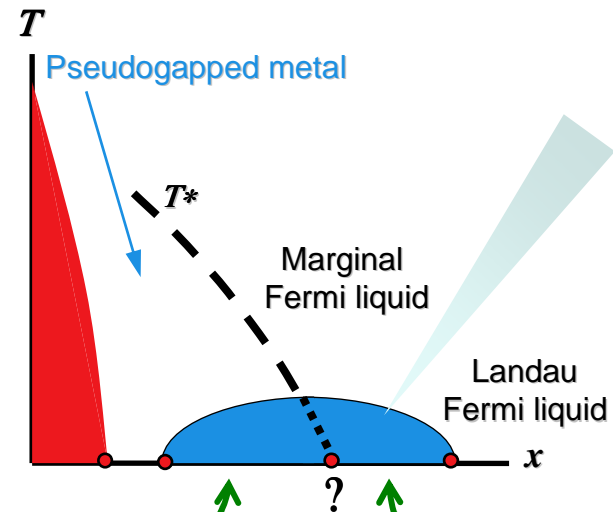
BCS:

K.E. increases  
P.E. decreases  
P.E. wins



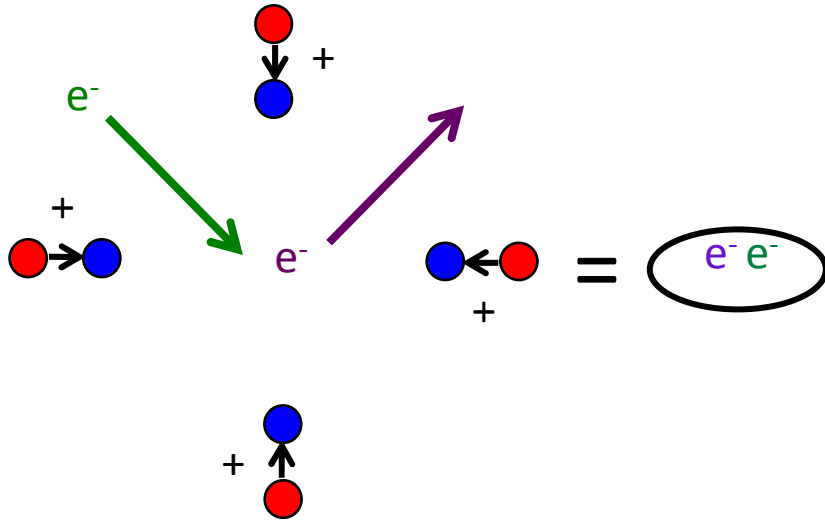
$$\Delta = \omega_D \exp(-1/N_0 g)$$

Large DOS ( $N_0$ )  $\rightarrow$  large  $\Delta$



High  $T_c$ :  
K.E. increases (overdoped)  
K.E. decreases (underdoped)  
who wins?

# Everything (?) You Wanted to Know About Pair Formation (The Tyranny of the "Glue")



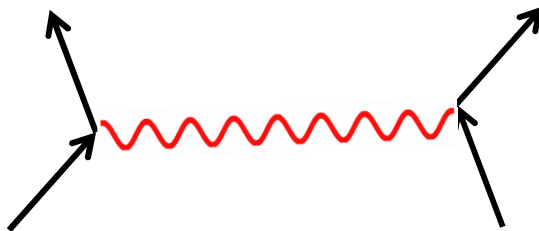
(the electron-phonon case)

1. 1st  $e^-$  attracts + ions
2. Ions shift position from red to blue
3. 1st  $e^-$  moves away
4. 2nd  $e^-$  sees + ion hole and moves to former position of 1st  $e^-$

$$\mathcal{H} = K.E. + g \int d\mathbf{r} \psi_{\uparrow}^{\dagger}(\mathbf{r})\psi_{\downarrow}^{\dagger}(\mathbf{r})\psi_{\downarrow}(\mathbf{r})\psi_{\uparrow}(\mathbf{r})$$

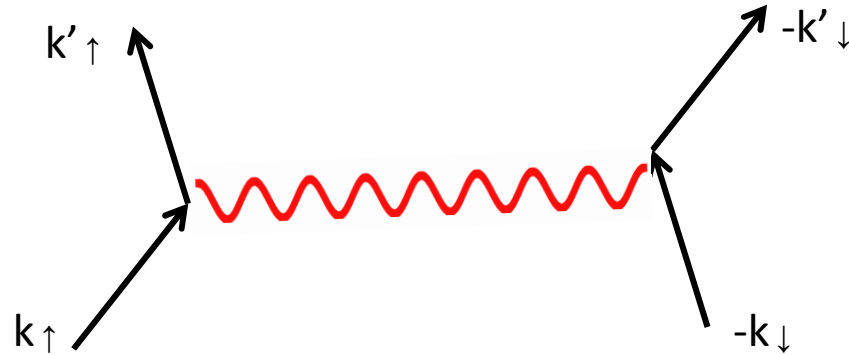
Interaction is local in space  
(s-wave singlet pairs,  $L=0, S=0$ )  
but retarded in time

or, Eliashberg's theory: phonon or spin fluctuation ( $L=2, S=0$ )



