New development in the physics of Van der Waals forces.
Moscow, June 20, 2008

L.P. Pitaevskii
Kapitza Institute for Physical Problems,
Moscow, Russia;
CNR INFM-BEC and Dipartment of Physics,
University of Trento,
Trento, Italy.

Van der Waals forces-theory.

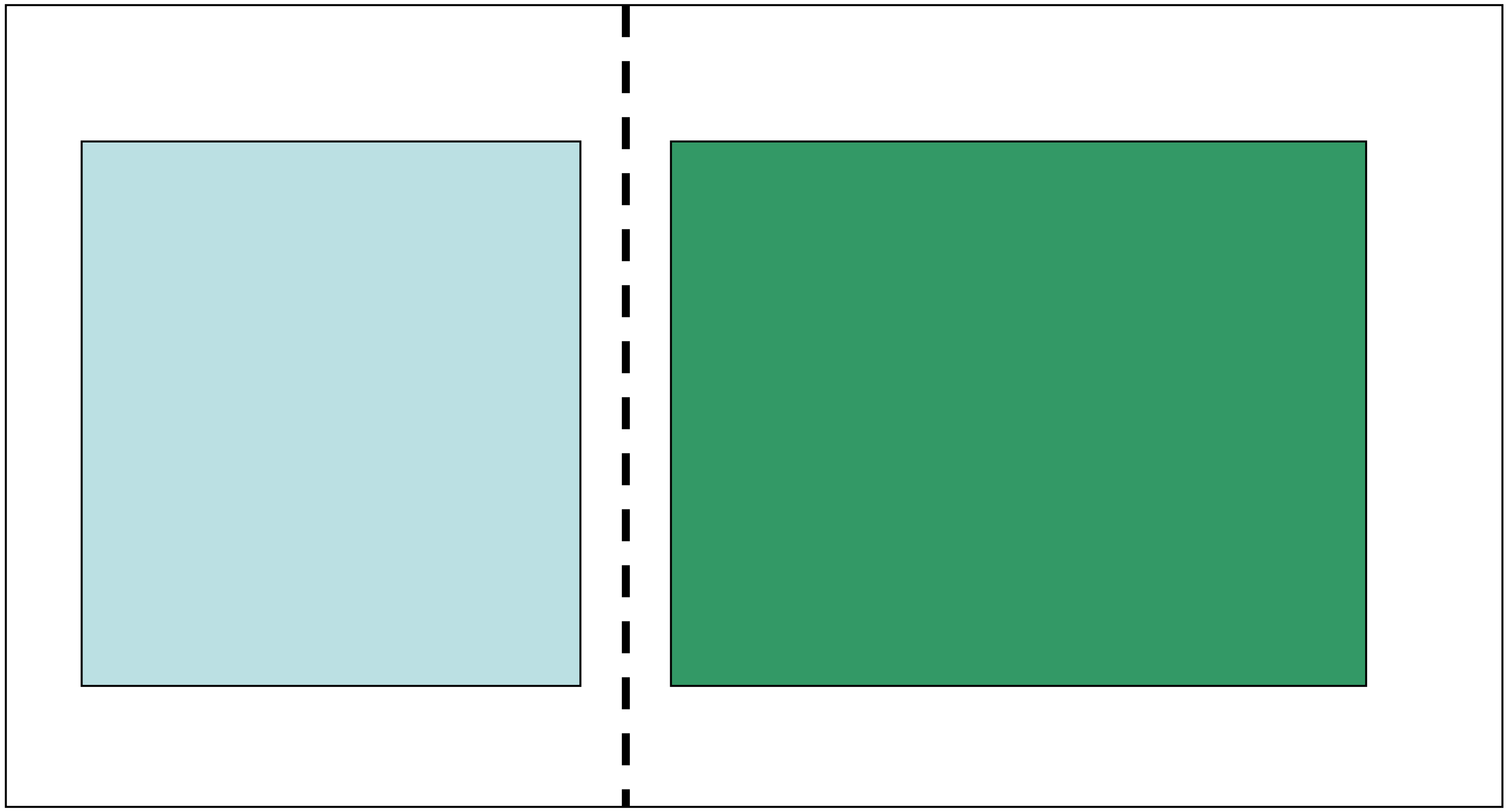
- Van der Waals, about 1890
- F. London (1928). QM derivation of $1/r^6$ law.
- H.D.B. Casimir, D. Polder (1948). Effects of returding, Macroscipoc approach to interaction of ideal metals.
- E.M. Lifshitz (1954). General theory of interation of dielectric bodies. Thermal effects.
- I.E. Dzyaloshinskii, E.M. Lifshitz, L.P. Pitaevskii (1959). Interaction through dielectric medium.Green's function representation.
- C.Henkel et al. (2002); M. Antezza, L.P. Pitaevskii, S.Stringari (2004). Non-equilibrium effects.

Correlations of random polarizations

$$\left\langle P_i(\mathbf{r}) P_k(\mathbf{r}_1)\right\rangle_\omega = \varepsilon''(\omega,\mathbf{r})\hbar \coth\left(\frac{\hbar\omega}{2T}\right)\delta(\mathbf{r}-\mathbf{r_1})\delta_{ik}$$

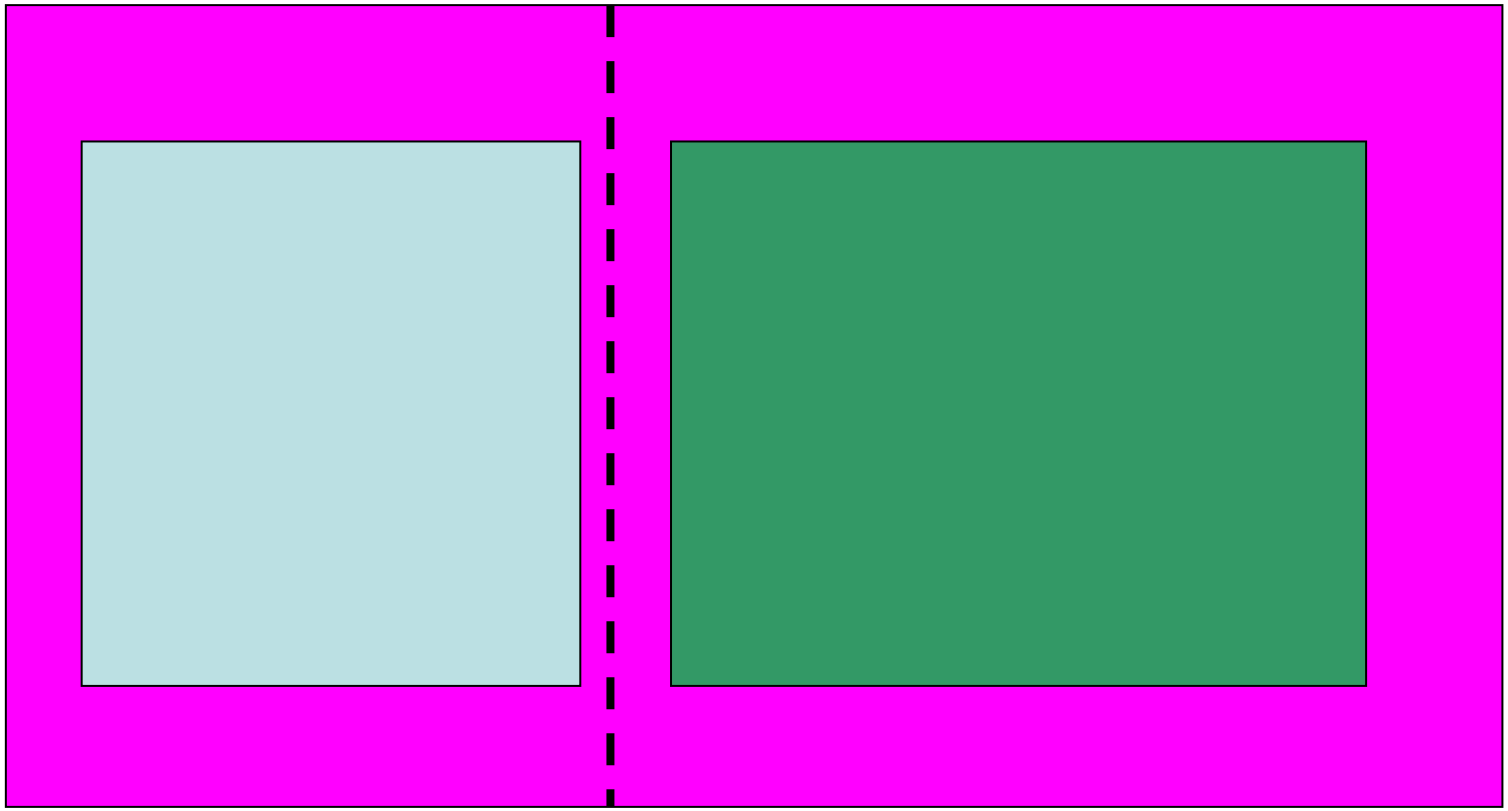
S.M.Rytov (1954); L.D.Landau, E.M.Lifshitz (1958).

