

# From Landau levels to quantum Hall effects

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# The Quantum Hall Effects

Large set of peculiar phenomena in **two-dimensional electron systems**, at low temperatures in **strong magnetic fields**.

**Usually:** electrons in **semiconductor structures**: e.g. electrons trapped in a thin layer of GaAs, surrounded by AlGaAs.

**Very recently:** QHE seen in **graphene**: single atomic layer of graphite.

Samples can differ widely in electron densities and freedom from defects. Magnetic fields range from 0.1 to 45 Tesla.

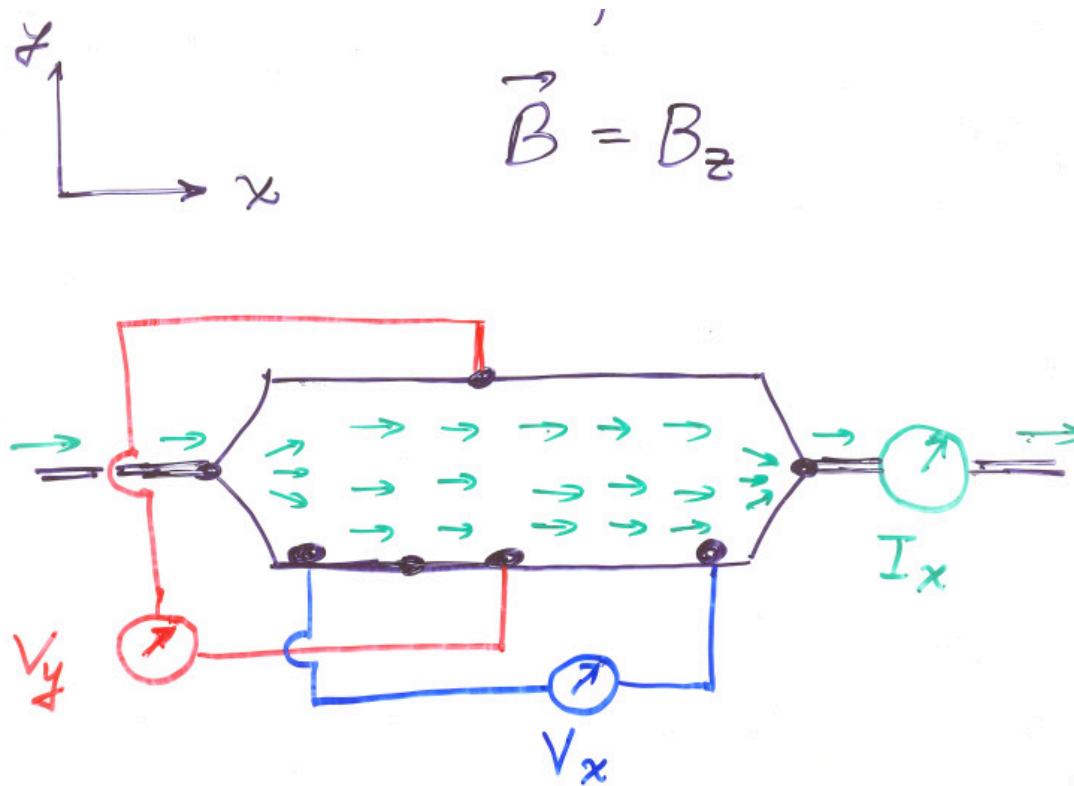
# Repeated surprises

Experiments have produced many surprises since the discovery of the Integer Quantized Hall Effect, in 1980.

Understanding has required concepts and mathematical techniques from all corners of theoretical physics, including some completely new ideas.

The **goal** of this talk is to review a **few aspects** of the quantized Hall effects, and show how our understanding of these phenomena stems from the work of **L. D. Landau**.

# Hall Geometry



Hall resistance:  $R_H = V_y / I_x$

Longitudinal Resistance:  $R_{xx} = V_x / I_x$

# Integer QHE

Discovered by Klitzing et al, 1980.