Large DOS ( $N_0$ )  $\rightarrow$  large  $\Delta$

## Eliashberg theory for retarded interaction



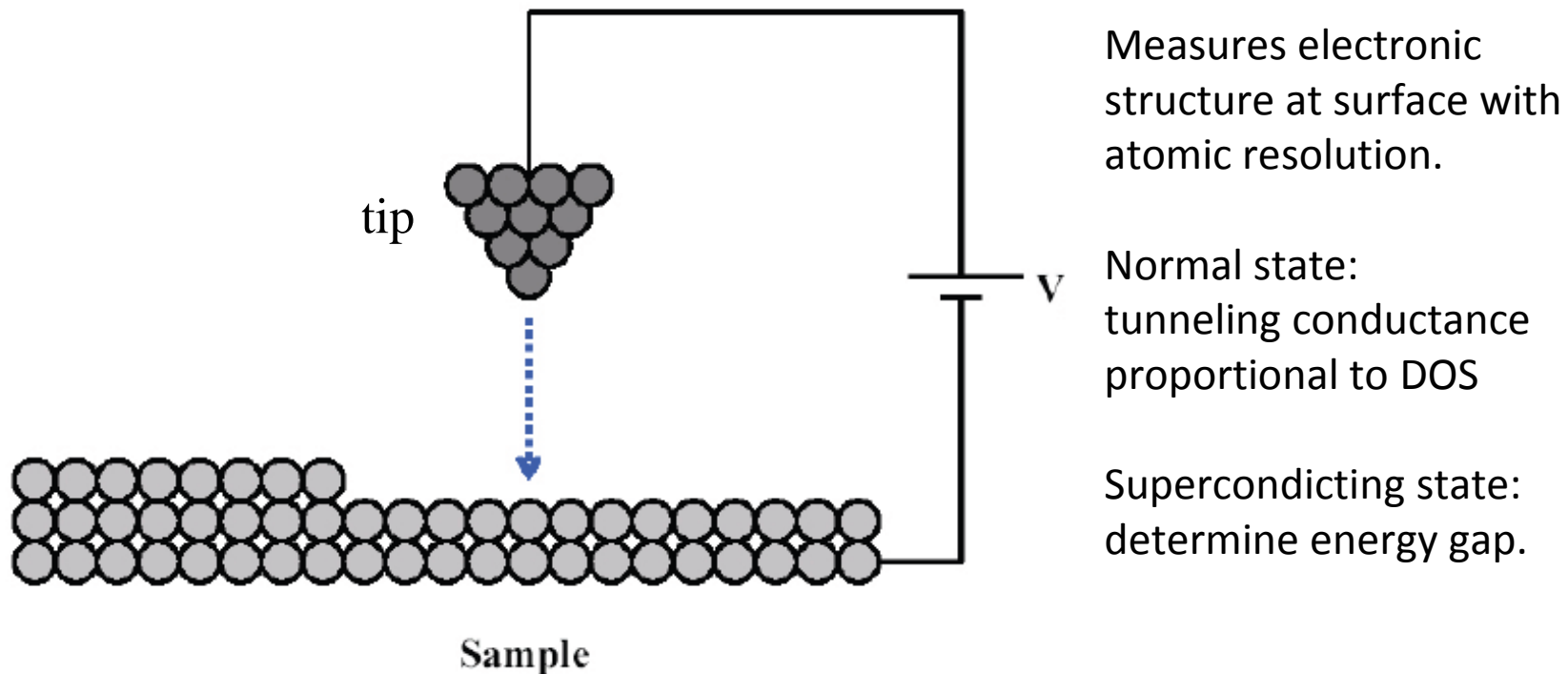
-Underlying well-defined normal state, e.g. Landau Fermi liquid quasiparticles and “pairing boson”.



-For cuprates, it would be antiferromagnetic spin fluctuation  $\rightarrow$  d-wave pairing.