Interaction through vacuum



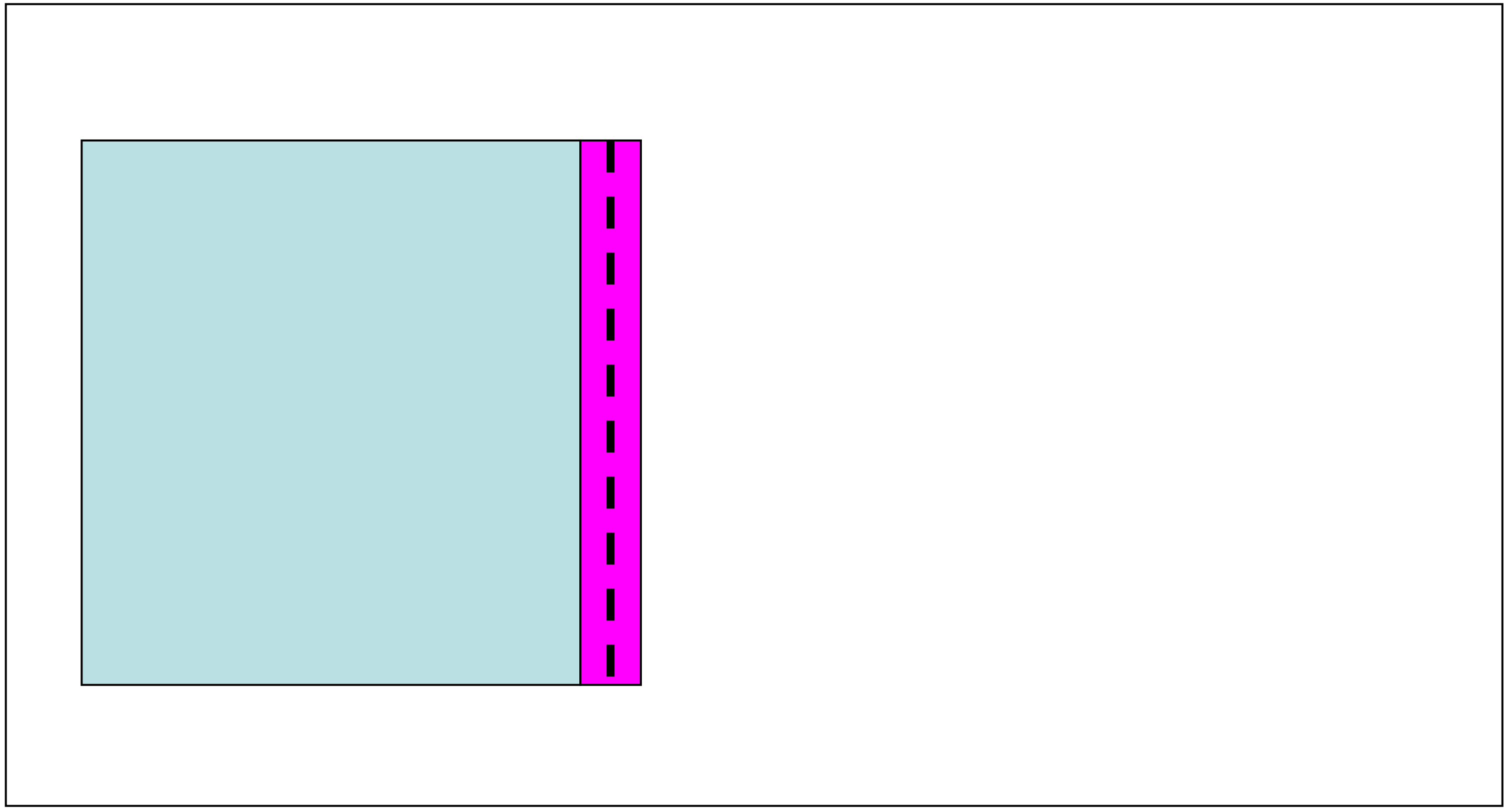
$$\sigma_{xx}$$

Interaction through medium



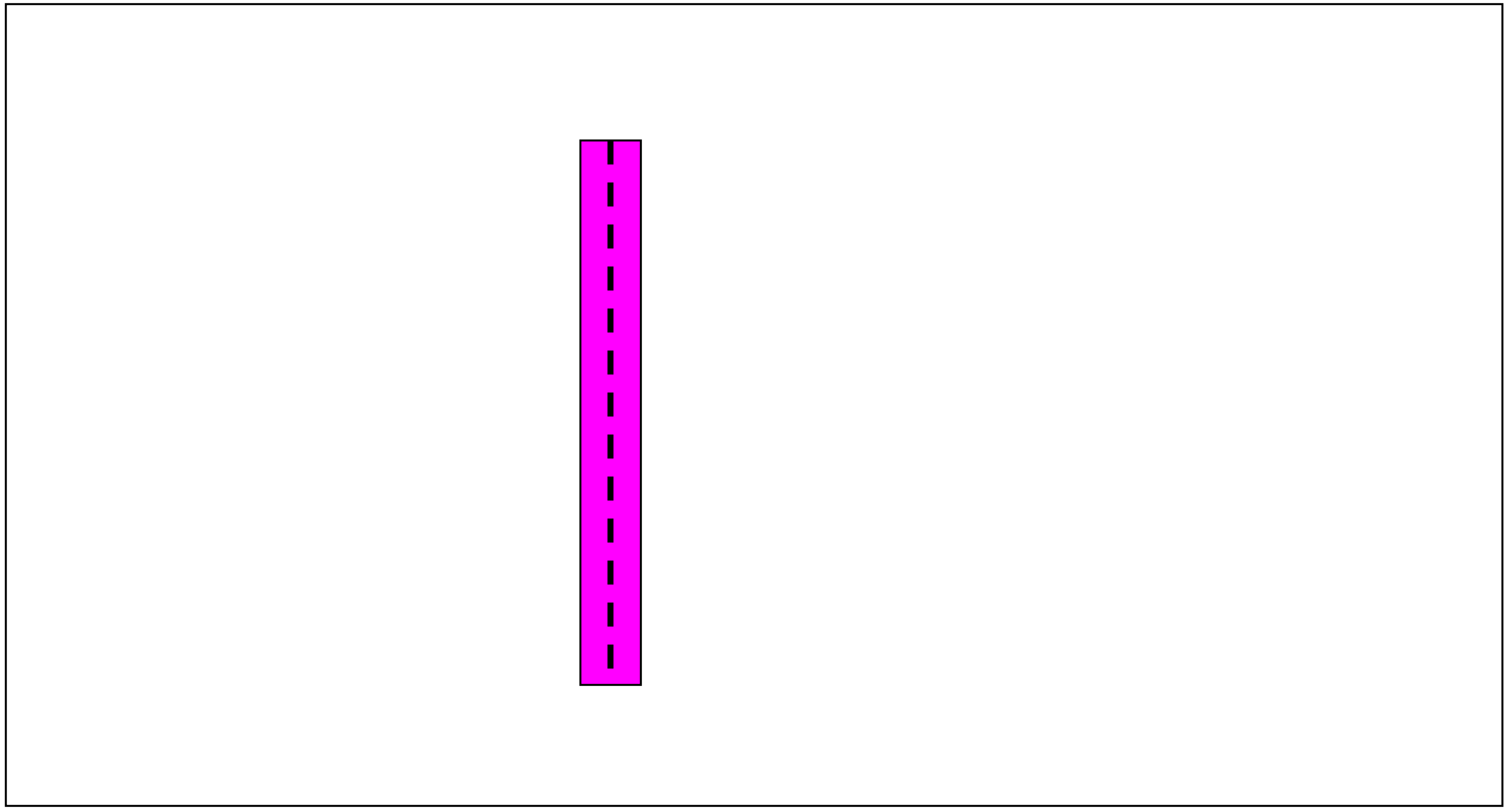
$$\sigma_{xx} = ?$$

Problem of a film



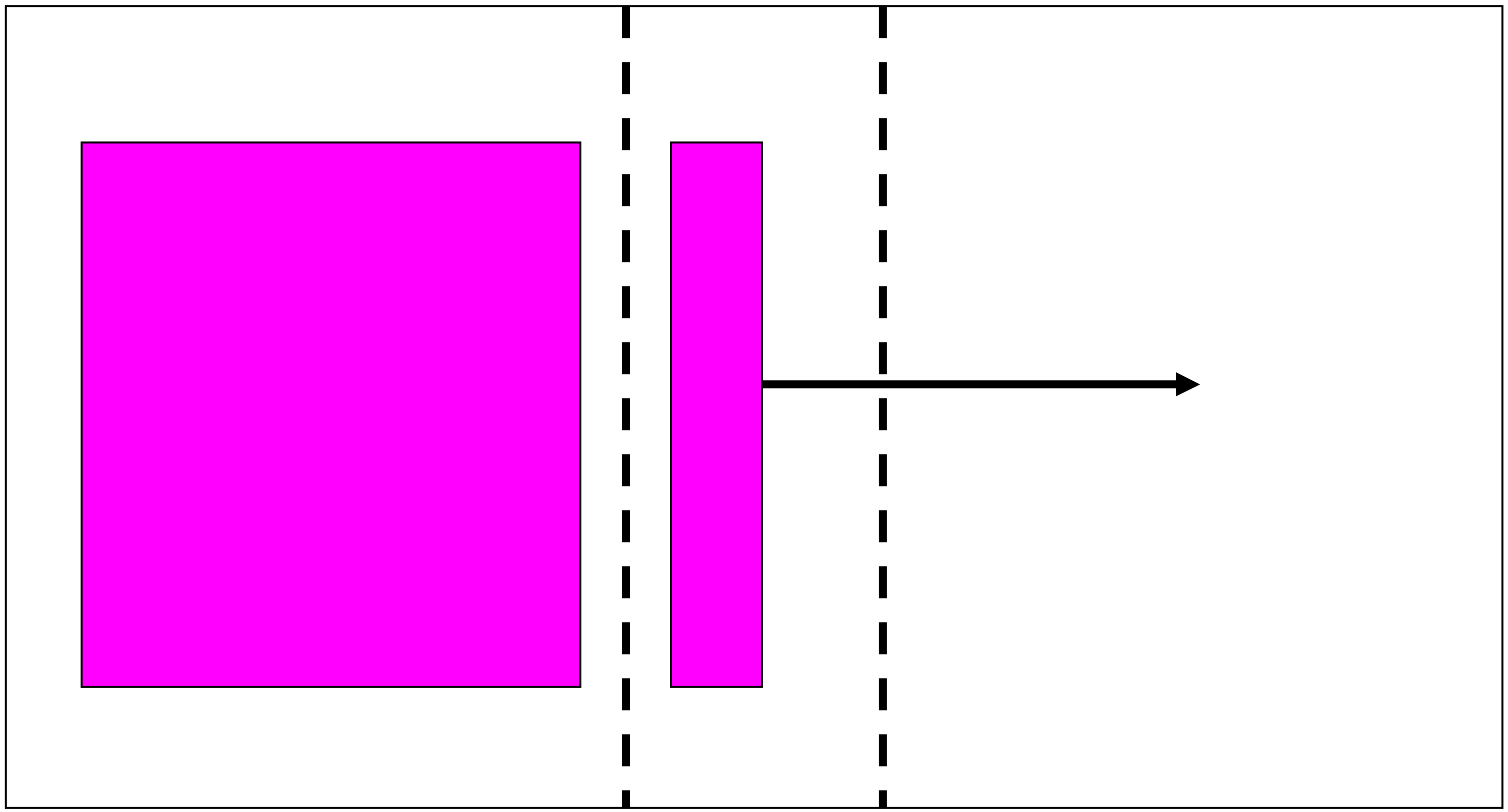
$$\mu(d) = \sigma_{xx}(d)$$

Problem of a film in vacuum



$$\mu(d) = \mu_\infty + \delta\mu(d)$$

Solution of the problem of a film



Free energy variation

Free energy variation :

$$\delta F = \frac{T}{4\pi\hbar}\sum_{s=0}^{\infty}{}' \int \mathrm{D}_{ll}^{E}\left(\zeta_s, \mathbf{r}, \mathbf{r}\right)\delta\varepsilon\left(i\left|\zeta_s\right|, \mathbf{r}\right)d^3x$$

(I. Dzyaloshinskii, L. Pitaevskii, 1959)

Energy of atom-surface interaction

Free energy variation :

$$\delta F = \frac{T}{4\pi\hbar}\sum_{s=0}^{\infty}{}' \int \mathrm{D}_{ll}^{E}(\zeta_s, \mathbf{r}, \mathbf{r})\,\delta\varepsilon(i|\zeta_s|, \mathbf{r})\,d^3x$$

$$\delta\varepsilon(i|\zeta_s|, \mathbf{r}) = 4\pi\alpha(i|\zeta_s|)\,\delta(\mathbf{r} - \mathbf{r}')$$

