From: Landau Fermi liquid & two-fluid hydrodynamics
To: physics of quantum vacuum & cosmology

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Landau Memorial Meeting, Moscow, June 19-20, 2008

1. effective (phenomenological) theories of hydrodynamic type

Landau two-fluid hydrodynamics

Einstein general relativity

2. effective (phenomenological) theories from p-space topology

Landau theory of Fermi liquid

Standard Model + gravity

3. extension of Landau ideas

quantum vacuum as Lorentz invariant medium
application to cosmology

3+1 sources of effective (**phenomenological**) theory of quantum liquids & relativistic QFT

I think it is safe to say that no one understands **Quantum Mechanics**

Richard Feynman

Thermodynamics is the only physical theory of universal content

Albert Einstein

Symmetry: conservation laws, translational invariance,
Galilean principle of relativity, ...

missing ingredient

Topology: you can't comb the hair on a ball smooth

1. Effective theories of hydrodynamic type

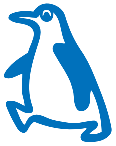
Landau two-fluid hydrodynamics

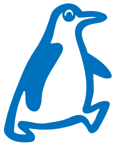
Einstein general relativity

classical low-energy property
of quantum liquids

classical low-energy property
of quantum vacuum

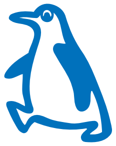
Landau equations





Einstein equations

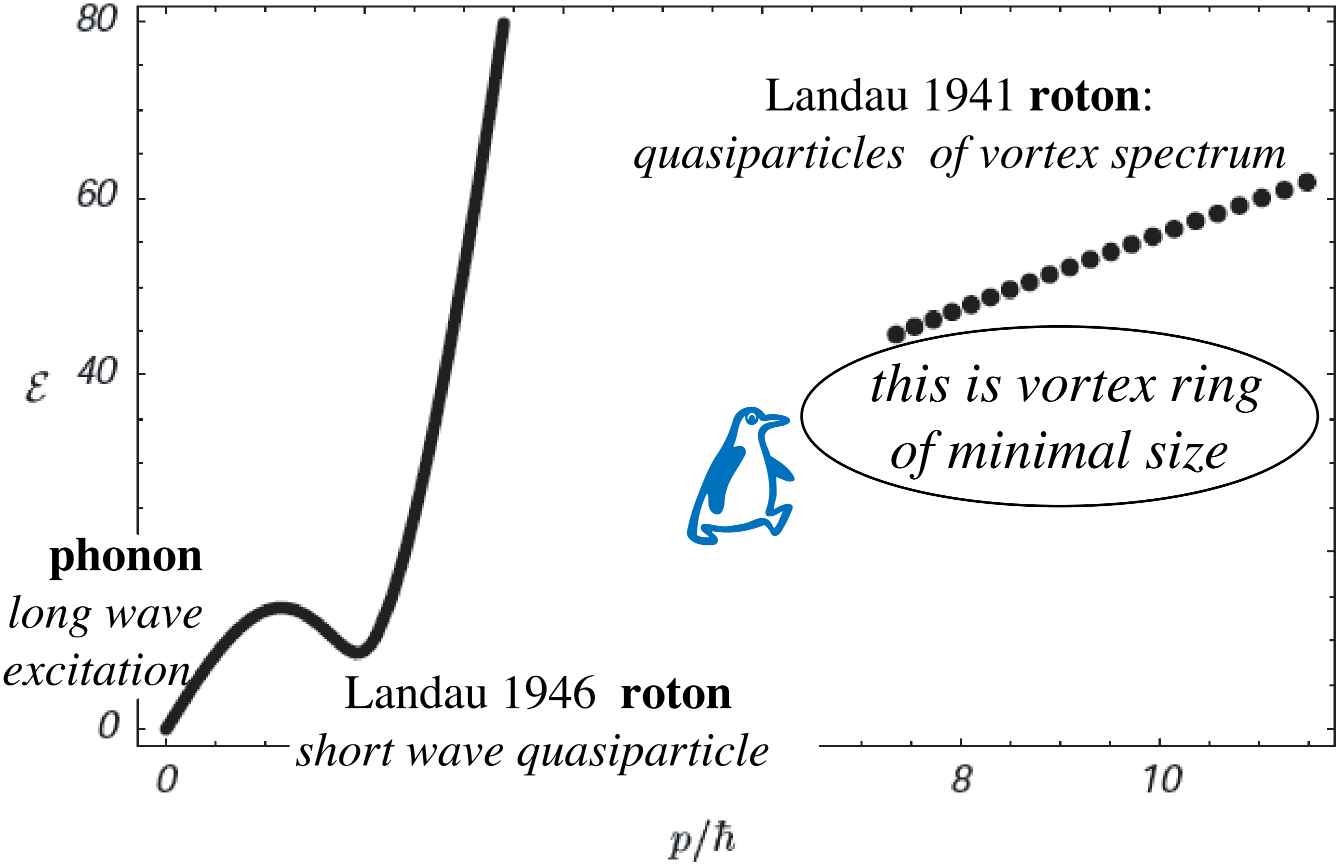
Landau equations & Einstein equations
are **effective theories**
describing dynamics of
metric field + **matter** (quasiparticles)



Landau quasiparticles

weakly excited state can be considered as system of "elementary excitations"

Landau, 1941



Landau 1941 **roton**:

quasiparticles of vortex spectrum

this is vortex ring of minimal size

phonon

long wave excitation

Landau 1946 **roton**

short wave quasiparticle

Landau gap in vortex spectrum

$$\boxed{\Delta \sim h^2 \rho^{2/3} m^{-5/3}}$$

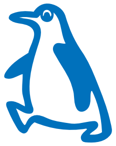
this is energy of vortex ring of minimal size

Landau estimate of vortex gap is correct !

N.G. Berloff & P.H. Roberts
Nonlocal condensate models of superfluid helium
J. Phys. **A32**, 5611 (1999)

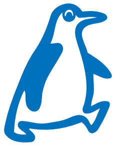
droplet of superfluid ^{4}He

superfluid
ground state
+
elementary
excitations



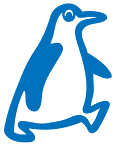
ground state = vacuum

quasiparticles =
elementary particles



Universe

vacuum
+
elementary
particles

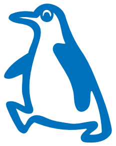


long-wave quasiparticles:
`relativistic' phonons

$$E(\mathrm{k}) = ck$$

c – speed of sound
or light

long-wave quasiparticles:
relativistic fermions & bosons



effective metric

$$g^{\mu\nu}k_\mu k_\nu = 0$$

general relativity

Effective metric in Landau two-fluid hydrodynamics

Doppler shifted phonon spectrum in moving superfluid

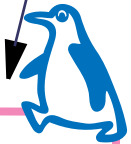
review:
Barcelo, Liberati & Visser,
Analogue Gravity
Living Rev. Rel. **8** (2005) 12

$$\boxed{E = cp + \mathbf{p}\cdot\mathbf{v}_s}$$

c speed of sound

$\mathbf{v}_s$ superfluid velocity

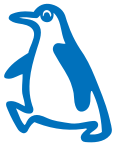
move $\mathbf{p}\cdot\mathbf{v}_s$ *to the left*



$$\boxed{E - \mathbf{p}\cdot\mathbf{v}_s = cp}$$

take square

$$\boxed{(E - \mathbf{p}\cdot\mathbf{v}_s)^2 - c^2 p^2 = 0} \longrightarrow \boxed{g^{\mu\upsilon} p_\mu p_\nu = 0}$$



$$p_\nu = (-E, \ \mathbf{p})$$

$$g^{00} = -1 \qquad g^{0\,i} = -v_s{}^i \qquad g^{ij} = c^2\delta^{ij} - v_s{}^i v_s{}^j$$

effective metric

$\downarrow$

inverse metric $g_{\mu\upsilon}$ determines effective spacetime
in which phonons move along geodesic curves

$$ds^2 = -\, c^2 dt^2 \ + \ (d\mathbf{r} - \mathbf{v}_s dt)^2 \longleftarrow ds^2 = \ g_{\mu\upsilon} dx^\mu dx^\nu$$

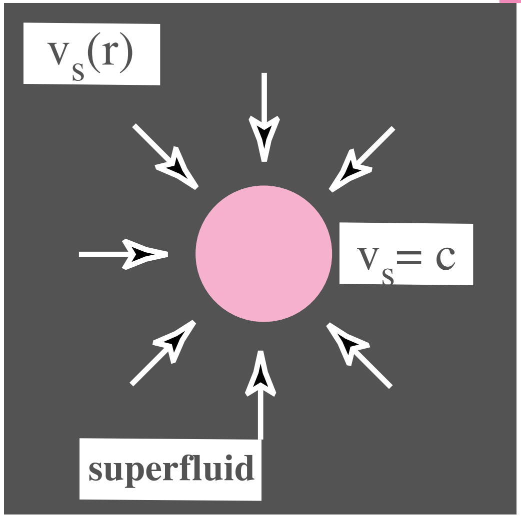
reference frame for phonon is dragged
by moving liquid

Landau critical velocity = black hole horizon

superfluid ^{4}He

acoustic horizon

$$v^2(r) = c^2 \frac{r_h}{r}$$



(Unruh, 1981)

Painleve-Gulstrand metric

$$\boxed{ds^2 = - dt^2\,(c^2 - v^2) \;+\; 2\,v\,dr\,dt \;+ dr^2 + \; r^2 d\Omega^2}$$

g_{00} g_{0r}

Schwarzschild metric

$$\boxed{ds^2 = - dt^2\,(c^2 - v^2) \;+\; dr^2\,/(c^2 - v^2) \;+\; r^2 d\Omega^2}$$

g_{00} g_{rr}

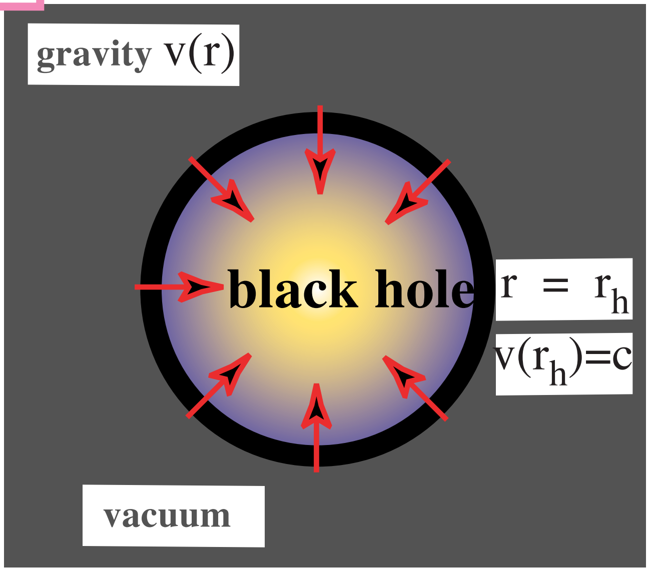
horizon at $g_{00} = 0$
where flow velocity reaches
Landau critical velocity

$$\boxed{v(r_h) = v_{\mathrm{Landau}} = c}$$

Gravity

black hole horizon

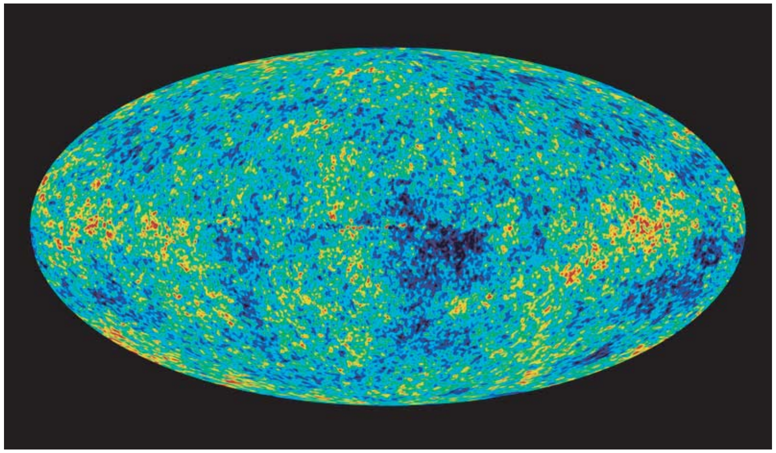
$$v^2(r) = \frac{2GM}{r} = c^2 \frac{r_h}{r}$$