-Large density of states  $\rightarrow$  large energy gap

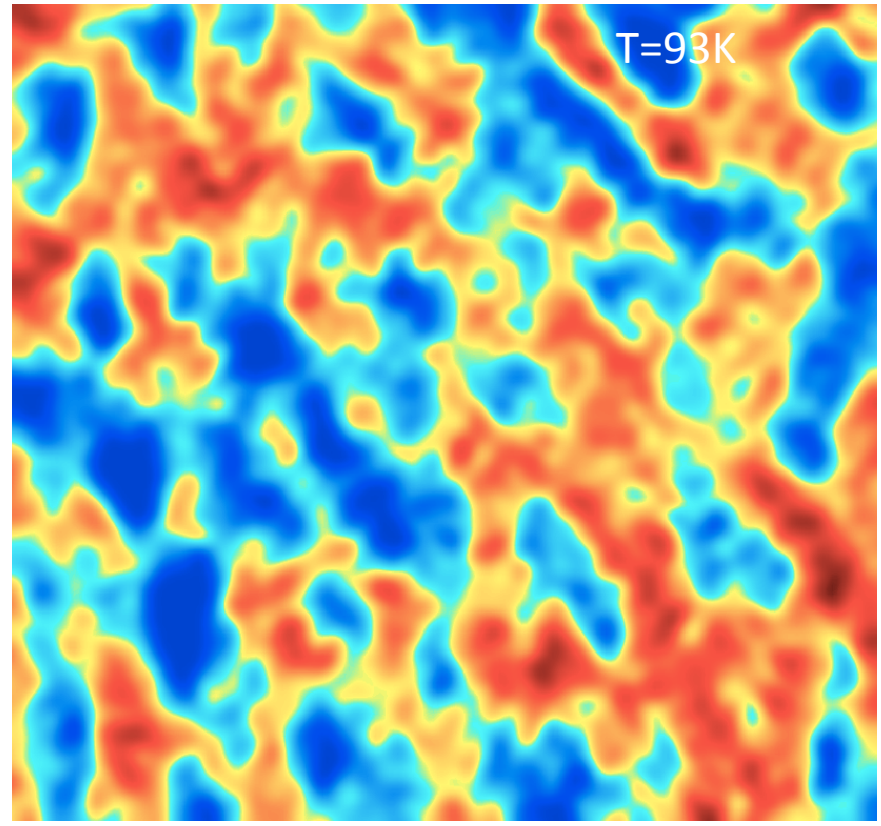
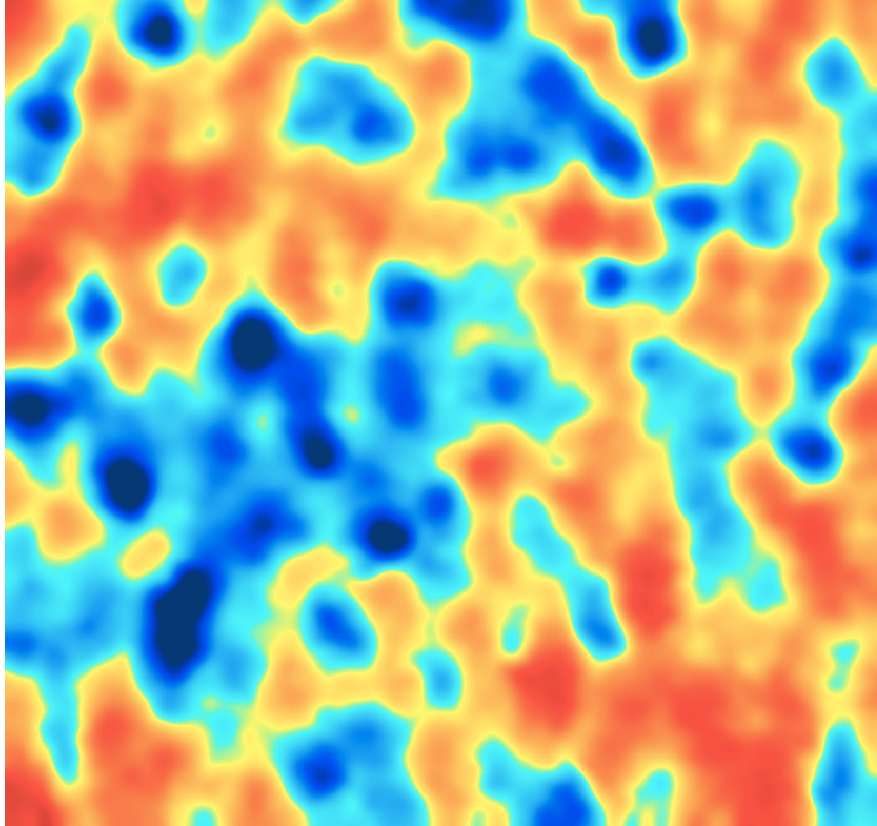
# Scanning tunneling microscope STM experiment



Measures electronic structure at surface with atomic resolution.