Atom - sutface potential :

$$V(l) = \frac{T}{\hbar}\sum_{s=0}^{\infty}{}' \alpha(i|\zeta_s|)\left[\mathrm{D}_{ll}^{E}(\zeta_s, \mathbf{r}, \mathbf{r})\right]_{\mathbf{r}\to\mathbf{r}\to\mathbf{r}_a}$$

Asymptotic in equilibrium Lifshitz theory

1. London regime: $z \ll \lambda_0 : \quad F = -A / z^4$

$$A = \frac{\hbar}{4\pi} \int_0^\infty \alpha(i\xi) \frac{\varepsilon(i\xi) - 1}{\varepsilon(i\xi) + 1} d\xi$$

2. Casimir-Polder regime:

$$\lambda_0 \ll z \ll \lambda_T = \hbar c / k_B T : \quad F = -B / z^5$$

$$B = \frac{3\hbar c}{2\pi} \alpha_0 \frac{\varepsilon_0 - 1}{\varepsilon_0 + 1} \varphi(\varepsilon_0); \quad \varepsilon_0 \equiv \varepsilon(\omega = 0)$$

3. Lifshitz regime:

$$\lambda_T \ll z : \quad F = -C / z^4 \quad C = \frac{3 k_B T}{4} \alpha_0 \frac{\varepsilon_0 - 1}{\varepsilon_0 + 1}$$

Relevant length scales

-Optical length λ_{opt} fixed by optical properties of the substrate (typically fractions of microns)

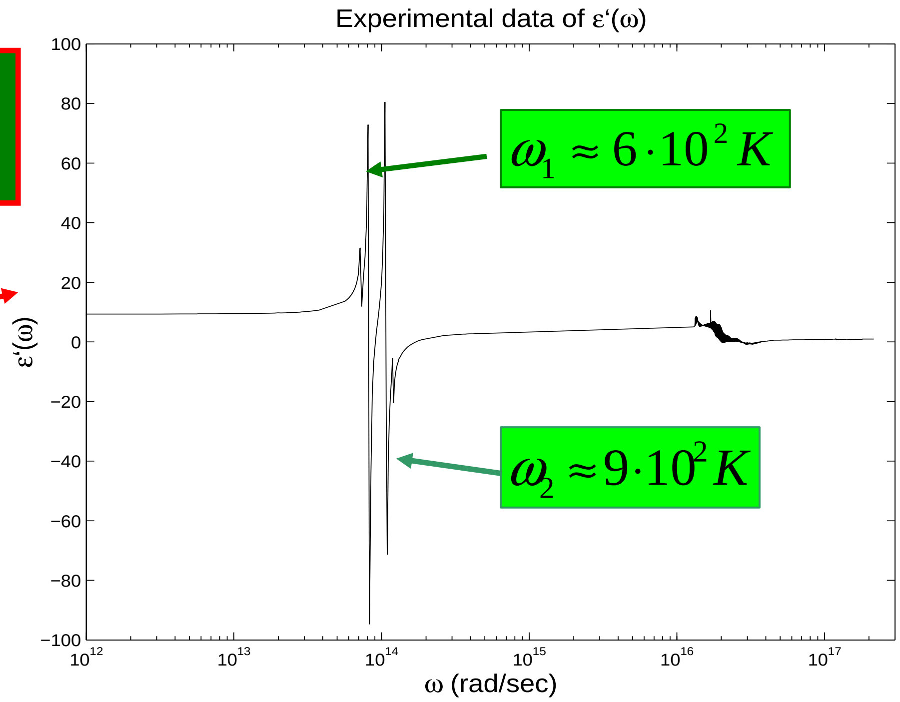
- Thermal photon wavelength ($\lambda_T = \hbar c / k_B T \approx 7.6\,\mu m$ at room temperature)

Casimir-Polder (Vacuum+retardation)

van der Waals-London (Vacuum) **Lifshitz (Thermal)**

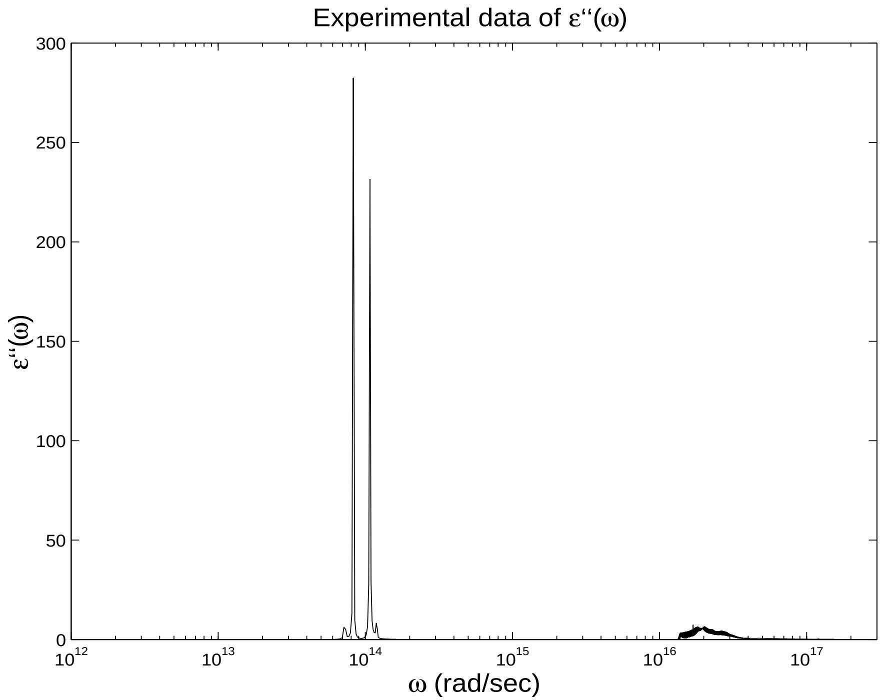
$\frac{1}{z^4}$ $\frac{1}{z^5}$ $\frac{1}{z^4}$