Hawking radiation is phonon/photon creation
above Landau critical velocity

Superfluids

Universe



acoustic gravity **metric theories of gravity** **general relativity**

geometry of effective space time for quasiparticles (phonons) $g_{\mu\upsilon}$ geometry of space time for matter

geodesics for phonons $ds^2 = g_{\mu\upsilon} dx^\mu dx^\nu = 0$ geodesics for photons

Landau two-fluid equations

$$\boxed{\begin{aligned} &\dot{\rho} + \nabla\cdot(\rho\mathbf{v}_s + \mathbf{P}^{\text{Matter}}) = 0 \\ &\dot{\mathbf{v}}_s + \nabla(\mu + \mathbf{v}_s{}^2/2) = 0 \end{aligned}}$$

equations for superfluid component

equation for normal component

$$\boxed{T^{\mu\nu}_{;\nu\,\text{Matter}} = 0}$$

dynamic equations for metric field $g_{\mu\upsilon}$

Einstein equations of GR

$$\boxed{\frac{1}{8\pi G}(R_{\mu\nu} - g_{\mu\nu}R/2) = T_{\mu\nu}^{\text{Matter}}}$$

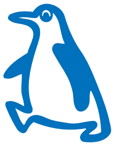
equation for matter

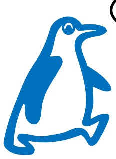
1/2 of GR

message from: Landau hydrodynamics to: gravity

is gravity fundamental ?

it may emerge as classical output of underlying quantum system





as hydrodynamics ?

superfluid helium

↓

classical 2-fluid hydrodynamics

underlying microscopic quantum system at high energy

↓

emergent low-energy effective theory

quantum vacuum

↓

classical general relativity

$$\dot{\rho} + \nabla\cdot(\rho\mathbf{v}_s + \mathbf{P}^{\mathrm{Matter}}) = 0$$

$$\dot{\mathbf{v}}_s + \nabla(\mu + \mathbf{v}_s{}^2/2) = 0$$

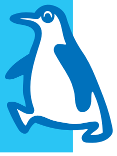
$$\nabla_\nu T^{\mu\nu}_{\mathrm{Matter}} = 0$$

$$\frac{1}{8\pi G}(R_{\mu\nu} - g_{\mu\nu}R/2) - \Lambda g_{\mu\nu} = T^{\mathrm{Matter}}_{\mu\nu}$$

2. Effective theories from momentum space topology

Landau theory of Fermi liquid

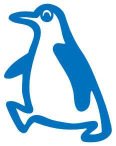
Standard Model + gravity



two major universality classes of fermionic vacua:
vacuum with Fermi surface & vacuum with Fermi point

gravity emerges from
Fermi point
analog of
Fermi surface

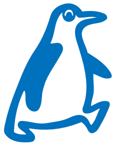
$$g^{\mu\nu}(p_\mu - eA_\mu - e\tau \cdot \mathbf{W}_\mu)(p_\nu - eA_\nu - e\tau \cdot \mathbf{W}_\nu) = 0$$



crossover from Landau 2-fluid hydrodynamics to Einstein general relativity

they represent two different limits of hydrodynamic type equations

equations for $g^{\mu\nu}$ depend on hierarchy of ultraviolet cut-off's:
Planck energy scale $E_{\rm Planck}$ vs Lorentz violating scale $E_{\rm Lorentz}$

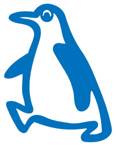


$E_{\rm Planck} >> E_{\rm Lorentz}$

Landau two-fluid hydrodynamic equations

$E_{\rm Planck} << E_{\rm Lorentz}$

Einstein equations of general relativity



^{3}He-A with Fermi point

Universe

$E_{\rm Lorentz} << E_{\rm Planck}$

$E_{\rm Lorentz} \sim 10^{-3} E_{\rm Planck}$

$E_{\rm Lorentz} >> E_{\rm Planck}$

$E_{\rm Lorentz} > 10^{15} E_{\rm Planck}$

what is common between cold liquid & Universe ???

high-energy physics and cosmology are extremely ultra-low temperature physics

characteristic high-energy scale in our vacuum
(analog of atomic scale in cond-mat) is Planck energy

$$E_P = (hc^5/G)^{1/2} \sim 10^{19}\ \mathrm{GeV} \sim 10^{32}\mathrm{K}$$

highest energy in accelerators

$$E_{\mathrm{ew}} \sim 1\ \mathrm{TeV} \sim 10^{16}\mathrm{K}$$

$$E_{\mathrm{ew}} \sim 10^{-16} E_{\mathrm{Planck}}$$

T of cosmic background radiation

$$T_{\mathrm{CMBR}} \sim 1\ \mathrm{K}$$

$$T_{\mathrm{CMBR}} \sim 10^{-32} E_{\mathrm{Planck}}$$

we live
at exremely low temperature?

this is natural for any cond-mat
if no energy flux from enviroment

How can we survive
in such a freezer ?

Right question !

cosmology is extremely ultra-low frequency physics

B. L. Hu
New View on Quantum Gravity and the Origin of the Universe

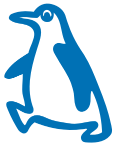
gr-qc/0611058

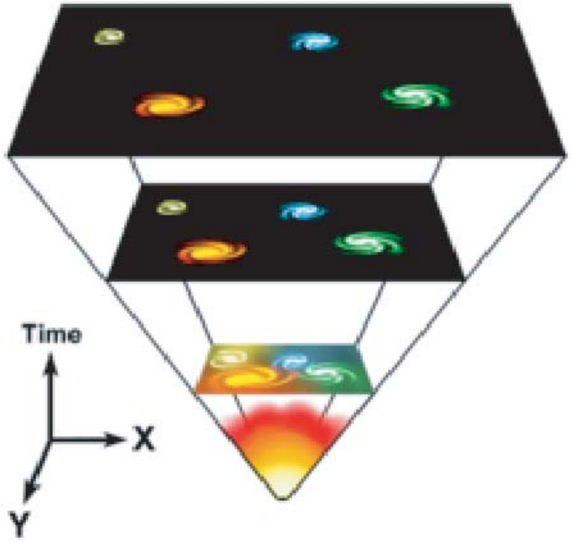
$$v(r) = Hr$$

expansion of Universe (Hubble parameter)

$$H \sim 10^{-60} E_{\mathrm{Planck}}$$

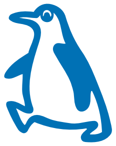
$$E_{\mathrm{P}} \sim 10^{19}\ \mathrm{GeV} \sim 10^{32}\mathrm{K}$$





our Universe is extremely close to equilibrium ?

this is also natural for any cond-mat if no energy flux from enviroment



Why no freezing at low T?

natural masses of elementary particles
are of order of characteristic energy scale
the Planck energy

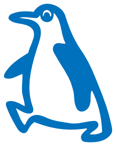
$$\boxed{m \sim E_{\text{Planck}} \sim 10^{19}\ \text{GeV} \sim 10^{32}\text{K}}$$

even at highest temperature
we can reach

$$\boxed{T \sim 1\ \text{TeV} \sim 10^{16}\text{K}}$$

everything should be completely frozen out

$$\boxed{e^{-m/T} = 10^{-10^{16}} = 0}$$



10^{-123}, $10^{-10^{16}}$
another great challenge?

main hierarchy puzzle

$$\boxed{m_{\mathrm{quarks}}\,,\; m_{\mathrm{leptons}} <<< E_{\mathrm{Planck}}}$$

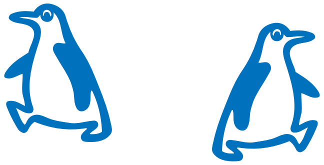
momentum-space topology solution:

$$\boxed{m_{\mathrm{quarks}} = m_{\mathrm{leptons}} = 0}$$

reason:

quasiparticles leaving near Fermi surface have no gap

quasiparticles leaving near Fermi point have no mass



cosmological constant puzzle

$$\boxed{\Lambda <<<<< E^4_{\mathrm{Planck}}}$$

effective theory solution:

$$\boxed{\Lambda = 0}$$

reason:

equilibrium condition in theories of hydrodynamic type:

Landau 2-fluid hydrodynamics & hydrodynamics of vacuum

Why no freezing at low T?

massless particles & gapless excitations are not frozen out

who protects massless excitations?

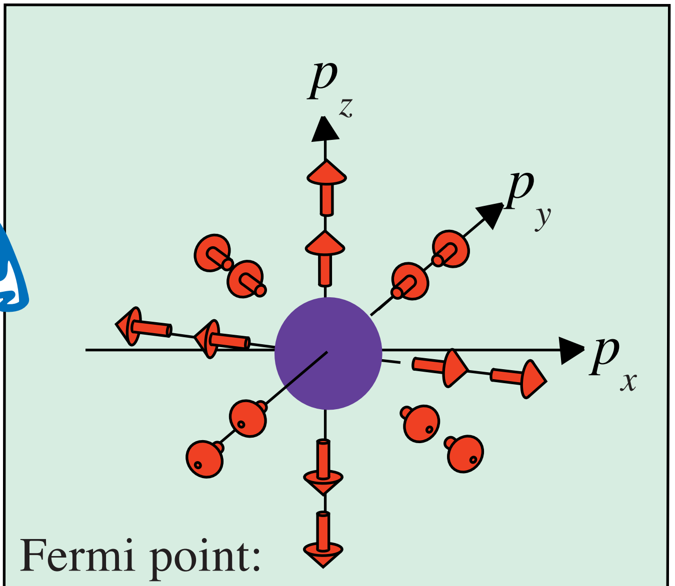
who protects massless fermions?