Normal state:  
tunneling conductance  
proportional to DOS

Superconducting state:  
determine energy gap.



# A possible resolution

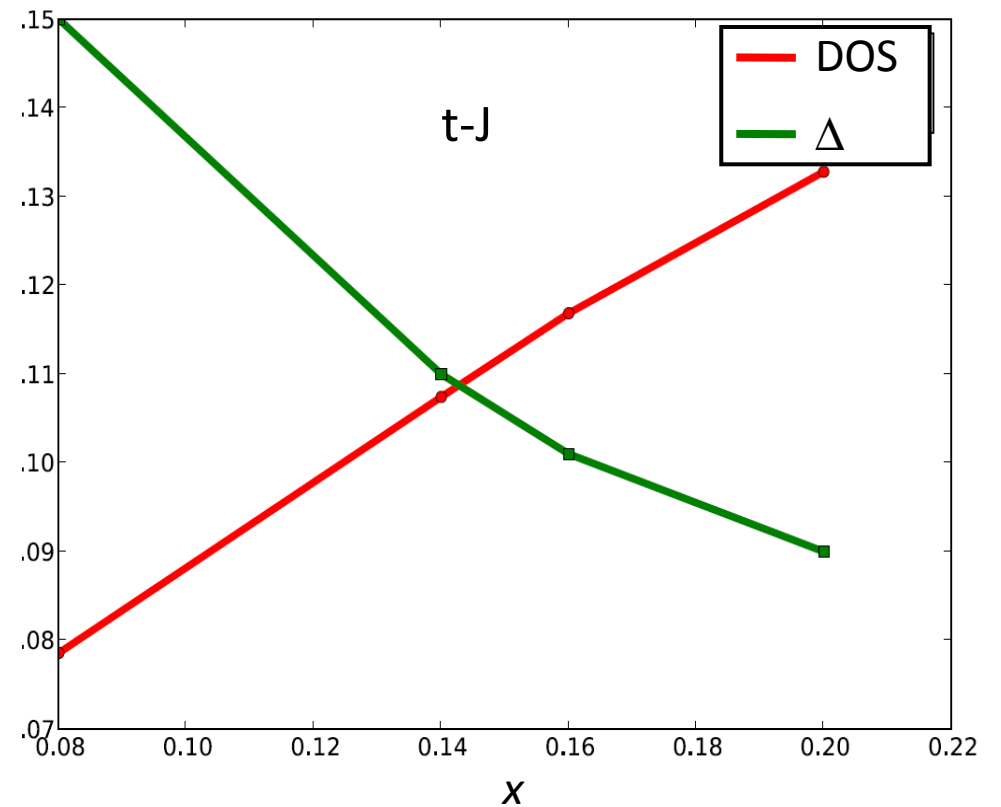
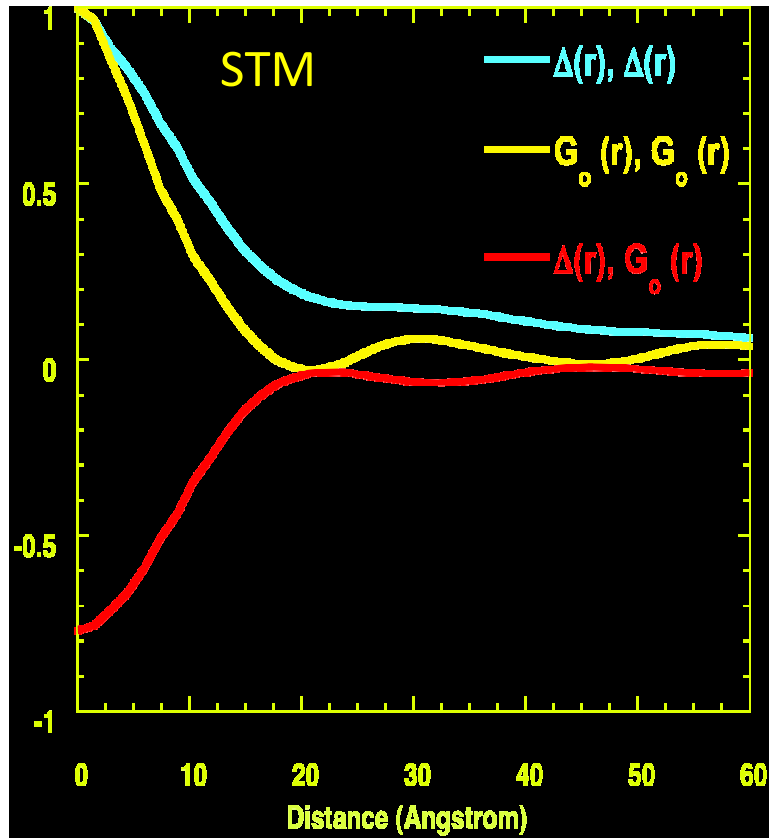
High-Tc cuprate superconductors are strongly-correlated superconductors.