Dynamic dielectric and polarizability functions



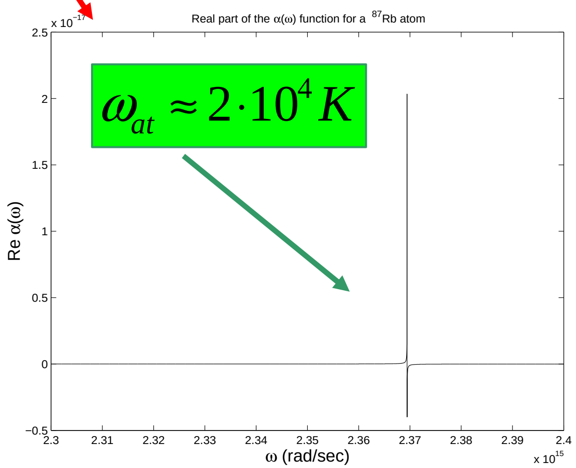
Sapphire substrate

$$\varepsilon(\omega) = \varepsilon'(\omega) + i\varepsilon''(\omega)$$

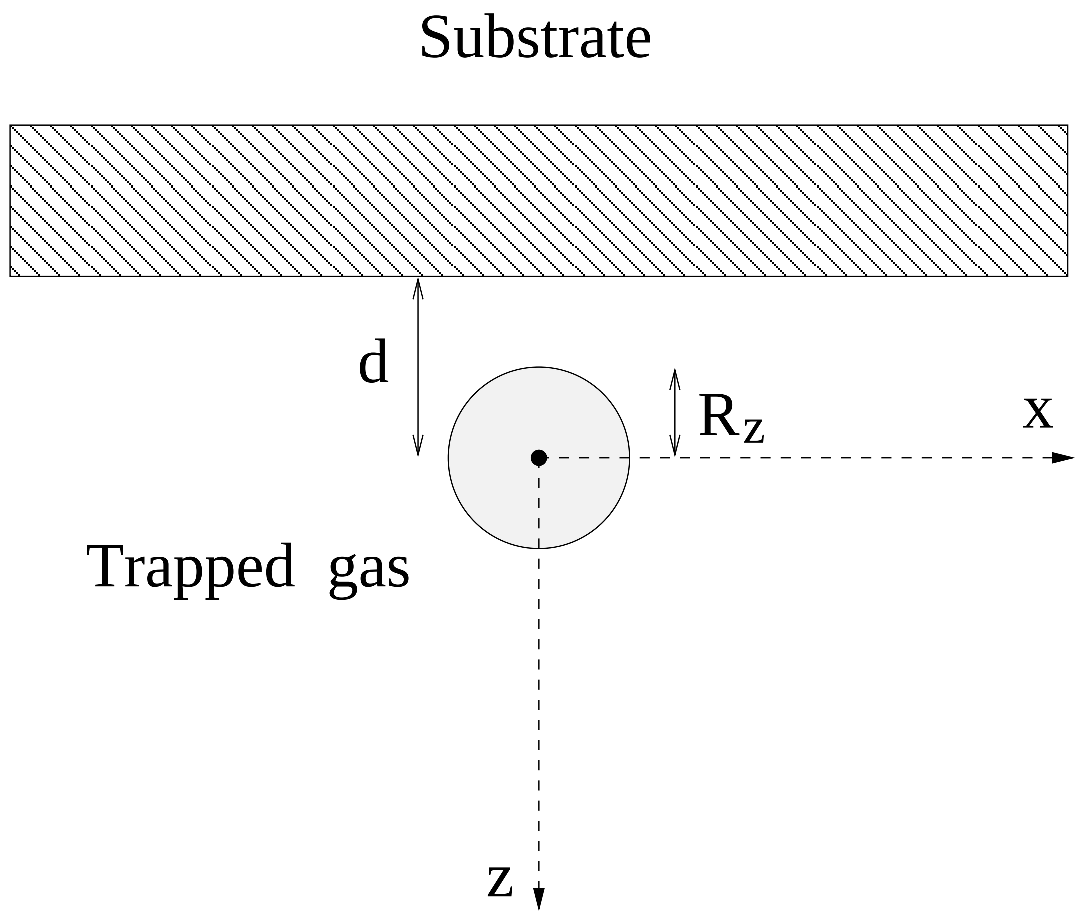


Rubidium atoms

$$\alpha(\omega) = \alpha'(\omega) + i\alpha''(\omega)$$

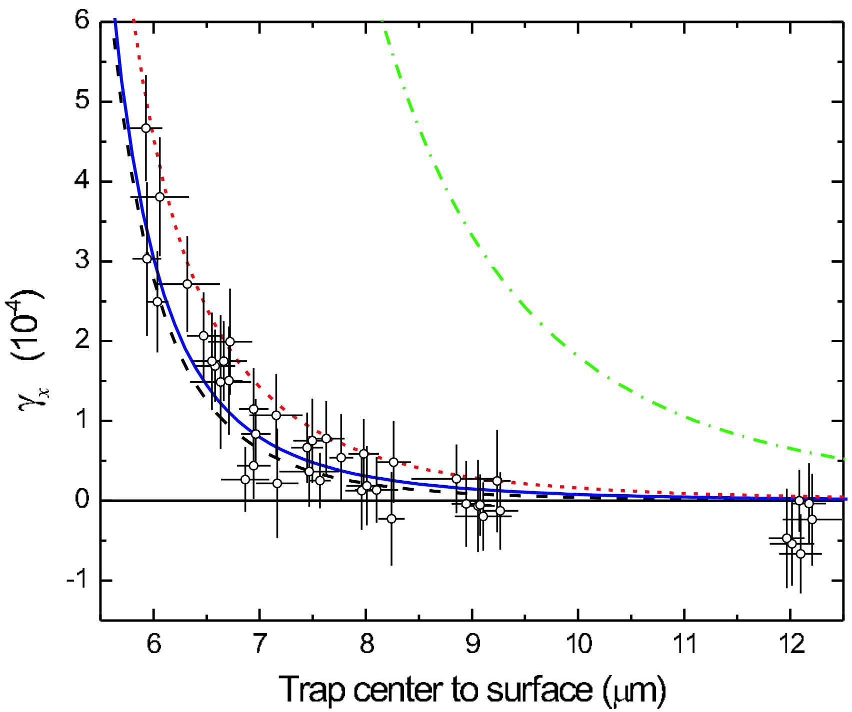


JILA experimental setup



JILA RESULTS

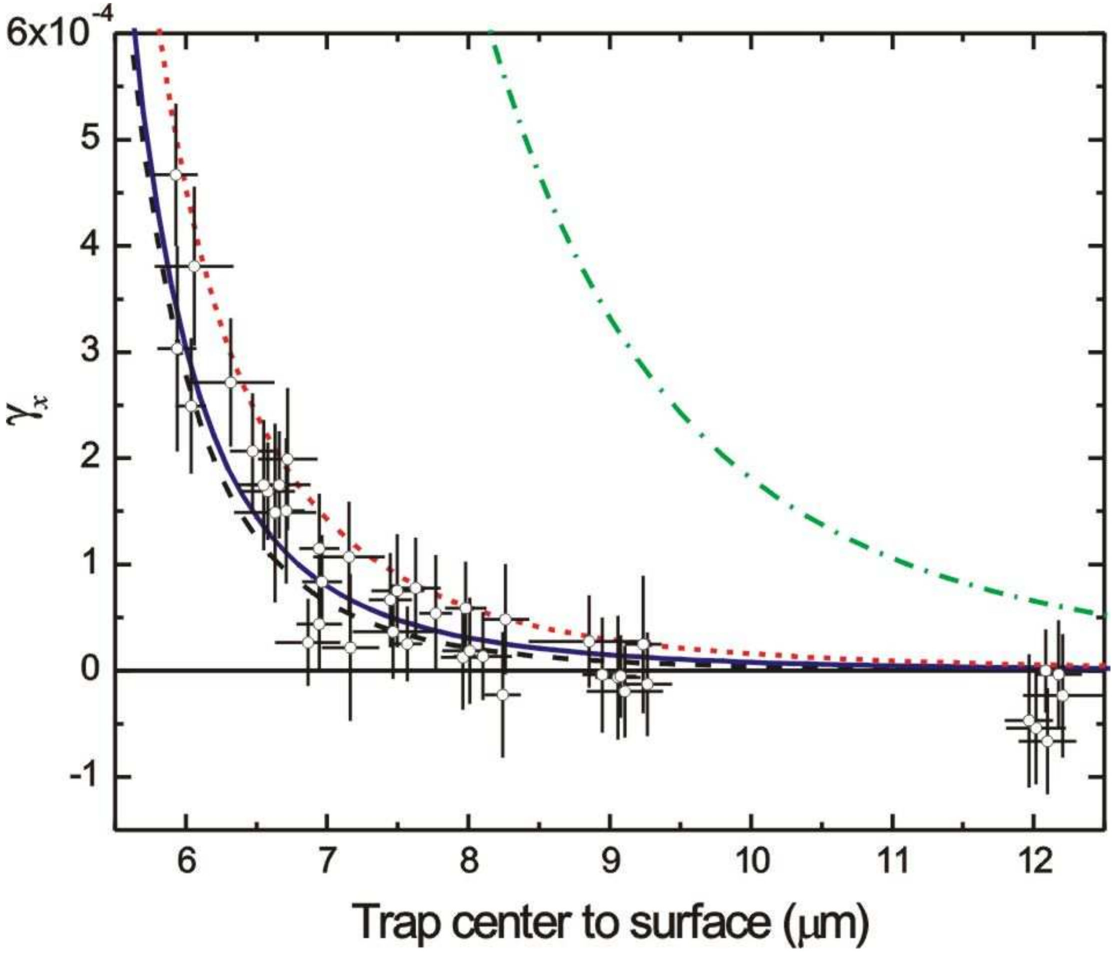
$$\gamma \equiv -\frac{\Delta\omega}{\omega} = -\frac{1}{2\omega m}\langle \partial_z F \rangle$$



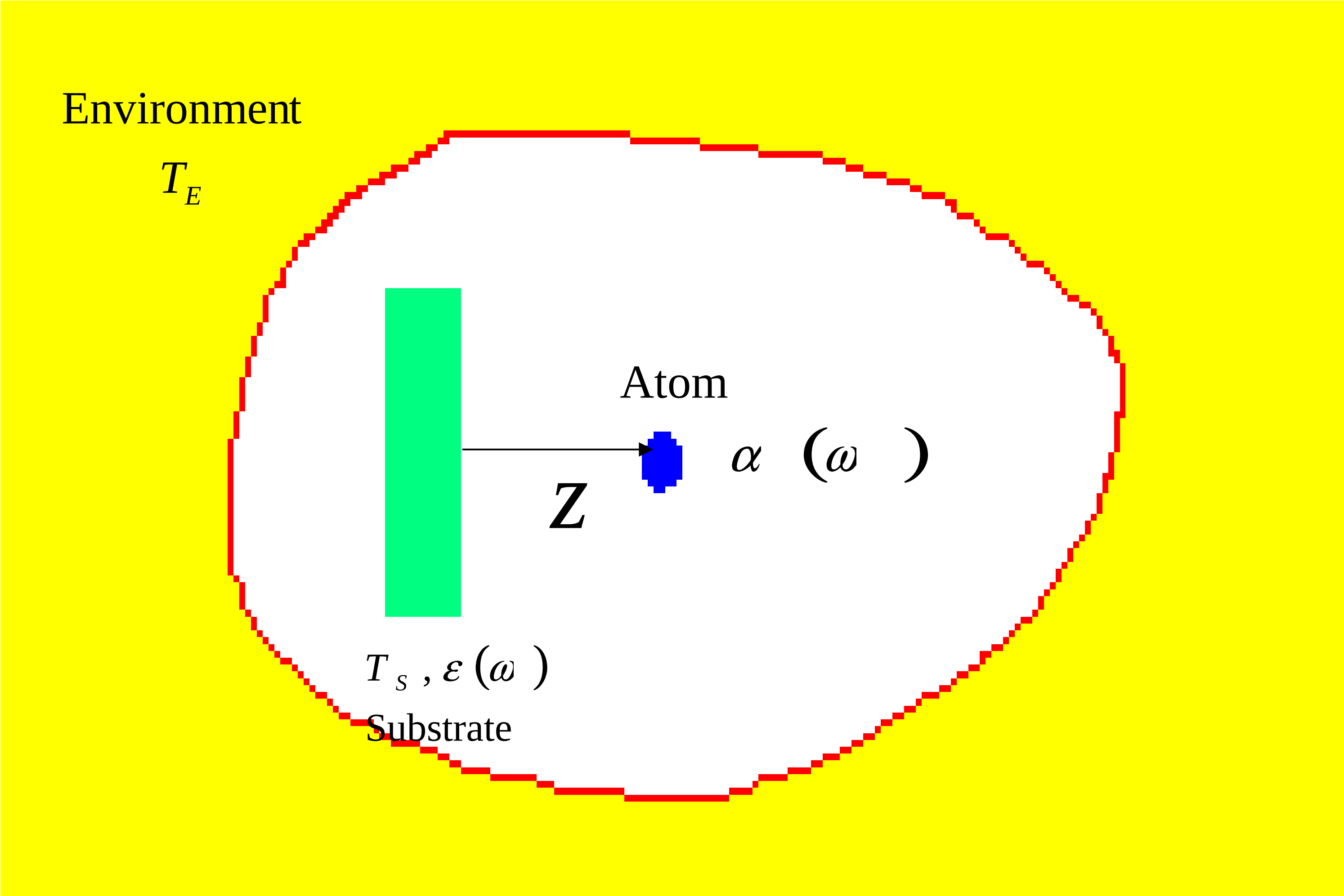
Theory: T=0-dash black; T=300K-solid blue; T=600K dot red; extrapolation of $1/d^3$ law-dash dot green

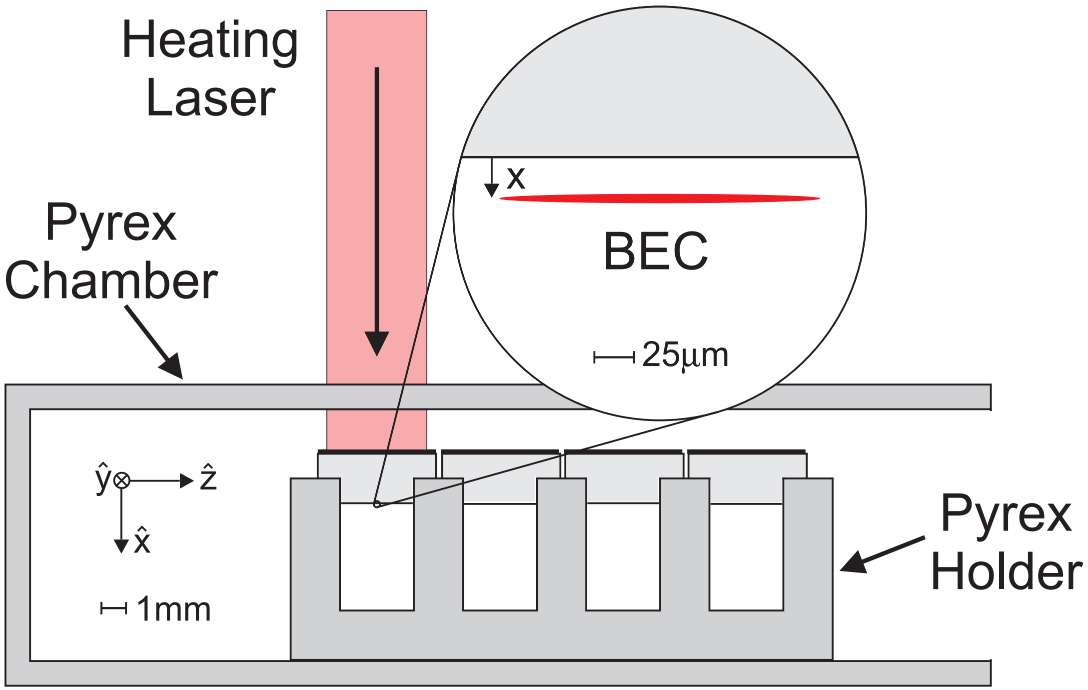
Equilibrium JILA results

$$\gamma \equiv -\frac{\Delta\omega}{\omega} = -\frac{1}{2\omega m}\langle \partial_z F \rangle$$