Topology

gapless fermions live
near
Fermi surface & Fermi point

no life without
topology ?

we live because
Fermi point is the hedgehog
protected by topology

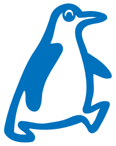


Fermi point:
hedgehog in momentum space

hedgehog is stable:
one cannot comb the hair
on a ball smooth

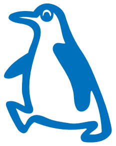
Universality classes of quantum vacua

physics at low T is determined by low-lying excitations



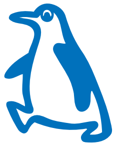
low-lying excitations live
near Fermi surface

in metals

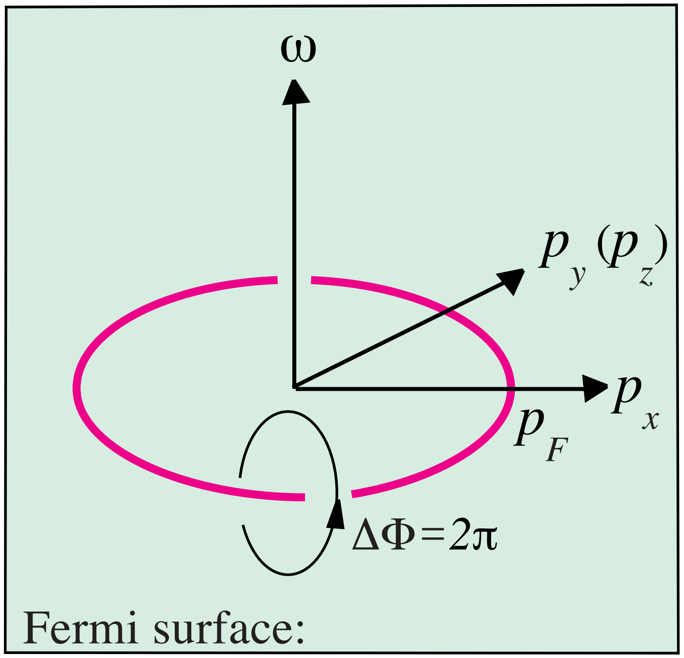


or near Fermi point

in our vacuum



Fermi surface class

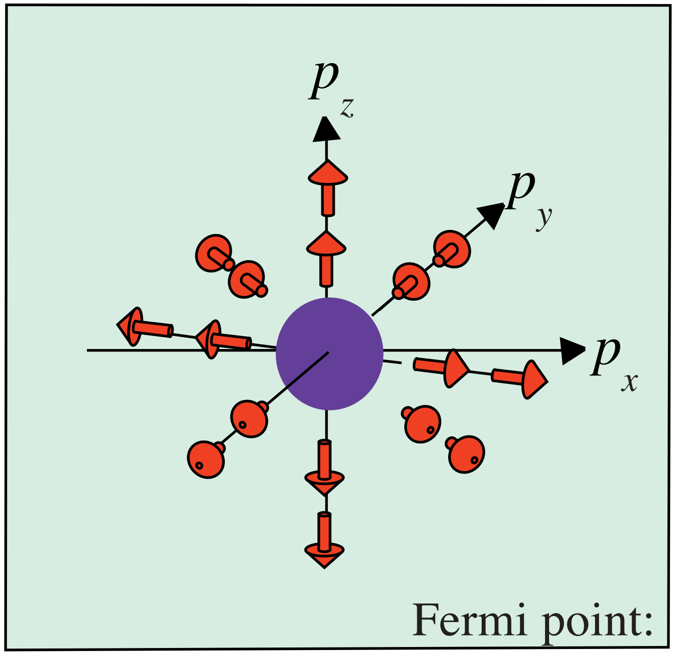


Fermi surface:
vortex in **p**-space

Topology in
momentum space

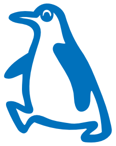
P. Horava,
Stability of Fermi surfaces
& K-theory
PRL **95**, 016045 (2005)

Fermi point class



Fermi point:
hedgehog in **p**-space

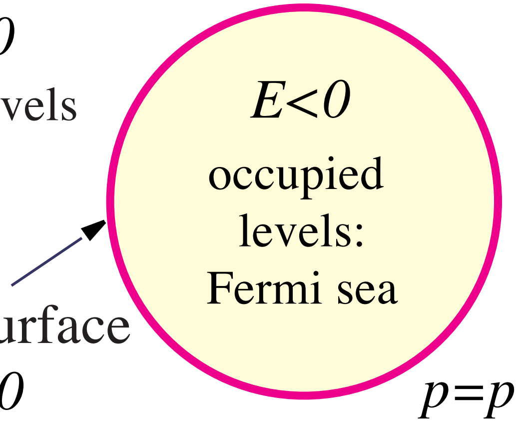
topology *protects vortices & hedgehogs:*
one cannot comb the hair on a ball smooth



Topological stability of Fermi surface

Energy spectrum of
non-interacting gas of fermionic atoms

$$E(p) = \frac{p^2}{2m} - \mu = \frac{p^2}{2m} - \frac{p_F^{\,2}}{2m}$$



$E>0$
empty levels

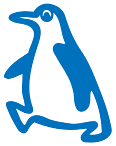
$E<0$
occupied
levels:
Fermi sea

Fermi surface
$E=0$

$p=p_F$

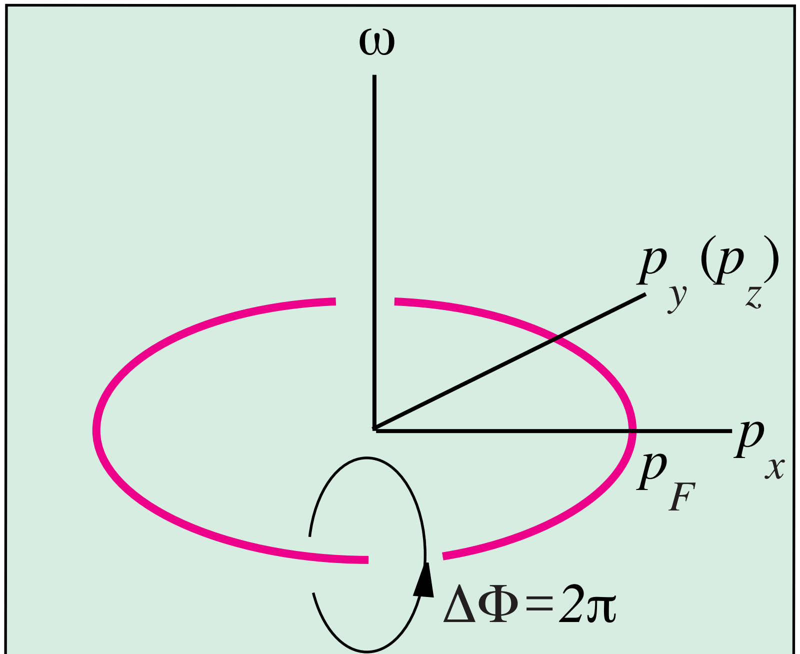
is Fermi surface a domain wall
in momentum space?

no!
it is a **vortex ring**



Green's function

$$G^{-1} = i\omega - E(p)$$



Fermi surface:
vortex ring in **p**-space

phase of Green's function
$$G(\omega,\mathbf{p})=|G|e^{\,i\Phi}$$
has winding number $N = 1$

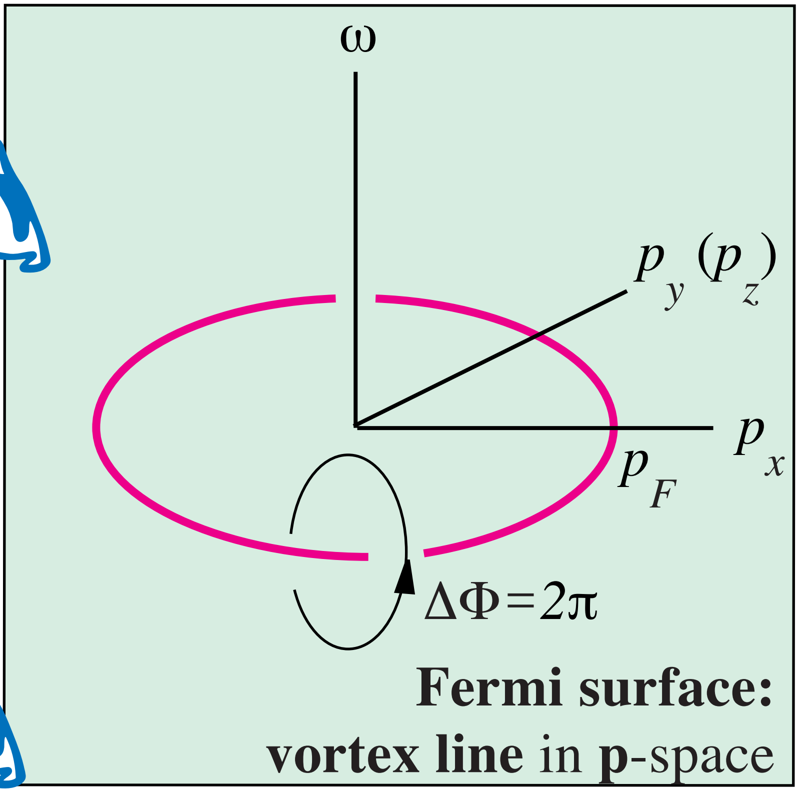
Route to Landau Fermi-liquid

is Fermi surface robust to interaction ?

Sure! Because of topology:
winding number $N=1$ cannot change continuously,
interaction cannot destroy singularity

then Fermi surface survives in Fermi liquid ?

Landau theory of Fermi liquid
is topologically protected & thus is universal



Fermi surface:
vortex line in **p**-space

$$G(\omega,\mathbf{p})=|G|e^{\,i\Phi}$$

all metals have Fermi surface ...

Not only metals.
Some superconductore too!

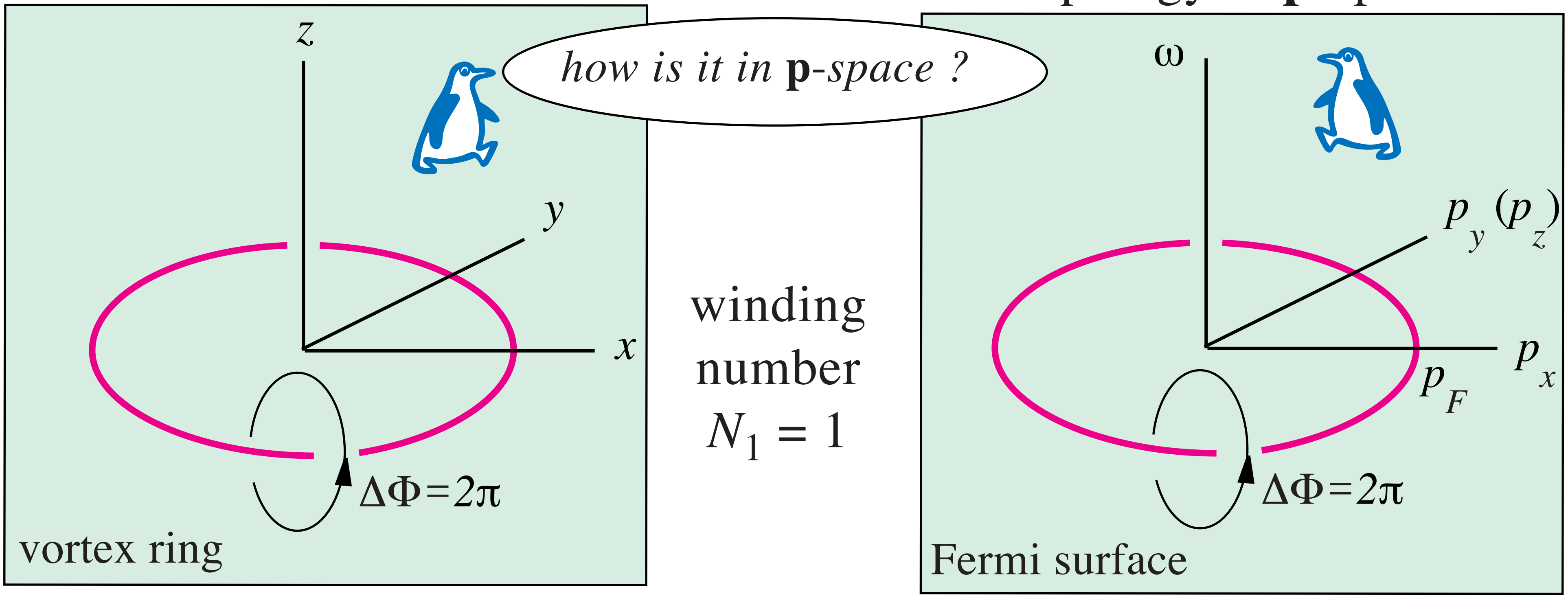
Stability conditions & Fermi surface topologies in a superconductor
Gubankova-Schmitt-Wilczek, *Phys.Rev.* **B**74 (2006) 064505

quantized vortex in **r**-space ≡ **Fermi surface** in **p**-space

homotopy group π_1

Topology in **r**-space

Topology in **p**-space



*how is it in **p**-space ?*

winding number $N_1 = 1$

$\Psi(\mathbf{r})=|\Psi|\, e^{\,i\Phi}$

scalar order parameter
of superfluid & superconductor

$G(\omega,\mathbf{p})=|G|e^{\,i\Phi}$

Green's function (propagator)