These figures from Paalanen et al, 1981

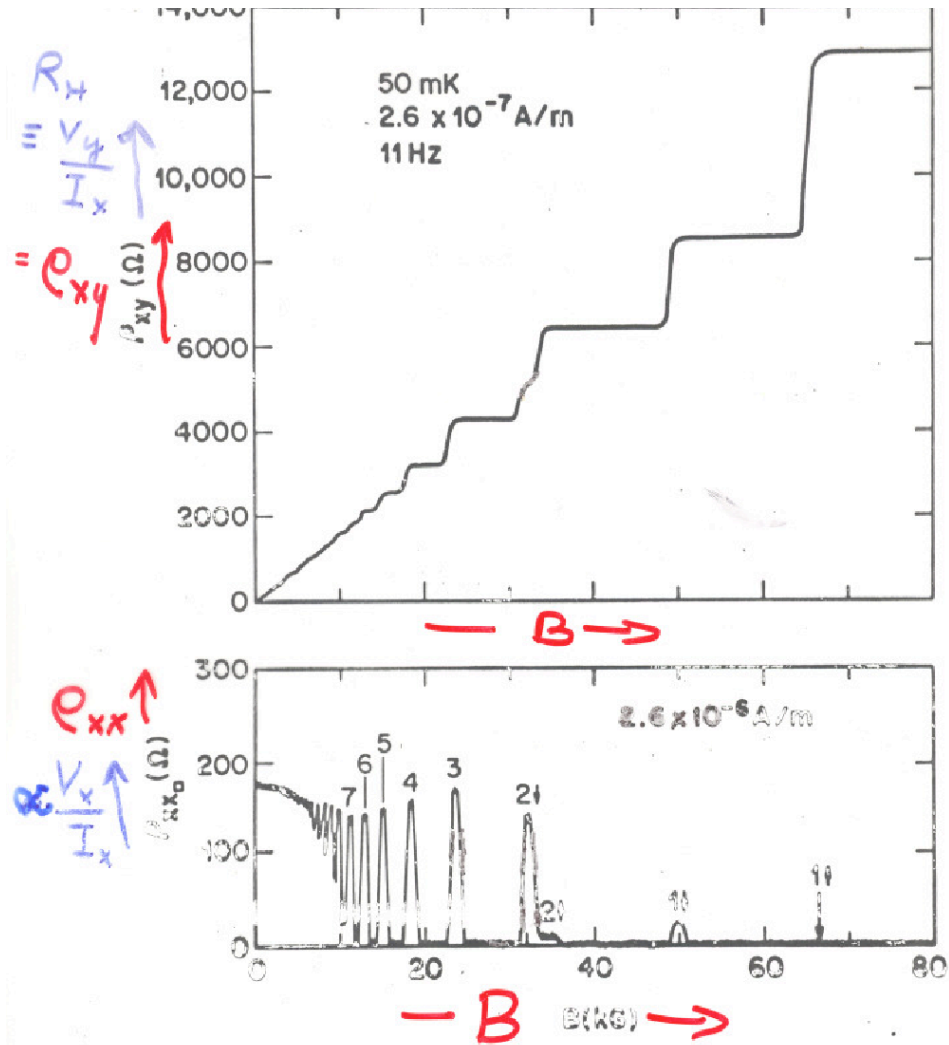


Figure 1

Paalanen, Tsui, & Gossard

## Integer QHE (continued)

On the plateaus:  $1/R_H = \nu e^2/h$

where  $\nu$  is an integer (different on different plateaus)

$h/e^2 = 25, 812.02$  ohms.

Independent of precise shape of sample, choice of material, etc.

# Fractional Quantized Hall Effect

Discovered by Tsui, Stormer and Gossard (1982)

In samples of very high quality, in very strong magnetic fields, one finds **additional plateaus**, where

$$R_H^{-1} = \nu e^2/h$$

But  $\nu$  is a **simple rational fraction**, usually with **odd denominator**.