Non-equilibrium setting




Heating Laser
Pyrex Chamber
x
BEC
25µm
ŷ ẑ
x̂
1mm
Pyrex Holder

Non-equilibrium force

The main assumption :

$$k_B T_S, k_B T_S \ll \hbar\omega_{at}; \alpha(\omega) \to \alpha_0$$

$$F = 4\pi\alpha_0 \frac{\partial}{\partial z} \frac{\langle E^2 \rangle}{8\pi}$$

Non-equilibrium fluctuations

$$\langle P_i(\mathbf{r}) P_k(\mathbf{r}_1)\rangle_\omega = \varepsilon''(\omega,\mathbf{r})\hbar\coth\left(\frac{\hbar\omega}{2T}\right)\delta(\mathbf{r}-\mathbf{r_1})\delta_{ik}$$

$$T \rightarrow T(\mathbf{r}) \quad ???$$

$$F_{th}^{neq}(T,0,z) =$$

$$\frac{\hbar\alpha}{2\pi^2}\int_0^\infty d\omega \frac{\varepsilon''(\omega)}{e^{\hbar\omega/T}-1}\mathrm{Re}\left[\int G_{ik}(\omega;\mathbf{r},\mathbf{r}_1)\partial_z G^*_{ik}(\omega;\mathbf{r},\mathbf{r}_1)\right]d^3\mathbf{r}_1$$

Simplifying assumption: substrate is transparent

$$k_B T_S, k_B T_S << \hbar\omega_{opt}$$

$$\varepsilon(\omega) \to \varepsilon(0) \equiv \varepsilon_0$$

Asymptotic of the non-equilibrium thermal force

The force is created by the evanescent waves. Only the radiation, which undergoes the total reflection, is important.

Asymptotically the evanescent waves near the total reflection angle ($\sin^2\theta_r=1/\varepsilon$) are important. They decay slowly on the vacuum side.

Energy density of evanescent waves

$$\frac{\langle \mathbf{E}^2 \rangle}{8\pi} \sim \int_{k_\perp=\omega/c}^{k_\perp=\sqrt{\varepsilon}\omega/c} \left| \mathbf{E}_\mathbf{k}(z=0) \right|^2 e^{-2z\kappa} \frac{2d^3\mathbf{k}_0}{(2\pi)^3}$$

$$\kappa = \sqrt{k_\perp{}^2 - \omega^2/c^2} \sim 1/z$$

$\mathbf{E}_\mathbf{k}(z=0)$ can be expressed in terms of field in the substrate using Freshnel equations

Energy density of evanescent waves

$$\frac{\langle \mathbf{E}^2 \rangle}{8\pi} = \frac{1+\varepsilon}{\varepsilon} \int_{k_\perp = \omega/c}^{k_\perp = \sqrt{\varepsilon}\omega/c} \frac{\hbar\omega}{e^{\hbar\omega/T_S} - 1} e^{-2z\kappa} \frac{2d^3\mathbf{k}_0}{(2\pi)^3}$$

$$\kappa = \sqrt{k_\perp{}^2 - \omega^2/c^2} \sim 1/z$$

Asymptotic of the energy

$$z \to \infty : k_\perp \to \omega / c,\ \kappa = \sqrt{k_\perp{}^2 - \omega^2 / c^2} \to 0$$

$$d^3\mathbf{k}_0 = 2\pi k_\perp dk_\perp dk_{0z} \approx \frac{2\pi\varepsilon}{c\sqrt{\varepsilon - 1}} \kappa\, d\kappa\, d\omega$$

$$U_E = \frac{\left\langle \mathbf{E}^2 \right\rangle}{8\pi} = \frac{1+\varepsilon}{2\pi^2 c\sqrt{\varepsilon - 1}} \int_0^\infty \frac{\hbar\omega\, d\omega}{e^{\hbar\omega/T_S} - 1} \int_0^\infty e^{-2z\kappa} \kappa\, d\kappa$$

$$= \frac{1}{48} \frac{1+\varepsilon}{z^2\sqrt{\varepsilon - 1}} \frac{k_B^2 T_S^2}{c\hbar}$$

Characteristic interval of angles

$$\frac{\omega^2}{c^2}\sin\theta_r\, d\theta \sim \kappa\, d\kappa$$

$$z \to \infty : \kappa \sim 1/z, \omega \sim k_B T/\hbar,$$

$$\Delta\theta \sim \frac{c^2}{(k_B T)^2 z^2} \sim \frac{\lambda_T^2}{z^2}$$

$$U_E \sim \frac{\lambda_T^2}{z^2} U_{BB} \sim \frac{1}{z^2}\frac{k_B^2 T_S^2}{c\hbar}$$

Physical meaning

Only incident waves in the small interval of solid angles of the order of $(c/Tz)^2$ near the total reflection angle are important. Hence $U_E \sim (c/Tz)^2 U_{BB}$ where U_{BB} is the energy of the black-body radiation.

Asymptotic of the non-equilibrium thermal force

$$F_{th}^{neq} = 4\pi\alpha_0 \frac{\partial}{\partial z}\left\langle \frac{E^2}{8\pi} \right\rangle =$$

$$= -\alpha_0 \frac{\pi}{6} \frac{1+\varepsilon}{z^3\sqrt{\varepsilon - 1}} \frac{k_B^2 T_S^2}{c\hbar}$$

Antezza , Pitaevskii , Stringari ,

PRL **95**, 113202 (2005)

Finite temperatures of substrate and environment

$$F^{neq}(T,0,z) \propto 1/z^3; F^{eq}(T,z) \propto 1/z^4;$$

$$z \to \infty : F^{neq}(T,0,z) + F^{neq}(0,T,z) = 0$$

$$z >> \lambda_T / \sqrt{\varepsilon_0 - 1} :$$

$$F^{neq}(T_S, T_E, z) = -\frac{\pi k_B^2 \left(T_S^2 - T_E^2\right)}{z^3 6c\hbar} \alpha_0 \frac{\varepsilon + 1}{\sqrt{\varepsilon - 1}}$$

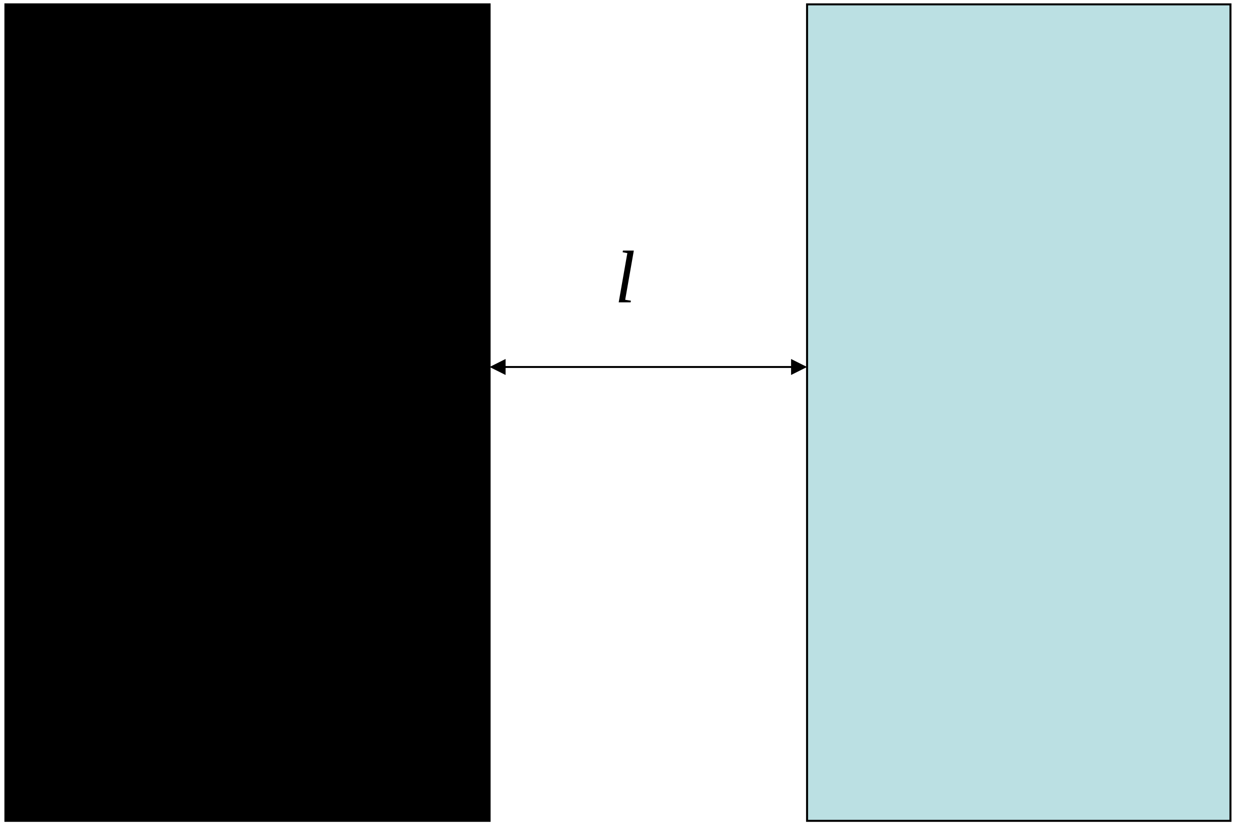
Metallic substrate

For a metallic substrate

$$\varepsilon(\omega) = i\frac{4\pi\sigma}{\omega}$$

$$F^{neq}(T_S, T_E, z) = -\frac{2.6k_B^{3/2}\left(T_S^{3/2} - T_E^{3/2}\right)}{z^3 c\sqrt{2\hbar}}\alpha_0\sqrt{\sigma}$$