classes of mapping $S^1 \rightarrow U(1)$

manifold of
broken symmetry vacuum states

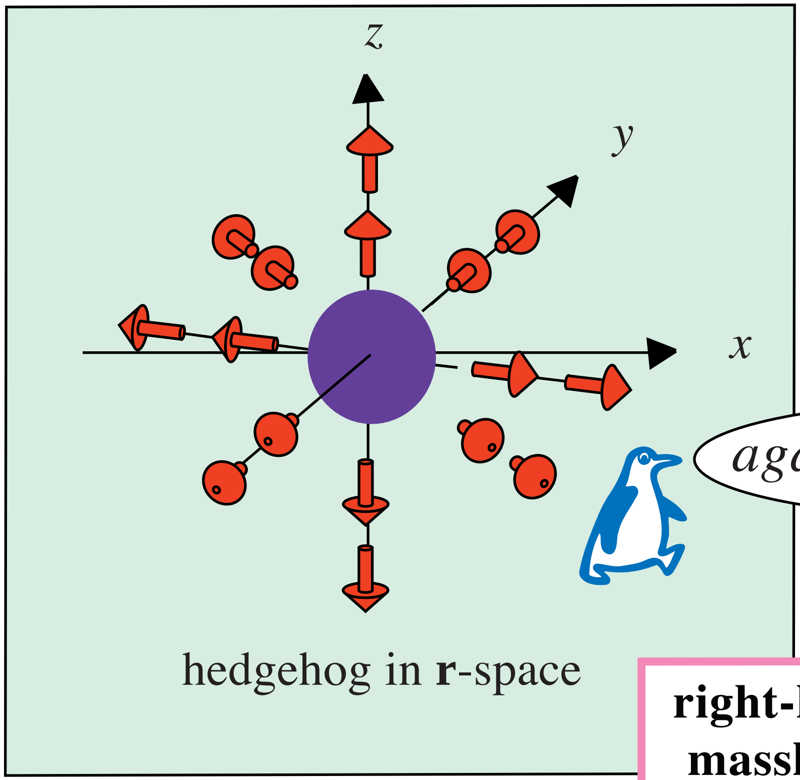
classes of mapping $S^1 \rightarrow GL(\mathrm{n,C})$

space of
non-degenerate complex matrices

From Landau Fermi-liquid to Standard Model

From Fermi surface to Fermi point

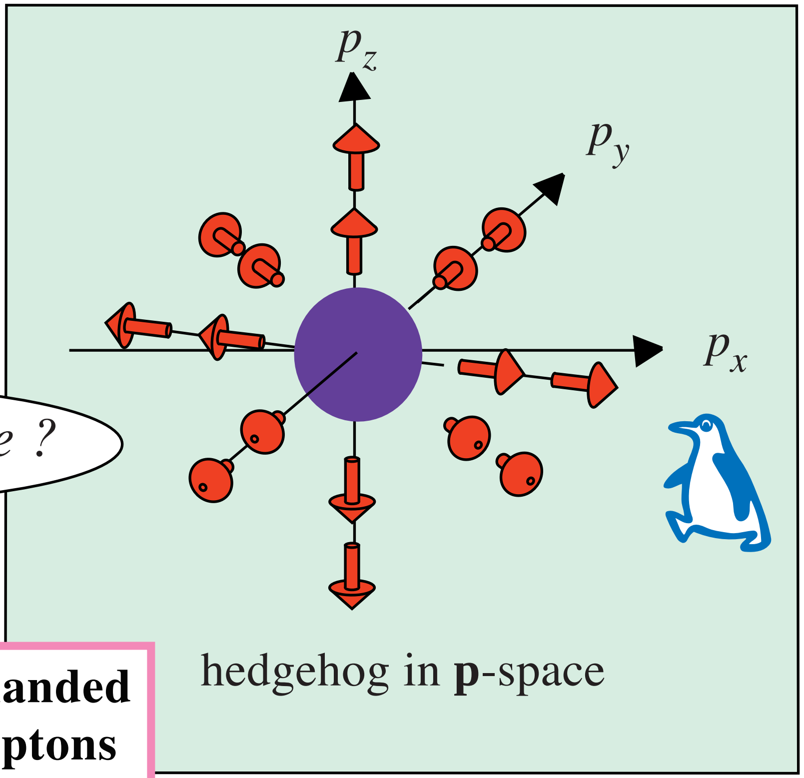
magnetic hedgehog vs right-handed electron



hedgehog in **r**-space

$$\sigma(\mathbf{r})=\hat{\mathbf{r}}$$

again no difference ?



hedgehog in **p**-space

$$\sigma(\mathbf{p}) = \hat{\mathbf{p}}$$

right-handed and left-handed massless quarks and leptons are elementary particles in Standard Model

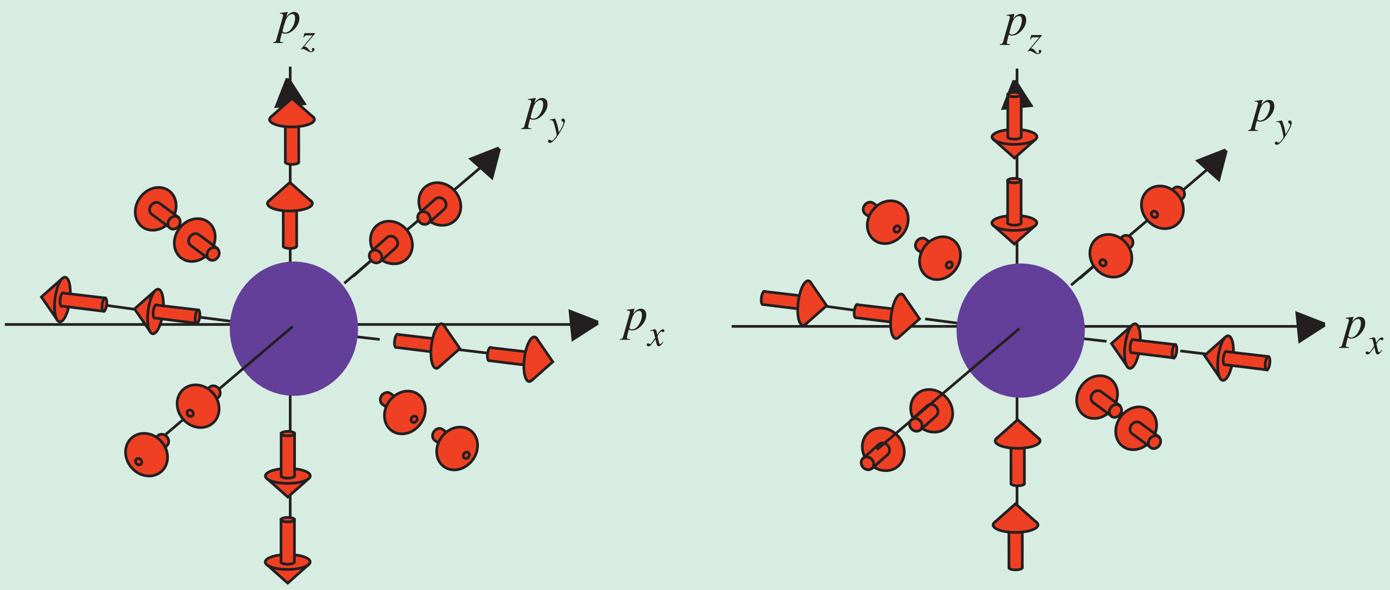
Landau CP symmetry is emergent

close to Fermi point

$$H = + c\, \sigma \cdot \mathbf{p}$$

right-handed electron =
hedgehog in **p**-space with spines = spins

Topological invariant for right-handed and left-handed elementary particles



hedgehog with spines (spins)
outward ($N_3 = +1$)

hedgehog with spines (spins)
inward ($N_3 = -1$)

right neutrino

$$H = +c\,\sigma\cdot\mathbf{p}$$
$$\mathbf{g}(\mathbf{p}) = +c\mathbf{p}$$

$$\boxed{H = \sigma\cdot\mathbf{g}(\mathbf{p})}$$

$$H = -c\,\sigma\cdot\mathbf{p}$$
$$\mathbf{g}(\mathbf{p}) = -c\mathbf{p}$$

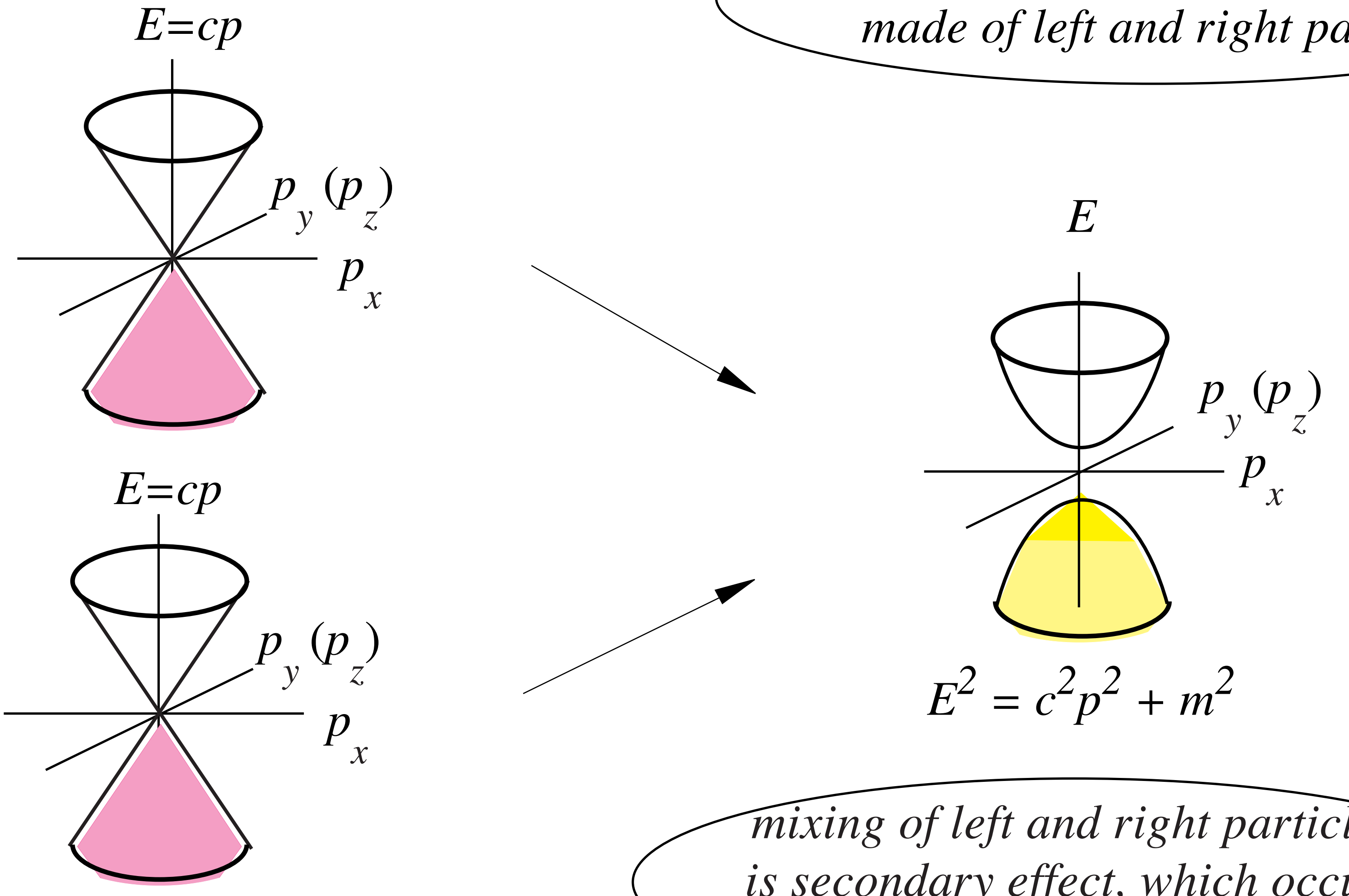
left neutrino

$$N_3 = \frac{1}{8\pi}\, e_{ijk} \int dS^i\, \hat{\mathbf{g}}\cdot(\partial^j \hat{\mathbf{g}} \times \partial^k \hat{\mathbf{g}})$$

over 2D surface
around Fermi point

where are Dirac particles?