Originally  $\nu = 1/3, 2/3$ . More recently include

$\nu = 4/3, 5/3, 1/5, 2/5, 3/5, 3/7, 4/7, 4/9, 5/9$  , others

## What about even denominator fractions?

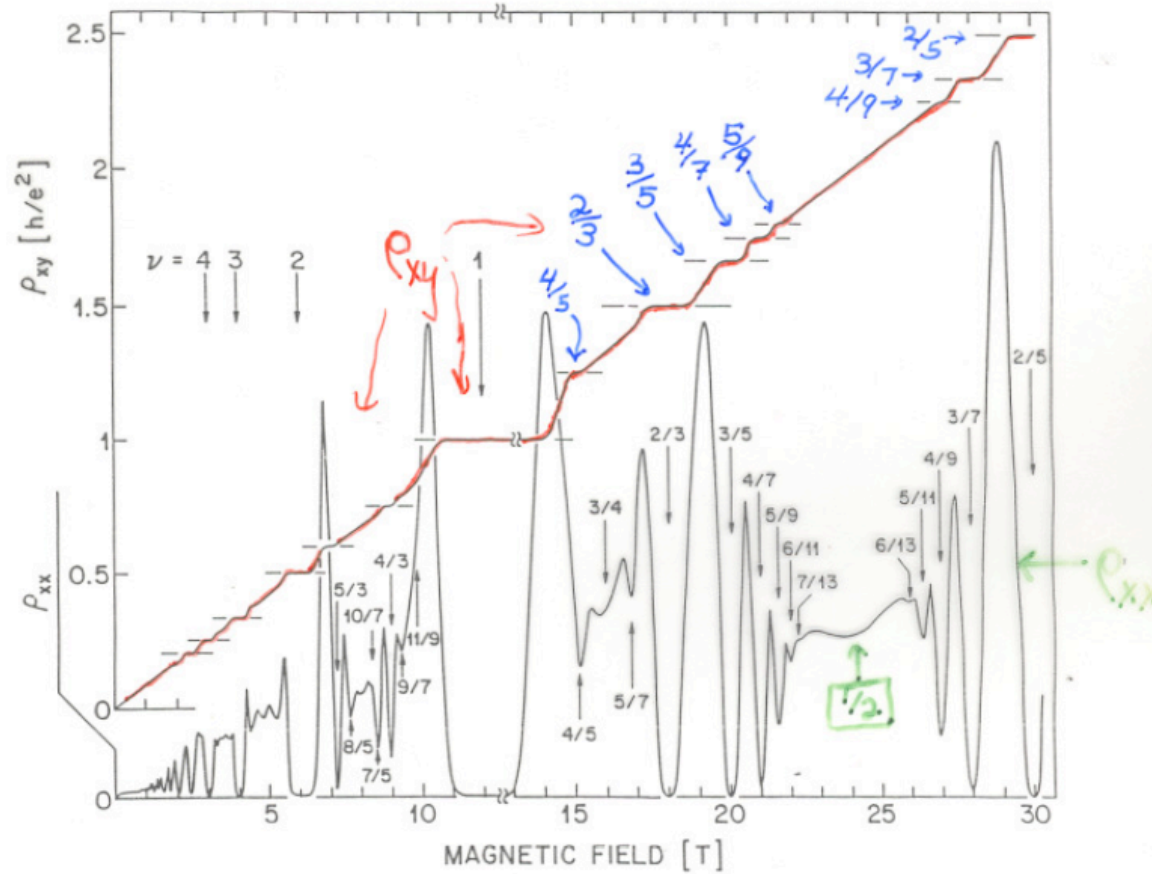
For most even-denominator fractions, quantized Hall plateaus are **not observed**. E.g., there is **no quantized Hall plateau** in a single layer sample at  $\nu = 1/2$ .  $R_H$  and  $R_{xx}$  vary smoothly with magnetic field. Yet, there are **strong anomalies seen in other properties**, such as the Surface Acoustic Wave velocity. So there is still something very strange occurring: **“Unquantized Quantum Hall Effect”**.

**Quantized Hall plateaus** have been found corresponding to a **few even denominator fractions**; e.g.,  $\nu = 5/2$ . The nature of this plateau is still a matter of debate, and active current research.