Atomic cloud as a limit of a rarefied body



$T, \varepsilon - 1 \sim 1$

$T_2, \varepsilon_2 - 1 << 1$

Atom-surface interaction as a limit of the interaction of a rarefied body

Equilibrium Lifshitz theory:

$$T = T_2, \text{at}\, \lambda_T \ll l, \varepsilon_2 - 1 \ll 1:$$

$$P^{eq} \propto \frac{k_B T}{l^3}\left(\varepsilon_2 - 1\right) = \frac{k_B T}{l^3} 4\pi\alpha N$$

$$U_E = -P / N$$

Non-additivity at large distances out of equilibrium

$$T \neq T_2 = 0, \varepsilon_2 - 1 << 1:$$

$$1.\ \text{At}\ \lambda_T << l << \lambda_T / \sqrt{\varepsilon_2 - 1}:$$

$$P^{neq} \propto \frac{(k_B T)^2}{l^2 c\hbar}(\varepsilon_2 - 1) = \frac{(k_B T)^2}{l^2 c\hbar} 4\pi\alpha N$$

Non-additivity at large distances

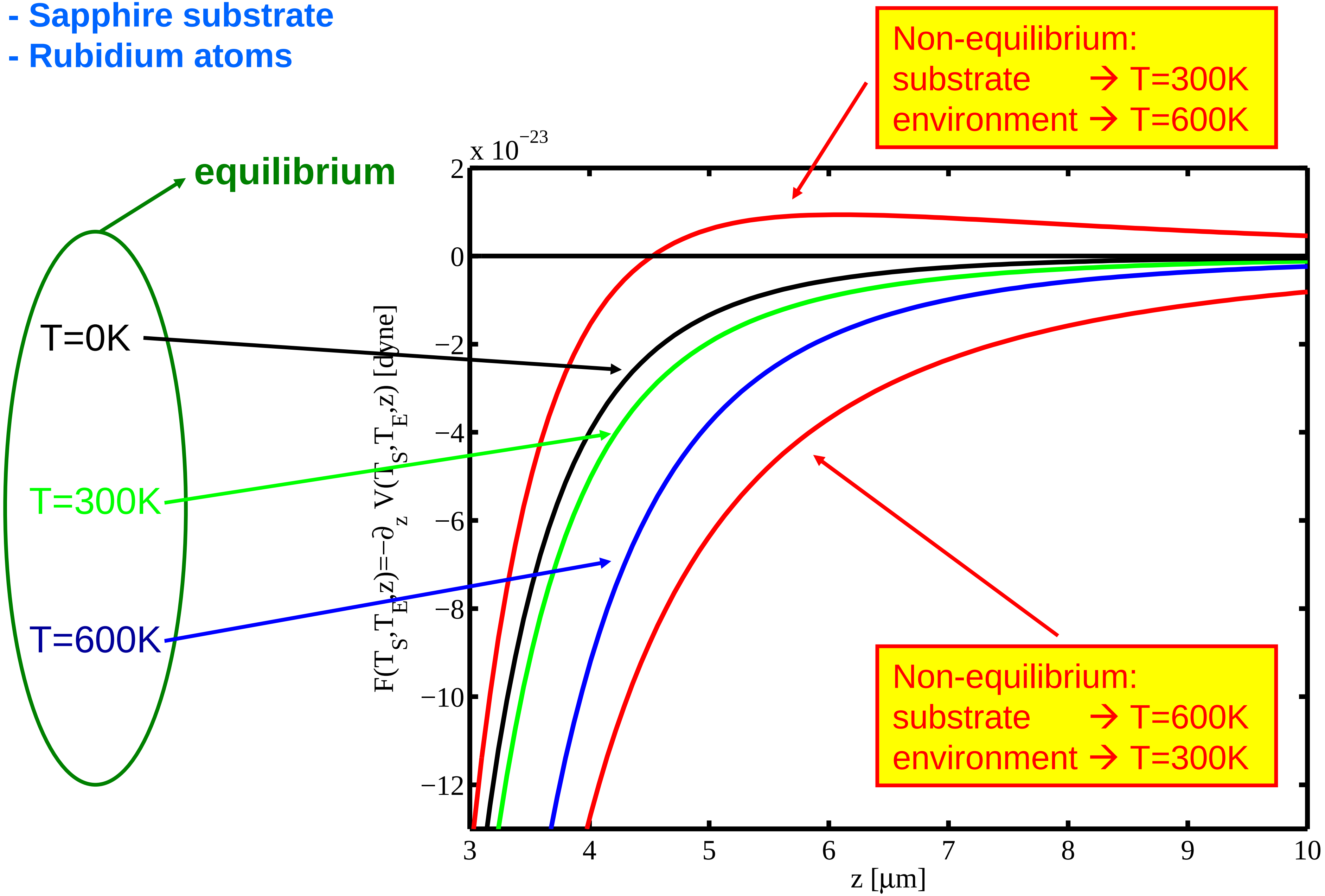
$$2.\text{At } \lambda_T / \sqrt{\varepsilon_2 - 1} << l:$$

$$P^{neq} \propto \frac{k_B T}{l^3} \sqrt{\varepsilon_2 - 1}$$

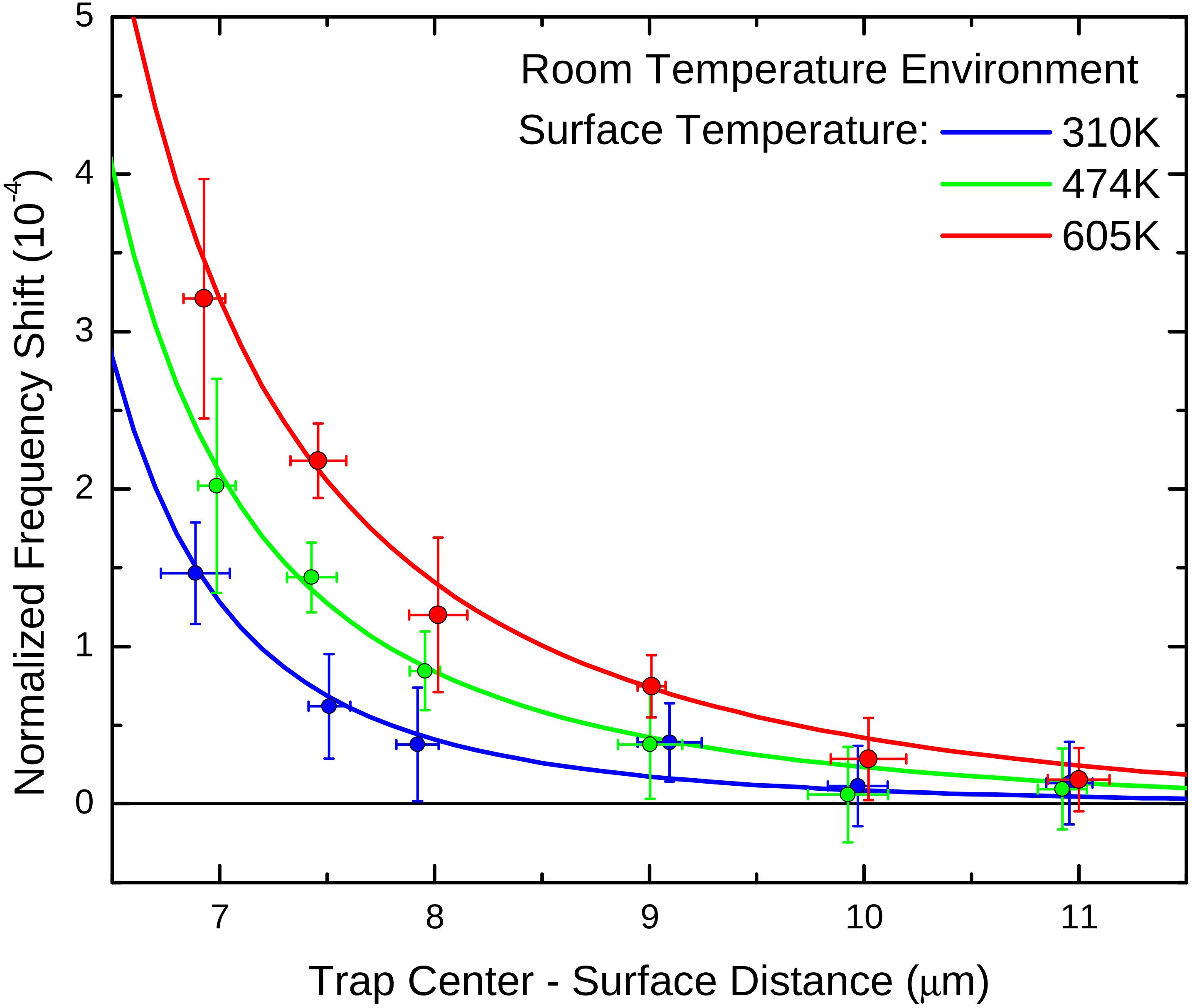
Non-additivity!