Dirac particle - composite object
made of left and right particles



$$E^2 = c^2p^2 + m^2$$

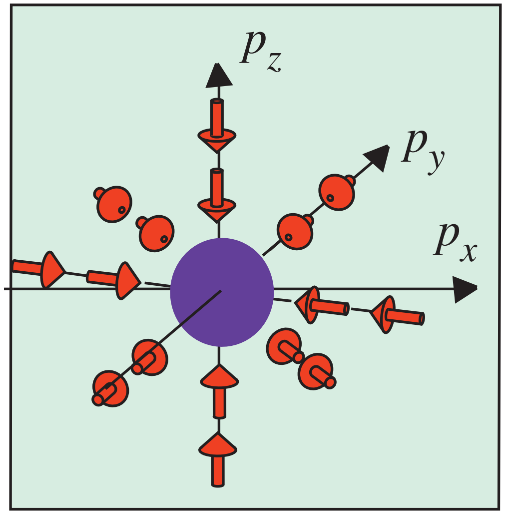
mixing of left and right particles
is secondary effect, which occurs
at extremely low temperature

$$T_{\mathrm{ew}} \sim 1\ \mathrm{TeV} \sim 10^{16}\mathrm{K}$$

Chiral fermions in Standard Model

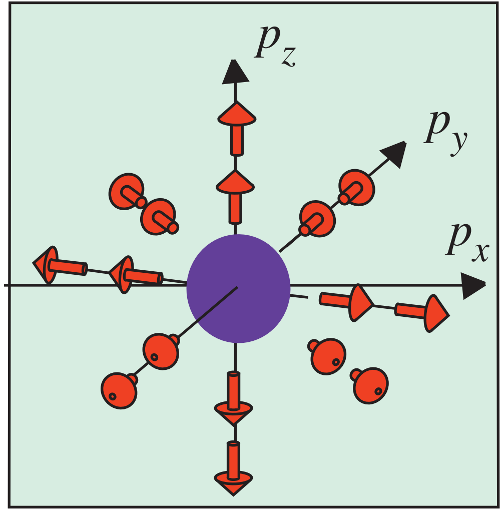
Family #1 of quarks and leptons

left particles



hedgehog with spines (spins) inward ($N_3 = -1$)

right particles



hedgehog with spines (spins) outward ($N_3 = +1$)

+2/3 u_L +1/6	-1/3 d_L +1/6	quarks	+2/3 u_R +2/3	-1/3 d_R -1/3
+2/3 u_L +1/6	-1/3 d_L +1/6	$SU(3)_C$	+2/3 u_R +2/3	-1/3 d_R -1/3
+2/3 u_L +1/6	-1/3 d_L +1/6		+2/3 u_R +2/3	-1/3 d_R -1/3
$SU(2)_L$				
0 ν_L -1/2	-1 e_L -1/2	leptons	0 ν_R 0	-1 e_R -1

$$H = -c\,\sigma\cdot\mathbf{p} \qquad H = +c\,\sigma\cdot\mathbf{p}$$

$$N_3 = -1 \qquad N_3 = +1$$

$$N_3 = \frac{1}{24\pi^2} e_{\mu\nu\lambda\gamma}\,\mathbf{tr} \int dS^\gamma\, \mathbf{G}^\mu\, \mathbf{G}^{-1}\, \mathbf{G}^\nu\, \mathbf{G}^{-1} \mathbf{G}^\lambda\, \mathbf{G}^{-1}$$

over 3D surface S in 4D momentum space

general topological invariant in terms of Green's function

life exists at low T because Fermi points are stable ?

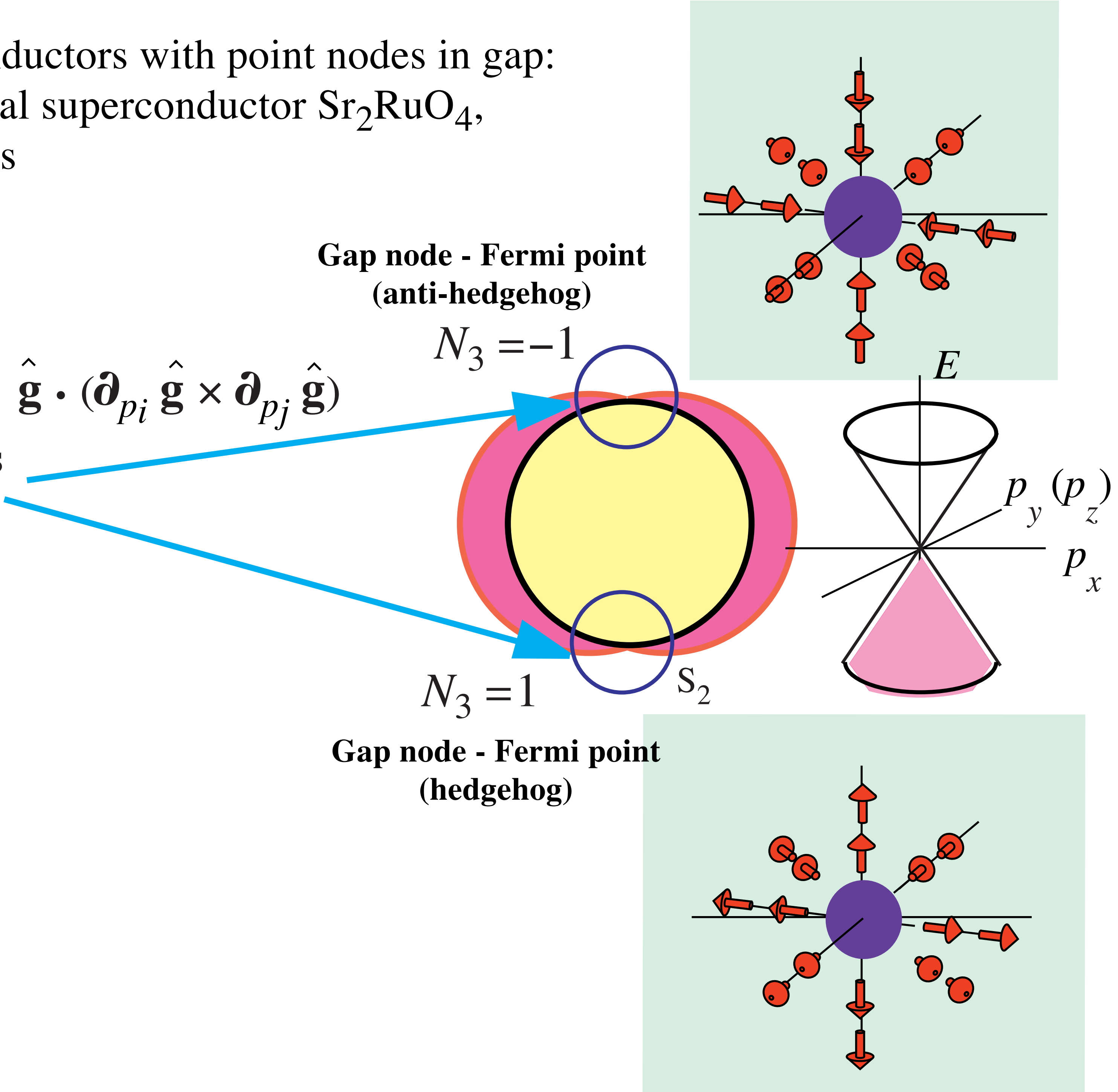
right !

examples of Fermi points in condensed matter

superfluids & superconductors with point nodes in gap:
superfluid ^{3}He-A, chiral superconductor Sr_2RuO_4,
triplet cold Fermi gases

$$N_3 = \frac{1}{8\pi} e_{ijk} \int dS^k \; \hat{\mathbf{g}} \cdot (\boldsymbol{\partial}_{p_i} \hat{\mathbf{g}} \times \boldsymbol{\partial}_{p_j} \hat{\mathbf{g}})$$

over 2D surface S
in 3D p-space



Gap node - Fermi point
(anti-hedgehog)

Gap node - Fermi point
(hedgehog)

emergence of relativistic particles

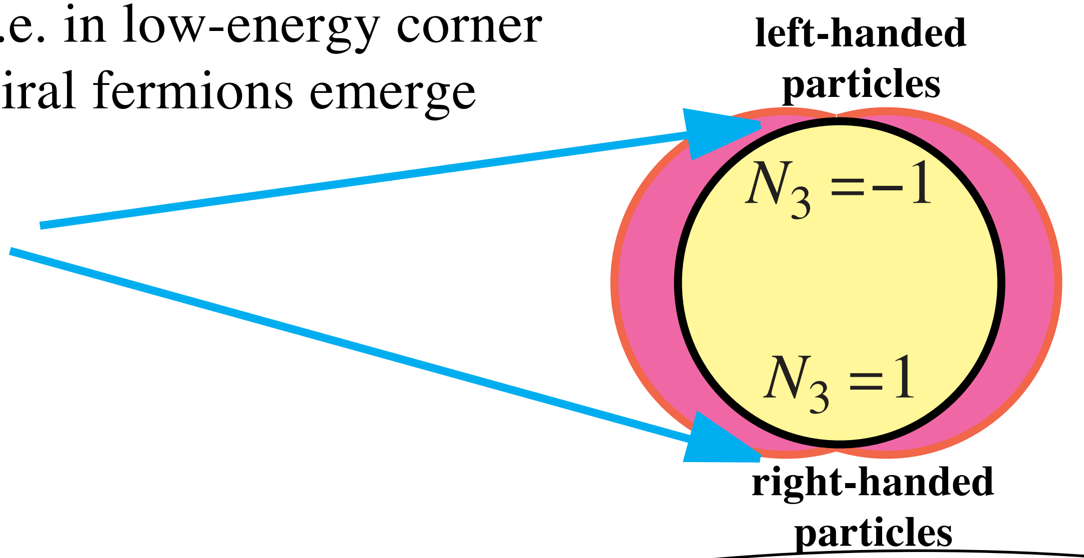
original non-relativistic Hamiltonian

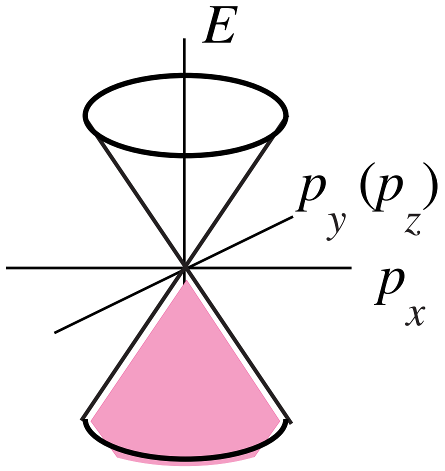
$$\mathrm{H} = \begin{pmatrix} \frac{p^2}{2m} - \mu & c(p_x + ip_y) \\ c(p_x - ip_y) & -\frac{p^2}{2m} + \mu \end{pmatrix} = \begin{pmatrix} g_3(\mathbf{p}) & g_1(\mathbf{p}) + i\, g_2(\mathbf{p}) \\ g_1(\mathbf{p}) - i\, g_2(\mathbf{p}) & -g_3(\mathbf{p}) \end{pmatrix} = \boldsymbol{\tau} \cdot \mathbf{g}(\mathbf{p})$$