Another example: the t-J model of strong correlation.

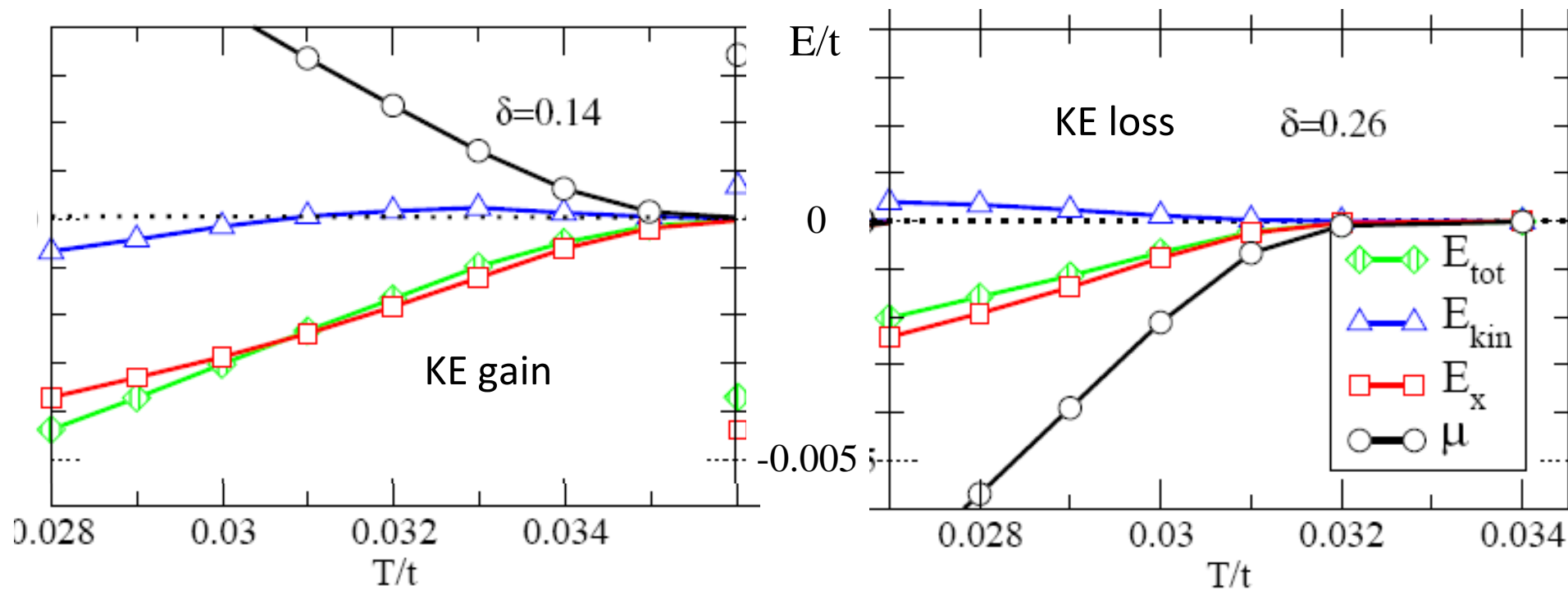
$$\mathcal{H} = -t P \sum_{\langle ij \rangle \sigma} (c_{i\sigma}^+ c_{j\sigma} + h.c.) P + J \sum_{\langle ij \rangle} S_i \cdot S_j$$

- Gives d-wave superconductivity, pseudogap, arcs, DOS-gap anticorrelation.
- There is no clear separation of kinetic and potential energies.
- Captures effects of proximity to Mott insulator at zero doping.

# Anticorrelation of normal state DOS and superconducting gap



Energy difference between the normal and superconducting state of the t-J model. (K. Haule 2006)



Condensation energy is exchange energy  
 Superconductivity without a coherent normal state

A new paradigm for superconductivity?