Thermal effects on the surface-atom force

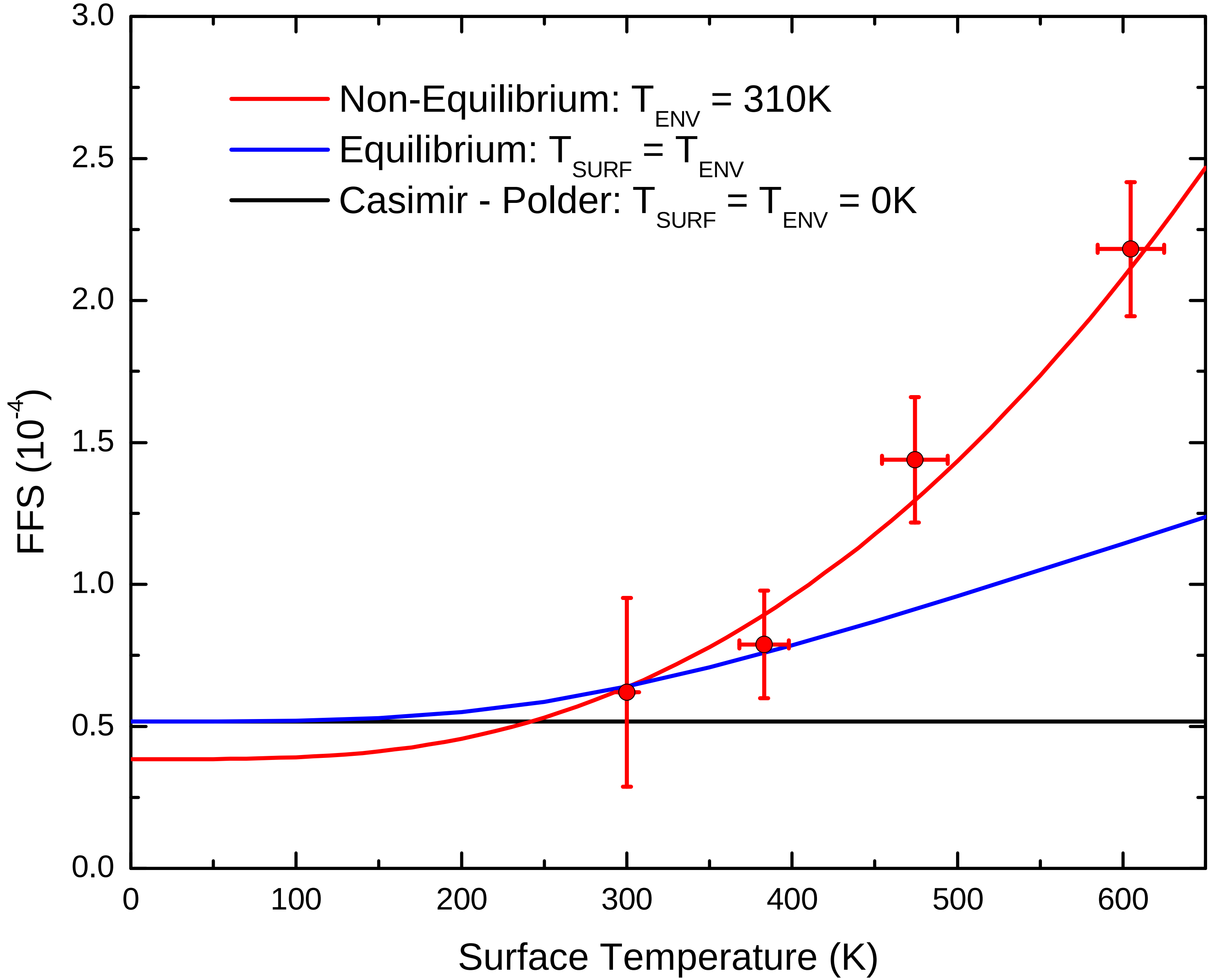
- Sapphire substrate
- Rubidium atoms



Recent Experimental results from JILA



J.M. Obrecht, R.J. Wild, M. Antezza, L.P. Pitaevskii, S. Stringari, and E.A. Cornell, PRL **98**, 063201 (2007).



Non-Equilibrium: T_{ENV} = 310K
Equilibrium: T_{SURF} = T_{ENV}
Casimir - Polder: T_{SURF} = T_{ENV} = 0K
FFS (10^{-4})
Surface Temperature (K)
0.0
0.5
1.0
1.5
2.0
2.5
3.0
0
100
200
300
400
500
600

Paradox of Lifshitz theory

$$l >> \hbar c / T$$

Only $s = 0$ term is important

$$V(l) = -\frac{T}{4l^3}\frac{\varepsilon_0 - 1}{\varepsilon_0 + 1}$$

$$\varepsilon(i\zeta) = \frac{4\pi\sigma}{\zeta} + \bar{\varepsilon}, V(l) = -\frac{T}{4l^3}$$

$$\sigma \to 0\ ???$$

Debye screening of the field

$$z < 0, \left[\Delta - \kappa^2\right]\varphi = 0$$

$$\kappa^2 = R_D^{-2} = \frac{4\pi e^2 n}{\bar{\varepsilon} T}$$

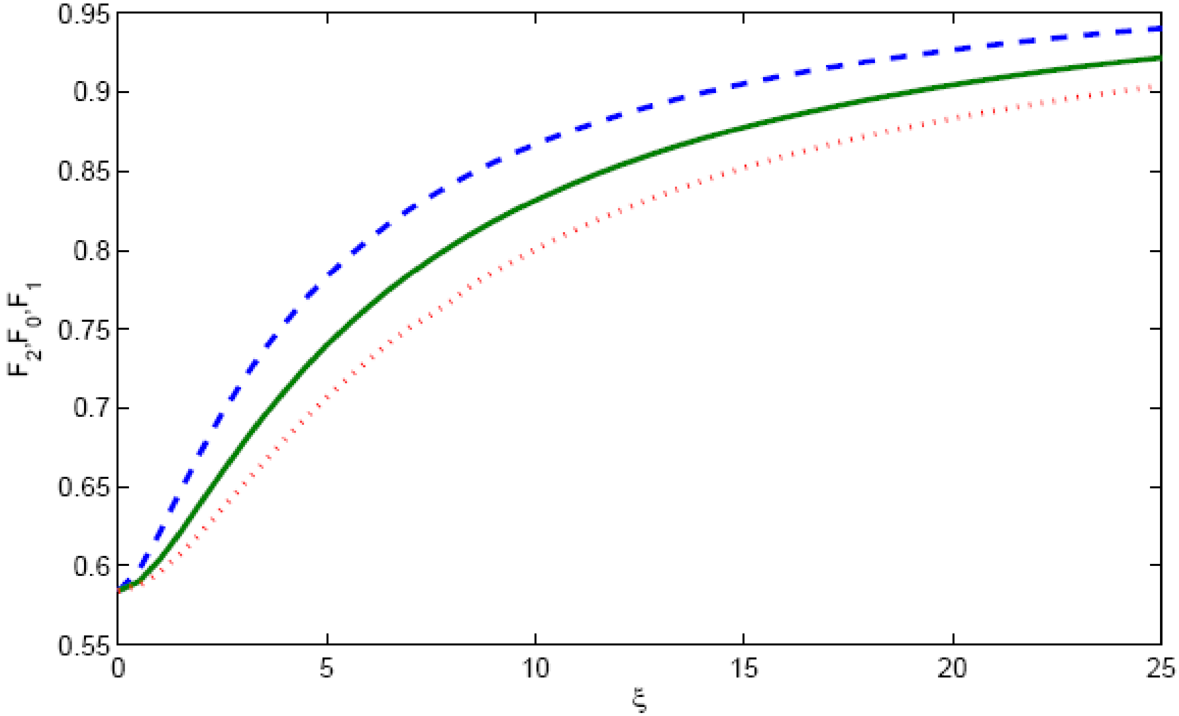
$$q = \sqrt{k^2 + \kappa^2}$$

$$V(l) = -T\alpha_0 \int_0^\infty \frac{\bar{\varepsilon} q - k}{\bar{\varepsilon} q + k} e^{-2kl} k^2 dk$$

$$\equiv -\frac{T\alpha_0}{4l^3} F_0(\xi = l / R_D)$$

L. Pitaevskii (2008)

Effect of the field penetration



Fused Silica: SiO_2

$$\bar{\varepsilon} = 3.81$$

$$\text{Conductivity is due to Na}^{+}$$

$$c_{Na} = (50 - 100)\text{ppb},$$

$$n_{Na} = \left(2.88 - 5.76\right) \times 10^{15}\, cm^{-3}$$

Fused Silica: T=605 K

$$n_{Na} = (2.88 - 5.76)\times 10^{15}\, cm^{-3}$$

$$R_D \le 6.2\times 10^{-2}\,\mu$$

$$\rho = 8.3\times 10^{10}\, ohm.cm = 9.3\times 10^{-2}\, s$$

$$\tau = \frac{\bar{\varepsilon}\rho}{4\pi} = 2.8\times 10^{-2}\, s$$

$$D = 8.5\times 10^{-10}\, cm^2 / s = bT$$

$$n_{Na}^{D} = \frac{T}{\rho D e^2} = 4.62\times 10^{15}\, cm^{-3}$$

Fused Silica: *T*=292 K

$$n_{Na} = (2.88 - 5.76) \times 10^{15} cm^{-3}$$

$$R_D \leq 4.3 \times 10^{-2} \mu$$

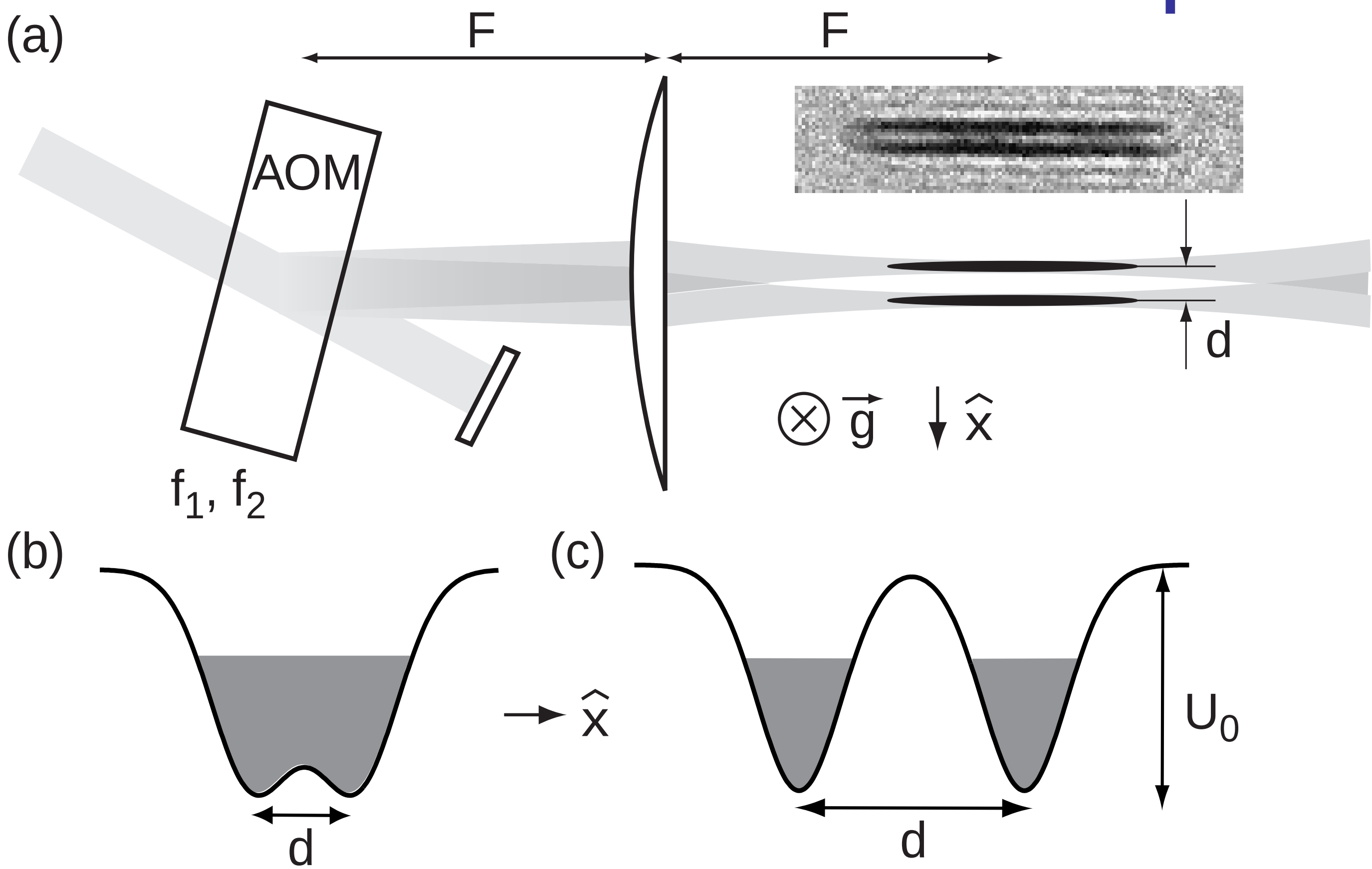
$$\rho = 10^{19} ohm.cm = 1.1 \times 10^{7} s \sim 900h$$