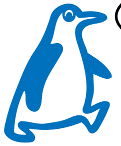
close to nodes, i.e. in low-energy corner
relativistic chiral fermions emerge

$$\mathrm{H} = N_3\, c\, \boldsymbol{\tau} \cdot \mathbf{p}$$

$$E = \pm\, cp$$

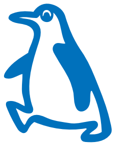




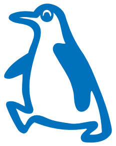


chirality is emergent ??

what else is emergent ?



top. invariant determines chirality in low-energy corner

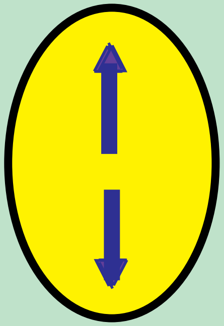


relativistic invariance as well

bosonic collective modes in two generic fermionic vacua

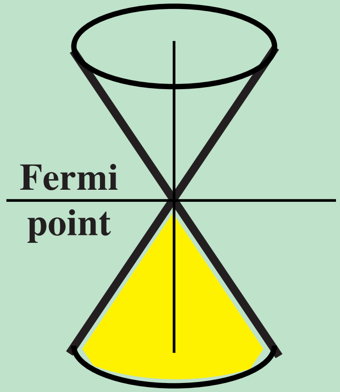
Landau theory of Fermi liquid

collective Bose modes
of fermionic vacuum:
propagating
oscillation of **shape**
of Fermi surface



Landau, ZhETF **32**, 59 (1957)

Standard Model + gravity

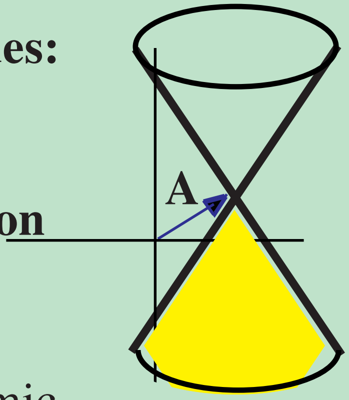


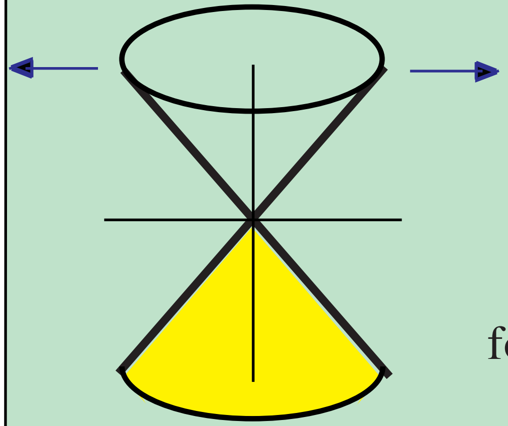
collective Bose modes:

propagating
oscillation of **position**
of Fermi point

$$\mathbf{p} \to \mathbf{p} - e\mathbf{A}$$

form effective dynamic
electromagnetic field

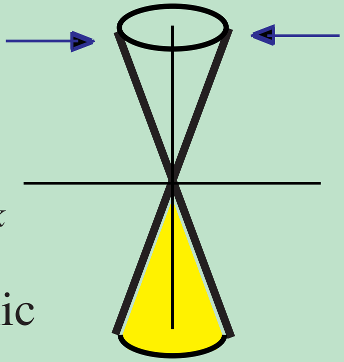




propagating
oscillation of **slopes**

$$E^2 = c^2p^2 \to g^{ik}p_i p_k$$

form effective dynamic
gravity field



two generic quantum field theories of interacting bosonic & fermionic fields

relativistic quantum fields and gravity emerging near Fermi point

Atiyah-Bott-Shapiro construction:
linear expansion of Hamiltonian near the nodes in terms of Dirac Γ-matrices

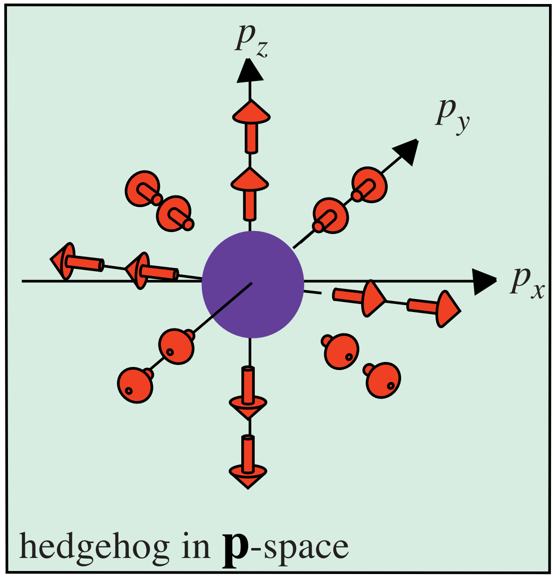
$$\boxed{E = v_F\,(p - p_F)}$$

linear expansion near
Fermi surface

emergent relativity

$$\boxed{H = e_i^k\,\Gamma^i \cdot (p_k - p_k^0)}$$

linear expansion near
Fermi point



hedgehog in **p**-space

$$\boxed{g^{\mu\nu}(p_\mu - eA_\mu - e\tau\cdot\mathbf{W}_\mu)(p_\nu - eA_\nu - e\tau\cdot\mathbf{W}_\nu) = 0}$$

effective metric:
emergent gravity

effective
electromagnetic
field

effective
$SU(2)$ gauge
field

effective
electric charge

$e = +1$ or -1

effective
isotopic spin

all ingredients of Standard Model :
chiral fermions & gauge fields
emerge in low-energy corner
together with spin, Dirac Γ−matrices, gravity & physical laws:
Lorentz & gauge invariance, equivalence principle, etc

gravity & gauge fields
are collective modes
of vacua with Fermi point

Extension of Landau idea:
electric charge purely from vacuum polarization

Landau proposal for QED

Landau, 1955

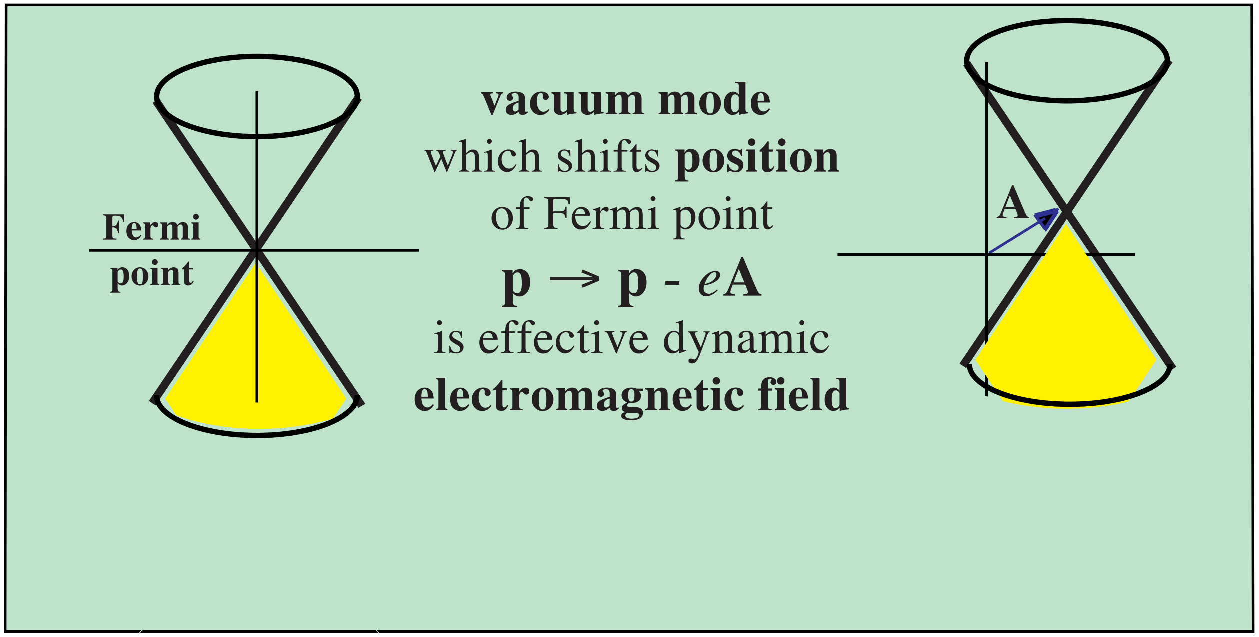
$$\boxed{\frac{1}{e^2} = \frac{\nu}{3\pi} \ln \left(E^2_{\rm Planck} / m^2\right)}$$

ν number of charged particles
$E_{\rm Planck}$ Planck energy
m electron mass

$$\boxed{\nu = 12}$$

$$\boxed{N_F = 4}$$

extension to Standard Model which as effective theory has two different cut-off for bosons & fermions



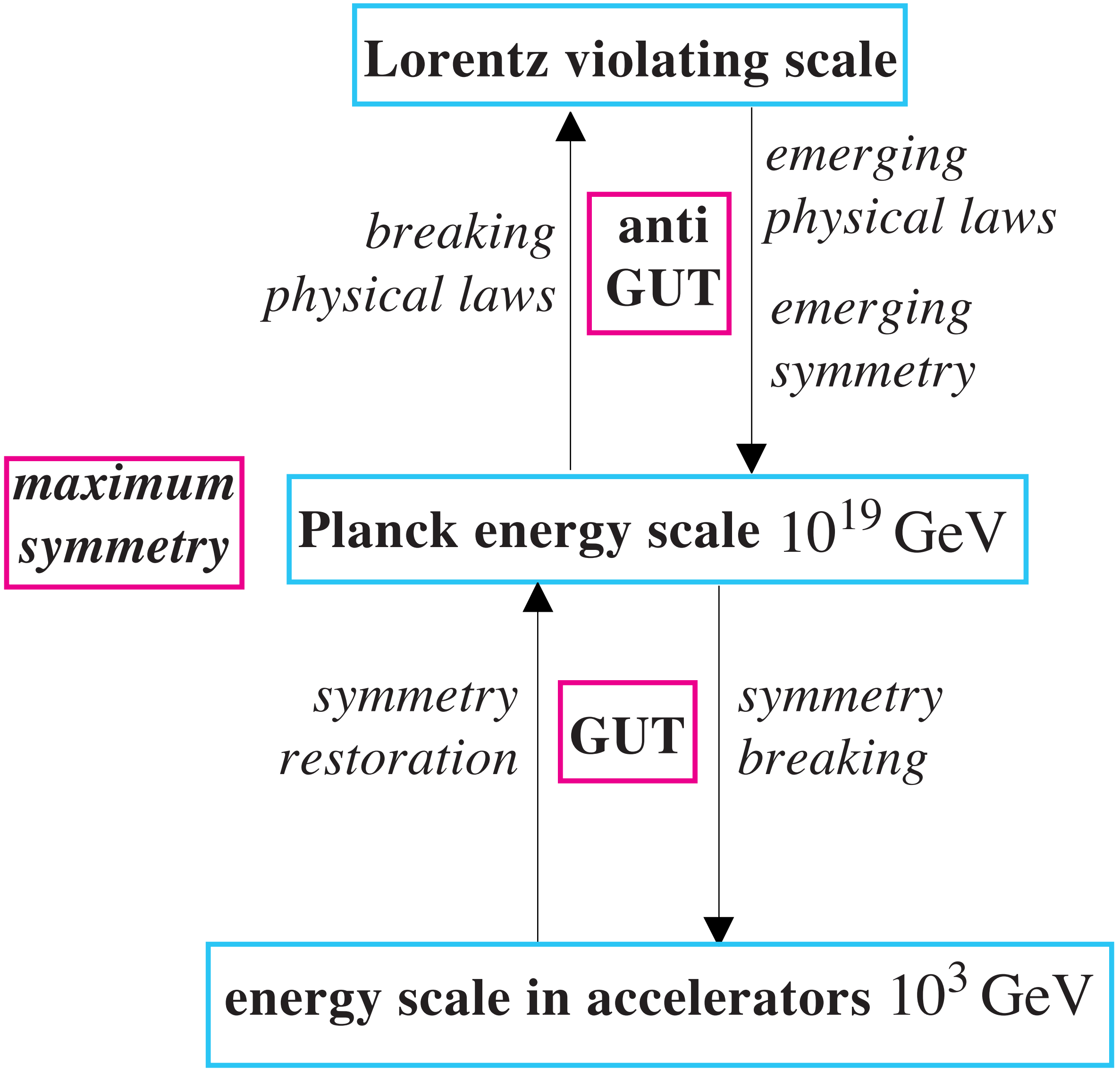
vacuum mode
which shifts **position**
of Fermi point
$\mathbf{p} \to \mathbf{p} - e\mathbf{A}$
is effective dynamic
electromagnetic field

$$\boxed{\frac{1}{e^2} = \frac{8N_F}{9\pi} \ln \left(E^2_{\rm UV} / m^2_{\rm Z}\right) - \frac{11}{6\pi} \ln \left(E^2_{\rm Planck} / m^2_{\rm Z}\right)}$$