GaAs/GaAlAs Heterostructure  
 $\mu = 1.3 \times 10^6 \text{ cm}^2/\text{Vsec}$

$n = 3.0 \times 10^{11} \text{ cm}^{-2}$



R Willett, J.P. Eisenstein, H.L. Störmer, D.C. Tsui, A.C. Gossard,  
 & J.H. English, Phys Rev. Lett. 59 1776 (1987)

## L. D. Landau, 1930\*

“Diamagnetism of Metals” (Z. Phys. 64, 629)

Solved the quantum mechanical problem of energy states of a charged particle in a uniform magnetic field.

Showed that in a **three-dimensional metal**, leads to **orbital diamagnetism**: on average the kinetic energy of the electrons is increased, by the magnetic field. Magnetic moment points in opposite direction to field.

At low temperatures, in very clean samples, there are **additional effects, oscillatory in the magnetic field**.

\* (Age 22)

# Landau Levels in 2-Dimensional Systems

Consider non-interacting electrons in uniform magnetic field  $B$  in 2D.

In quantum mechanics, energy levels are quantized into “Landau levels”, with

$$E_n = h v_C (n+1/2), \quad n = 0, 1, 2, 3, \dots$$

The number of independent orbits, in each Landau level is equal to the number of flux quanta:  $N_B \equiv B e \text{Area} / h$

Define Landau level filling factor  $f = N_e / N_B = (n_e/B)(h/e)$ .

If  $f$  is an integer, Fermi level is in an energy gap, between two Landau levels.

# Conditions for Quantized Hall Effect (Integer or Fractional)

In general, the conditions for a 2D electron system to have a quantized Hall plateau, with  $R_H^{-1} = \nu_s e^2/h$  are that

1. The **Fermi level** of an ideal system, without impurities, should lie in **an energy gap** when  $f = \nu_s$ , and
2. The amount of disorder in the actual sample should be sufficiently small that the energy gap has not been wiped out.

For non-interacting electrons, this **explains why there should be a quantized Hall effect for integer values of  $\nu$** , in high-mobility samples **But why are there fractional quantized Hall plateaus?**

# Fractional quantized Hall states

Fermi energy is the *middle* of a partially filled Landau level. For non-interacting electrons there is no energy difference between the highest filled states and the lowest empty states. *Energy gap* must come from *electron-electron interactions*.

Detailed explanation: Laughlin (1983) and subsequent work by many people.

FQHE states are *strongly-correlated many-body* states with very peculiar properties. E.g., elementary charged excitations are *quasiparticles* with *fractional electric charge*.

Also obey *fractional statistics* (properties intermediate between bosons and fermions). In some cases perhaps *non-abelian* statistics, even weirder and more interesting.

# Explanations for the Fractional Quantized Hall Effect

Many approaches.

Earliest methods: Trial wave functions:

Laughlin (1983); ..... Jain (1989)

Here discuss:

**Fermion-Chern-Simons approach**

Used by Lopez and Fradkin and others for Fractional Quantized Hall states (~1990); Halperin, Lee and Read (1992) for Unquantized Quantum Hall effect at  $f=1/2$ . (Based on Jain's ideas of "Composite Fermions".)

# Fermion-Chern-Simons Approach

Begins with a **unitary transformation** (singular gauge transformation which leads to a transformed Hamiltonian (units where  $e=\hbar=1$ ):

$$H = \sum_j |\mathbf{p}_j + \mathbf{A}_j - \mathbf{a}_j|^2 / 2m + V_{\text{Coulomb}}$$

where  $\mathbf{A}_j \equiv \mathbf{A}(\mathbf{r}_j)$  is the vector potential due to the external magnetic field, felt by particle  $j$ , and  $\mathbf{a}_j$  is a **fictitious “Chern-Simons” vector potential**, introduced by the transformation, which **depends on the positions of all the other electrons**, in such a way that

$$\mathbf{b}(\mathbf{r}) \equiv \nabla \times \mathbf{a}(\mathbf{r}) = 4\pi \sum_j \delta(\mathbf{r}-\mathbf{r}_j) .$$

This attaches two quanta of fictitious flux to each electron. Transformation preserves Fermi statistics.