$$\tau = \frac{\bar{\varepsilon}\rho}{4\pi} = 3.3 \times 10^{6} s$$

$$D = 2.2 \times 10^{-20} cm^{2} / s = bT$$

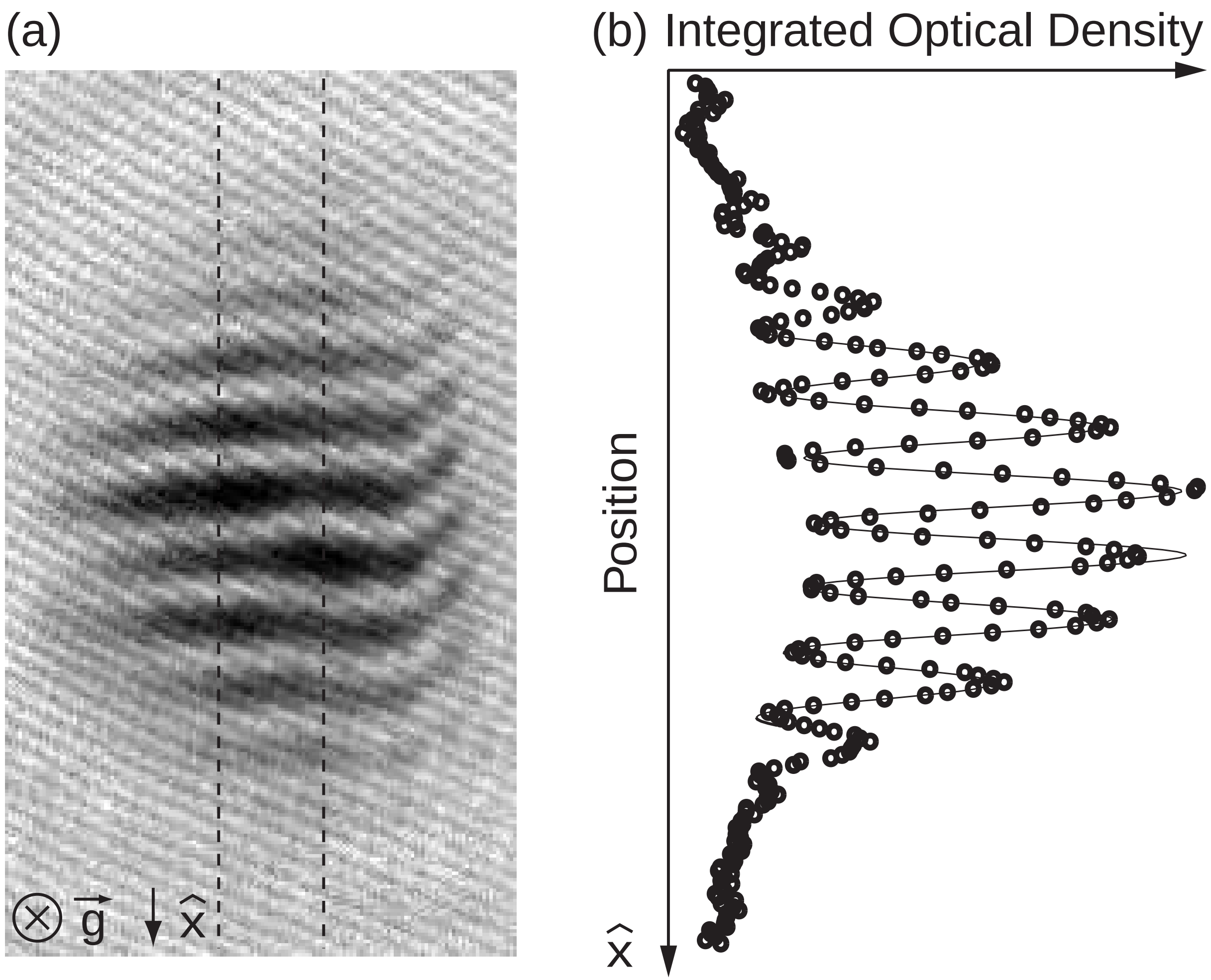
$$n_{Na}^{D} = \frac{T}{\rho D e^{2}} = 7 \times 10^{17} cm^{-3} \textbf{???}$$

MIT double-well setup



(a) Setup for double-well potential. (b), (c): d=6, 13mcm

Y. Shin, M. Saba, T. Pasquini, W. Ketterle, D. Pritchard, and A. Leanhard, PRL **92**, 050405 (2004).



Interference of condensates released from the double-well trap [1].

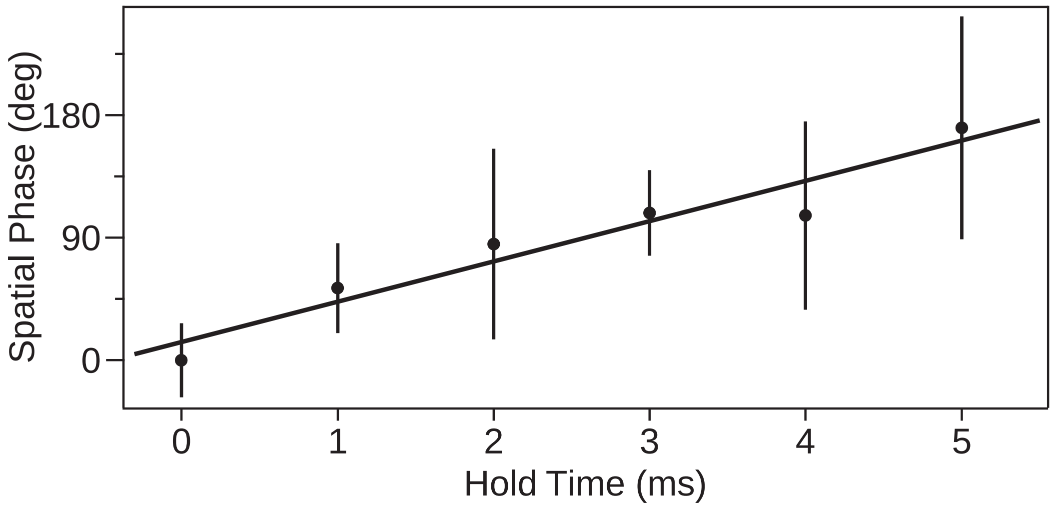
Measurement of the phase difference.

Interference of expanding condensates.

Density profile after expansion :

$$n(x,t) = \left[n_a + n_b + 2\sqrt{n_a n_b} \cos\left(\frac{md}{\hbar t} x + \Phi \right) \right]$$

$$\Phi = \Delta \mathrm{t} \langle U \rangle, U = -\int_z^{\infty} F dz$$



Time dependence of the relative phase in presence of chemical potentials difference [1].

Measurement of the phase difference. II

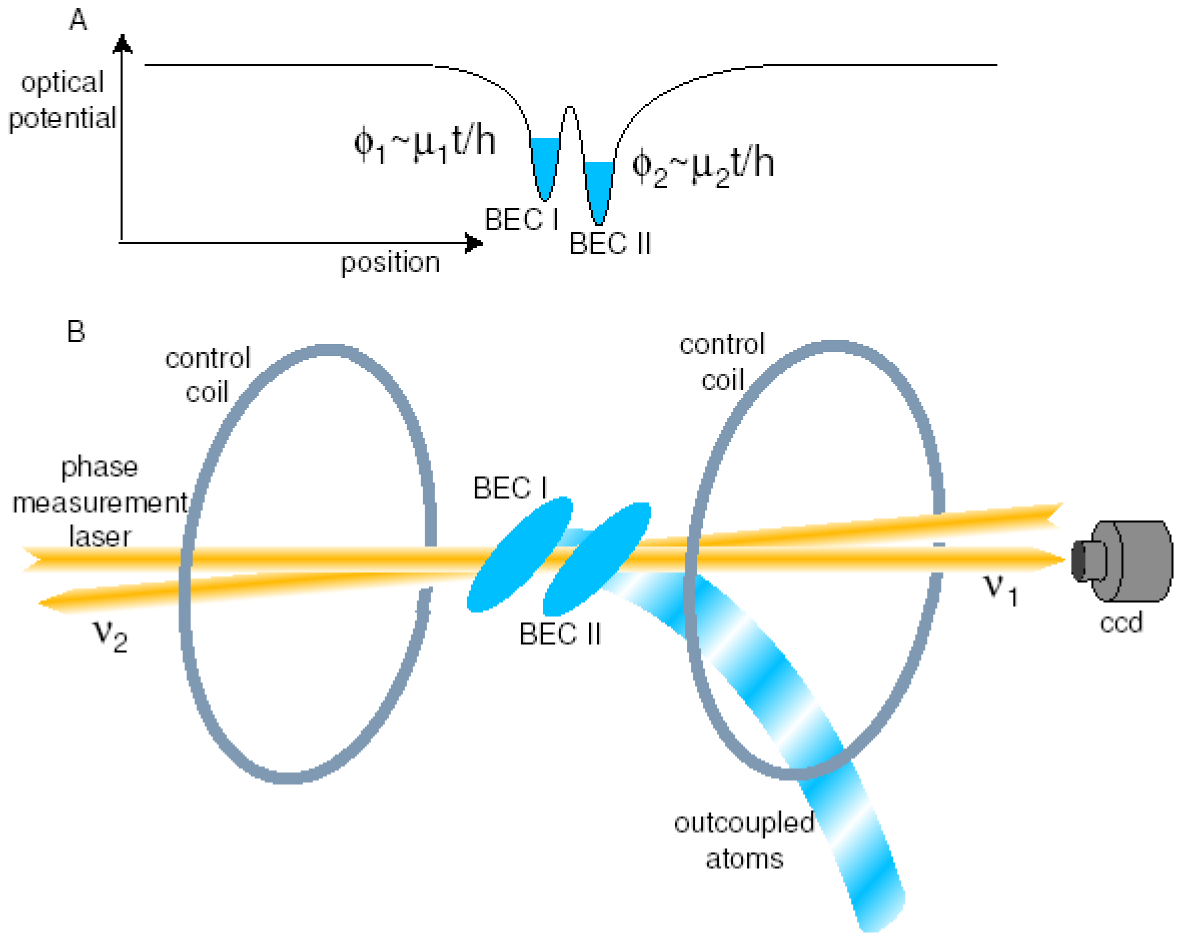
"Interference in momentum space"

Momentum distribution of two identical condensates in a double-well trap:

$$n(p_x) = 2\left[1 + \cos\left(\frac{p_z d}{\hbar} + \Phi\right)\right] n_a(p_x)$$

Measurement can be almost non-destructive!

L. Pitaevskii and S. Stringari, 1999.



Setup for continuous phase measurement.
M. Saba, T. Pasquini, C. Sanner, Y. Shin, W. Ketterle, and D. Pritchard, Science (2005).