N_F number of families
$E_{\rm UV}$ ultraviolet cutoff
$m_{\rm Z}$ mass of Z-boson

Klinkhamer-Volovik
JETP Lett. **81** (2005) 551

physics at the intermediate Planck scale is Lorentz invariant



Lorentz violating scale

breaking physical laws

anti GUT

emerging physical laws

emerging symmetry

maximum symmetry

Planck energy scale 10^{19} GeV

symmetry restoration

GUT

symmetry breaking

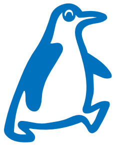
energy scale in accelerators 10^{3} GeV

Theory of quantum vacuum at Planck scale should be Lorentz invariant

crossover from Landau 2-fluid hydrodynamics to Einstein general relativity

they represent two different limits of hydrodynamic type equations

equations for $g^{\mu\nu}$ depend on hierarchy of ultraviolet cut-off's:
Planck energy scale E_{Planck} vs Lorentz violating scale E_{Lorentz}

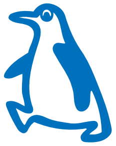


$E_{\text{Planck}} >> E_{\text{Lorentz}}$

emergent Landau two-fluid hydrodynamics

$E_{\text{Planck}} << E_{\text{Lorentz}}$

emergent general covariance & general relativity



^{3}He-A with Fermi point

$E_{\text{Lorentz}} << E_{\text{Planck}}$

$E_{\text{Lorentz}} \sim 10^{-3}\ E_{\text{Planck}}$

Universe

$E_{\text{Lorentz}} >> E_{\text{Planck}}$

$E_{\text{Lorentz}} > 10^{9}\ E_{\text{Planck}}$

3. Extension of Landau ideas

vacuum as Lorentz invariant medium & application to cosmology

Thermodynamics & dynamics of quantum vacuum

superfluid He

$N = 10^{23}$ atoms

Galilean principles

quantum vacuum & superfluid helium
are macroscopic systems;
they obey macroscopic
thermodynamic & hydrodynamic laws
which follow from their symmetry

quantum
vacuum

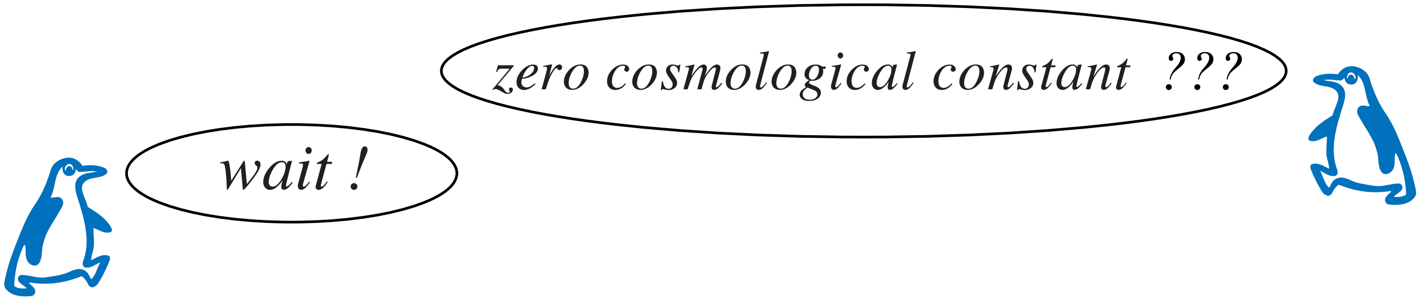
$N = 10^{180}$
"Planck units"

Lorentz invariance

Cosmological constant - weight of ether (vacuum vacuum)

> *"ether should not be thought of as endowed with the quality characteristic of ponderable media ... The idea of motion may not be applied to it."*

Einstein, 1920



perfect vacuum is weightless

"ether should not be thought of as endowed with the quality characteristic of ponderable media ... The idea of motion may not be applied to it."

Einstein, 1920

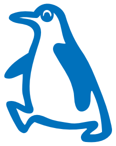
quantum vacuum

$N = 10^{180}$

"Planck units"

Lorentz invariance is guiding principle

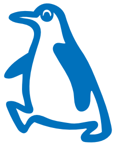
Lorentz invariant vacuum has no momentum



$$\begin{tikzcd}
\mathbf{P}_{\text{vac}} = 0 \arrow[d] & & \\
\mathbf{P}_{\text{vac}} = E_{\text{vac}}\mathbf{v}_{\text{vac}} \arrow[r] & E_{\text{vac}} = \int dV\, \varepsilon_{\text{vac}} = 0 \arrow[r] & \Lambda = \varepsilon_{\text{vac}} = 0
\end{tikzcd}$$

in Lorentz invariant vacuum cosmological constant $\Lambda = 0$

vacuum has no weight ***???***



wait !

weight of quantum vacuum

from two-fluid hydrodynamics of quantum vacuum

superfluid
ground state
+
elementary
excitations

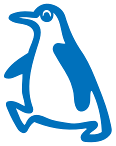
superfluid & normal
components

matter & other perturbations
violate Lorentz invariance
(preferred frame)
in the presence of matter
vacuum becomes ponderable

vacuum
+
elementary
particles
(matter)

"superfluid" & normal
components

vacuum has weight now:
its mass depends on matter



vacuum
+
non-gravitating
matter

$$\Lambda = \varepsilon_{vac} = P_{matter}$$

$$\mathbf{P}_{total} = \mathbf{P}_{vac} + \mathbf{P}_{matter} = \mathbf{v}\,\varepsilon_{total}$$
$$\varepsilon_{total} = \varepsilon_{vac} + \varepsilon_{matter}$$

vacuum + matter
obey Lorentz invariance,
while separately
they do not obey

vacuum
+
gravitating
matter

$$\Lambda = \varepsilon_{vac} = 0.5\,\varepsilon_{matter} + 1.5\,P_{matter}$$

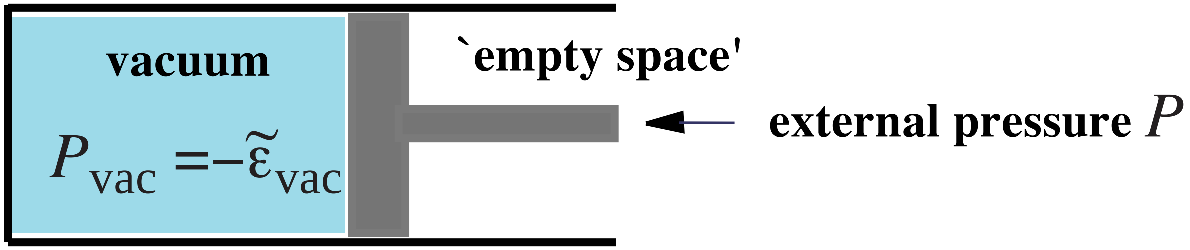
$$\mathbf{P}_{vac} = 0$$

Einstein static
closed Universe

vacuum momentum is still zero

extension of Landau ideas:

vacuum as Lorentz invariant medium & application to cosmology



F.R. Klinkhamer, G.E. Volovik
Self-tuning vacuum variable & cosmological constant
PRD **77**, 085015 (2008);
Dynamic vacuum variable & equilibrium approach in cosmology
arxiv: 0806.2805

$$\Lambda = \tilde{\varepsilon}_{\rm vac} = -P_{\rm vac}$$

energy density of vacuum **pressure of vacuum**

$$P_{\rm vac} = -\, dE/dV = -\, \tilde{\varepsilon}_{\rm vac}$$

$$\chi_{\rm vac} = -(1/V)\, dV/dP$$

compressibility of vacuum

$$\langle(\Delta P_{\rm vac})^2\rangle = T/(V\chi_{\rm vac})$$

$$\langle(\Delta\Lambda)^2\rangle = \langle(\Delta P)^2\rangle$$

pressure fluctuations

conclusions

natural value of Λ *determined by macroscopic physics*

$$\boxed{\Lambda \sim 0}$$

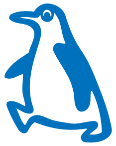
natural value of $\chi_{\rm vac}$ *determined by microscopic physics*

$$\boxed{\chi_{\rm vac} \sim E^{-4}_{\rm Planck}}$$

volume of Universe is large:

$$\boxed{V > T_{\rm CMB}/(\Lambda^2\chi_{\rm vac})}$$

$$V > 10^{28}\ V_{\rm hor}$$

thermodynamics & dynamics of Lorentz invariant vacuum

energy density $\boxed{\varepsilon_{\rm vac}\,(u^\mu{}_\nu)}$ **of vacuum is function of** $\boxed{u^\mu{}_\nu = \nabla_\nu u^\mu}$

equilibrium vacuum is obtained from equation

$$\boxed{\delta\varepsilon_{\rm vac}\,/\delta u^\mu = \nabla_\nu\,(\delta\varepsilon_{\rm vac}\,/\delta u^\mu{}_\nu)=0}$$

equilibrium solution**:**

$$\boxed{u_{\mu\nu} = u\, g_{\mu\nu}} \qquad u = const$$

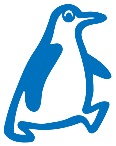
macroscopic vacuum energy:
from energy momentum tensor

$$\boxed{T_{\mu\nu} = \delta S\,/\delta g^{\mu\nu} = (\ \varepsilon_{\rm vac}\,(u) - u\, {\rm d}\varepsilon_{\rm vac}/{\rm d}u)\ g_{\mu\nu}}$$