# Mean-Field Approximation

(Hartree Approximation)

Replace true Chern-Simons magnetic field  $\mathbf{b}$  by its average value  $\langle \mathbf{b} \rangle = 4 \pi n_e$ .

Replace **Coulomb interaction** by the **average electrostatic potential**, which is just a **constant** for a homogeneous system. (We can choose constant = 0).

Get free fermions in an effective magnetic field

$$\Delta B = B - 4 \pi n_e .$$

Define effective filling  $\rho \equiv 2\pi n_e / \Delta B$ . Compare to true filling factor  $f = 2 \pi n_e / B$ .

Find  $\rho^{-1} = f^{-1} - 2$ , or:  $f = \rho / (2\rho + 1)$ .



If "effective filling factor"  $\rho$   
is an integer :

Mean field theory  $\Rightarrow$  stable state !  
 $\Rightarrow$   $|p|$  filled Landau levels

Energy gap = cyclotron frequency in  
effective field  $\Delta B$  :

$$E_g = \frac{|\Delta B|}{m} = \frac{4\pi n e}{m p}$$

$\Rightarrow$  Original electron system has a stable  
state at filling factor  $f = \frac{p}{2p+1}$

# Landau Levels within Landau Levels

So Fractional Quantized Hall States are understood as filled Landau Levels of particles in a fictitious magnetic field, within a partially-full Landau Level of the true magnetic field.

The fictitious magnetic field is an emergent phenomenon, an effective interaction that emerges from the collective behavior of the electrons.

Analogous to the way electrons in a solid can interact with each other by means of phonons.

If the electron filling fraction is  $\nu=1/2$

2 flux quanta of actual magnetic field per electron.

Effective magnetic field =  $B - \langle b \rangle = 0$ .

Mean-field ground state = filled Fermi Sea  $k_F = (4 \pi n_e)^{1/2}$

If this is correct, then there is **no energy gap**, **no QHE**.

Should be able to calculate all properties of  $\nu=1/2$  state using **perturbation theory**, starting from the mean field state.

Perturbations include effects of  $v(\mathbf{r}_i - \mathbf{r}_j)$  and **fluctuations in the Chern Simons field**  $\Delta b_i \equiv b_i - \langle b \rangle$ .

Fluctuations are crucial in calculating transport and dynamic properties, as well as for understanding the energy scale for excitations

# Why should perturbation theory work?

For the **quantized Hall states**, where the mean field theory predicts an **energy gap** between the ground state and excited states, it is reasonable that perturbation theory should converge: **if perturbation is not too strong, will not destroy the energy gap or change the character of the ground state.** We find that fluctuations reduce the energy gap significantly, but do not generally drive it to zero.

**But what about  $\nu=1/2$ , where there is no energy gap?**

# Landau's theory of Fermi liquids

ZhETF 30, 1058 (1956); 32, 59 (1957); 35, 97 (1958).

Interacting Fermi liquids (in 3D, no magnetic field) can behave in many ways like weakly-interacting fermions. Elementary excitations are *quasiparticles*, with well-defined momenta, related adiabatically to the original fermions, but with **renormalized effective mass**. Lifetimes are very long for low energy quasiparticles (momenta close to the Fermi surface). **There is a sharp Fermi surface, whose volume is not renormalized.** **Residual interactions** among the quasiparticles are **important for collective response** to an external perturbation (such as a magnetic field coupling to the spins) **and for dynamics: can produce collective modes such as zero sound.**

## Landau's theory of Fermi liquids (continued)

Other features:

There is a finite overlap between wave function for a quasiparticle and wave function for a bare particle added to the ground state.

At least in the case of short-range interactions, the charge of the quasiparticle is the same as that of the bare particle.

Reason perturbation theory works is that even though there is no energy gap in the unperturbed system, the density of states for particle-hole excitations is very small at low energies. So if interactions are not too strong, & not too singular at small momentum transfer, scattering of low-energy particles can be very small.

# Fermi liquid at $f=1/2$

Chern-Simons gauge interactions between transformed fermions at  $f=1/2$  are singular at small momentum transfers. Find that a sharp Fermi surface can and does exist, but Fermi liquid is “marginal”, and has many peculiar properties.