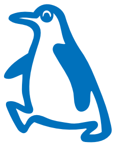
It is $T_{\mu\nu}$ **which is gravitating,**
thus cosmological constant is $\boxed{\Lambda=\varepsilon_{\rm vac}\,(u) - u\, {\rm d}\varepsilon_{\rm vac}/{\rm d}u}$

microscopic vacuum energy

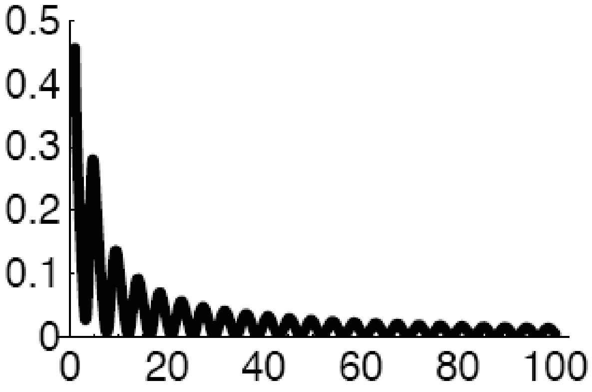
$$\boxed{\Lambda=\varepsilon_{\rm vac}\,(u) \sim E^4_{\bf Planck}???}$$



naive estimation of vacuum energy
& cosmological constant
highly disagrees with observations
(Cosmological Constant Problem)



dynamics of cosmological constant:
from Planck scale to present value



microscopic vacuum energy has **natural Planck scale:**

$$\boxed{\varepsilon_{\mathrm{vac}}(u) \sim E^{4}_{\mathbf{Planck}}}$$

macroscopic vacuum energy

$$\boxed{\varepsilon_{\mathrm{vac}}(u) - u\, \mathrm{d}\varepsilon_{\mathrm{vac}}/\mathrm{d}u = -P_{\mathrm{vac}}}$$

$$\boxed{\Lambda = \varepsilon_{\mathrm{vac}}(u) - u\, \mathrm{d}\varepsilon_{\mathrm{vac}}/\mathrm{d}u = -P_{\mathrm{vac}} = 0}$$

two huge quantities naturally cancel each other

due to **thermodynamics**

macroscopic energy & **cosmological constant**
have **natural zero value**

compare with
any condensed matter

$$\boxed{\varepsilon(\rho) \textit{ is atomic}}$$

$$\boxed{\varepsilon(\rho) - \rho\, d\varepsilon/d\rho = -P \quad \textit{is macroscopic}}$$

in the absence of environment:

$$\boxed{\varepsilon(\rho) - \rho\, d\varepsilon/d\rho = -P = 0}$$

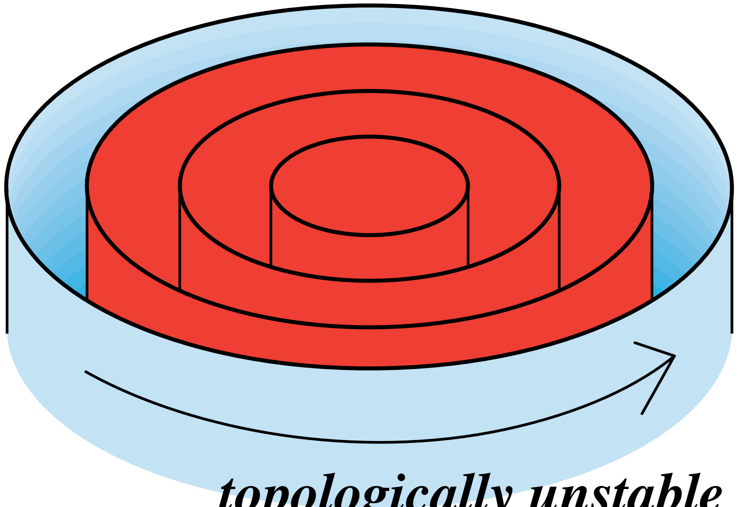
two microscopic quantities cancel each other

due to **thermodynamics**

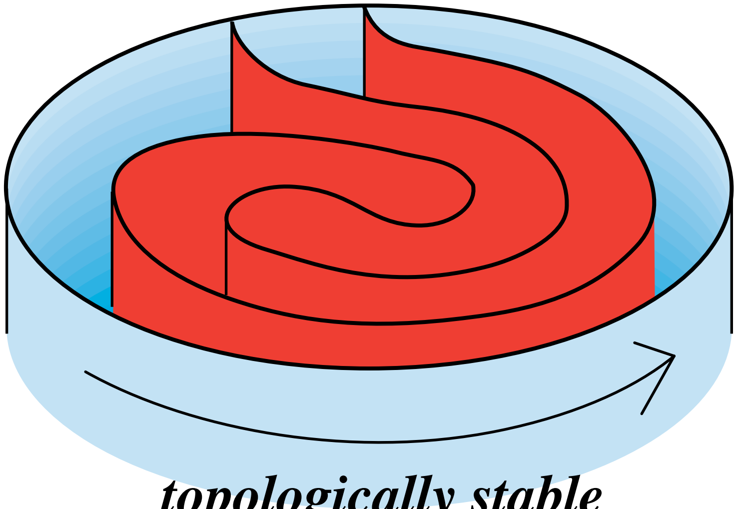
missing topology

Landau-Lifshitz vortex sheet
suggested for rotating superfluid 4He
DAN 100 (1955) 669

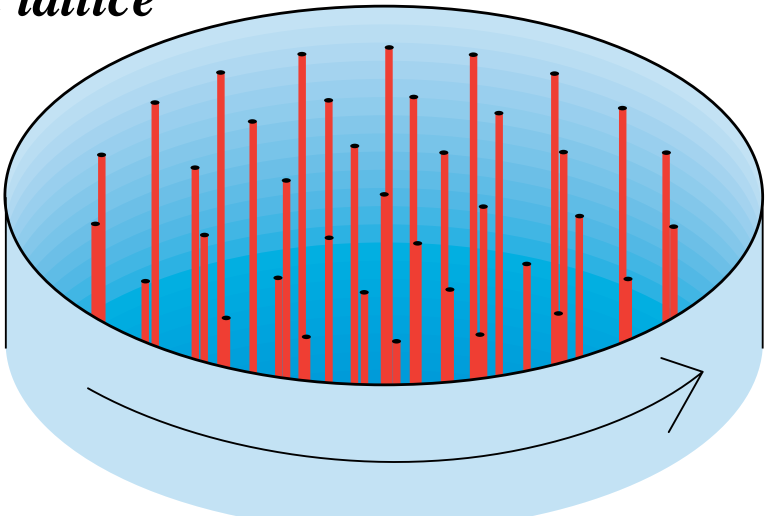
Landau-Lifshitz vortex sheet
observed in rotating superfluid 3He-A
PRL 72 (1994) 3839



topologically unstable
towards vortex lattice

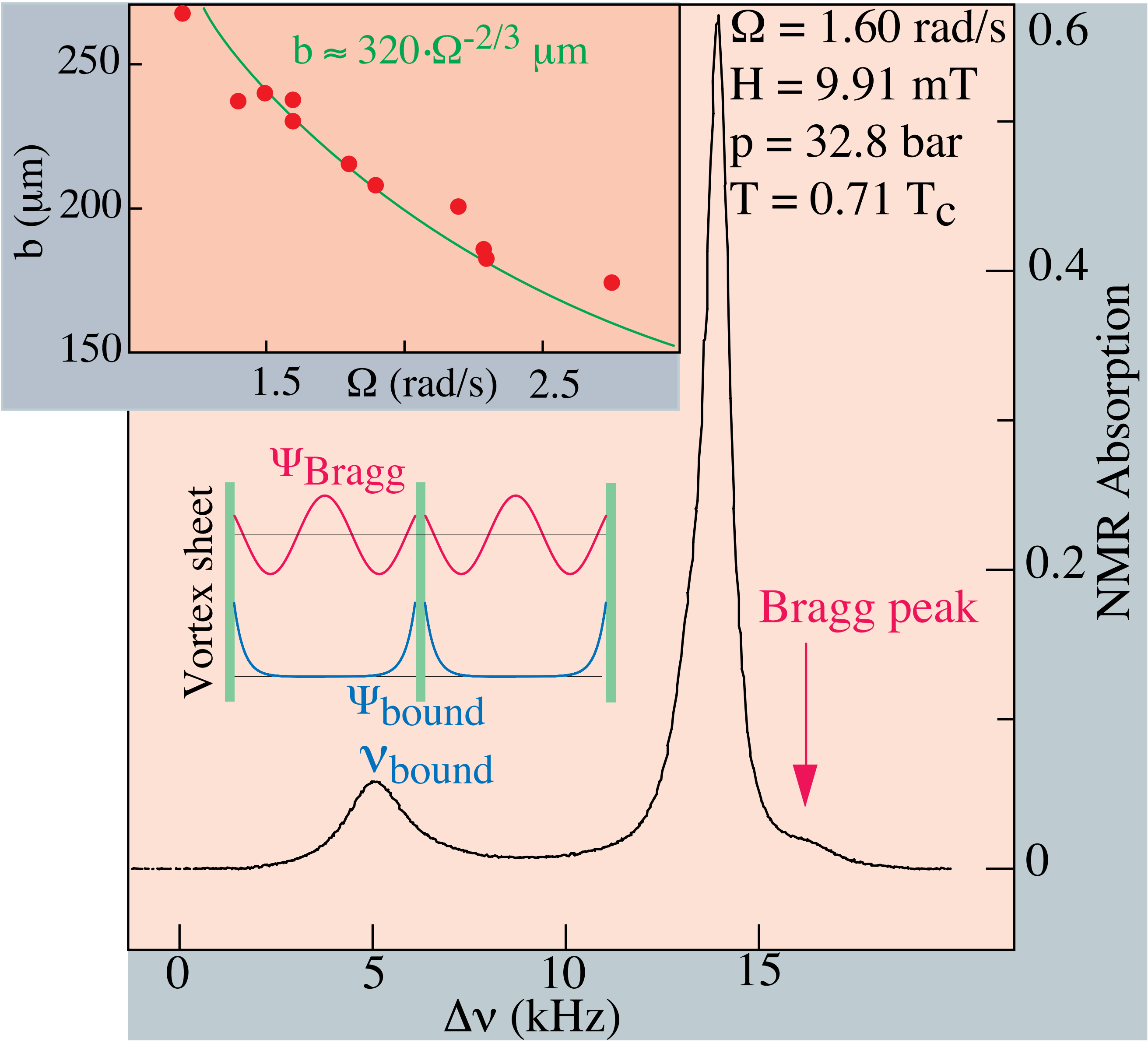


topologically stable



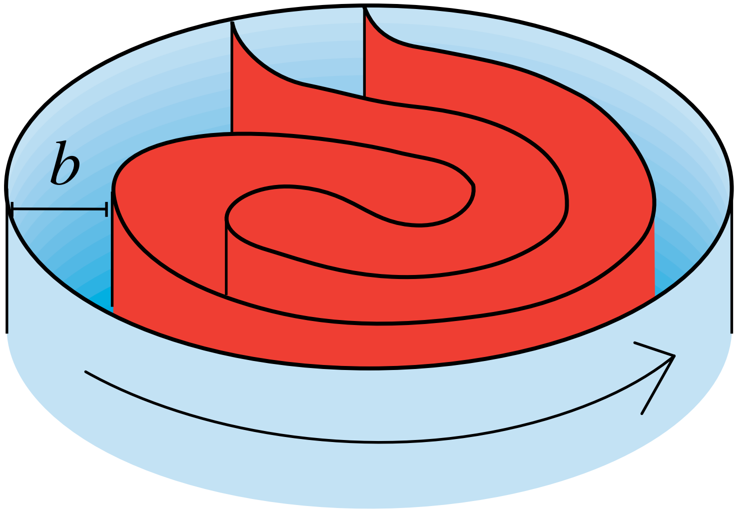
topologically stable

Helsinki NMR experiments:
satellite peaks from spin waves localized
in & between Landau-Lifshitz vortex sheets in ^{3}He-A



Landau-Lifshitz formula:
distance *b* between the sheets
as function of rotation velocity Ω

$$b = (3\sigma/\rho_s\Omega^2)^{1/3}$$



Theoretical value
from LL formula

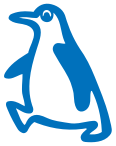
$b \approx 360 \cdot \Omega^{-2/3}$ μm

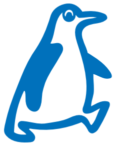
Helsinki experiment

$b \approx 320 \cdot \Omega^{-2/3}$ μm

Conclusion

Landau two-fluid hydrodynamics
& Einstein general relativity
are effective hydrodynamic theories:
they are two different extreme limits
of parameters in underlying microscopic theory





Landau theory of Fermi liquid
& Standard Model of electroweak & strong interactions
are effective theories
for two major classes of fermionic vacua:
vacua with Fermi surface
(normal 3He and metals)
& vacua with Fermi point
(relativistic quantum vacuum & superfluid 3He-A)

Landau ideas first applied to quantum liquids
are applicable to quantum vacuum -
the modern aether