Effective mass  $m^*$  is predicted to diverge (logarithmically) at low energies.

Electric charge of quasiparticles is renormalized to zero, but quasiparticles have an electric dipole moment

$$\mathbf{d} = h \boldsymbol{\varepsilon}_z \times \mathbf{p} / Be$$

It costs no energy to add a constant  $\mathbf{K}$  to the momenta of all particles. In Landau theory, this energy is

$\delta E = N_e K^2 (1+F_1) / 2m^*$  , where  $F_1$  is the Landau interaction coefficient for  $l=1$ . So we have  $1+F_1=0$  (marginal case).

## Results for $\nu=1/2$ (with no impurities)

Electron system is compressible.

Fluctuations in the electron density relax very slowly at long wavelengths. Relaxation rate obeys

$\gamma \propto q^2$ , for unscreened Coulomb interactions

$\gamma \propto q^3$ , for short range interactions.

Longitudinal electrical conductivity at finite wave vector obeys (in absence of impurities)

$$\sigma_{xx}(q) \sim (e^2/4h) (q / k_F) \quad (\text{Nonlocal})$$



# Longitudinal conductivity $\sigma_{xx}(q)$ is measured in a Surface Acoustic Wave experiment

$q$ = wavevector of sound wave.

Anomaly in SAW propagation at  $f=1/2$ , was observed by Willett and coworkers in 1990 and subsequent experiments.

**Find:** conductivity increases linearly with  $q$  when  $1/q$  is smaller than mean free path for scattering of composite fermions due to impurities -- as predicted by Fermion+Chern-Simons theory.

Quasiparticles can travel in a straight line for distances of order of a micron, 100 times larger than the cyclotron radius for electrons in the applied magnetic field

# What about quantized Hall plateau seen at $\nu=5/2$ ?

Possible explanation proposed by Moore and Read, (1991), based on “Pfaffian” trial wave function, implies very peculiar properties, including quasiparticles with “non-abelian statistics.” Ground state in presence of  $2N$  quasiparticles has degeneracy  $2^{N-1}$ . States are multiplied by non-commuting unitary transformations when positions of quasiparticles are interchanged. Supported by numerical calculations, but little experimental evidence that this description is correct.

How can we understand Moore-Read state in terms of Fermion-Chern-Simons Approach?

At  $f=1/2$ , depending on the short-distance interactions between fermions the Fermi surface may be unstable, e.g., to **formation of p-wave superconductivity**

If a superconducting energy gap forms at the Fermi surface, then state is stabilized at precisely  $f = 1/2$ . Deviations in filling fraction  $\Rightarrow B_{\text{eff}} \neq 0 \Rightarrow$  requires vortices, costs finite energy.

Then would get plateau in Hall conductance at  $f = 1/2$  : **fractional quantized Hall state, with non-abelian statistics.**

**Apparently:** Superconductivity **does not occur** for electrons in the **lowest Landau level** ( $f = 1/2$  ) but **does occur** for electrons in the **second Landau level** (Total  $f = 5/2$ ).

# Moore-Read quasiparticle $\Leftrightarrow$ vortex in superconductor

By Meissner effect, vortex must bind  $1/2$  quantum of magnetic flux to have finite energy. With a Chern-Simons gauge field, the source of magnetic flux is charge, rather than current.

$1/2$  quantum of Chern-Simons flux requires  $1/4$  electric charge.

Quasiparticles acquire **non-abelian statistics** due to zero-energy states of vortices.

# Credits

The ideas presented in this talk were developed over several decades by a large number of people. For the topic of the Fermi liquid at  $f=1/2$ , special credit belongs to

Nick Read, Patrick Lee, Ady Stern, Steve Simon, Felix von Oppen, Leonid Levitov, Andrey Shytov, (all collaborators with BIH on one or more occasion)

Ana Lopez, Eduardo Fradkin, Shoucheng Zhang, V. Kalmeyer, Duncan Haldane, V. Pasquier, Ed Rezayi, Dung-Hai Lee, Jainendra Jain, R. Shankar, Ganpathy Murthy

