Microscopic solitons in correlated electronic systems: theory versus experiment.

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Abstract. Symmetry broken electronic states give rise to topological defects: from extended domain walls - "stripes" as solitonic lattices to microscopic solitons as anomalous quasi-particles and instantons in their dynamics. We shall collect and interpret experimental evidences on existence of microscopic solitons, and their determining role in electronic processes of quasi-1D electronic crystals. Thus, the ferroelectric charge ordering in organic conductors gives access to several types of solitons observed in conductivity (holons) and in permittivity (polar kinks), to solitons' bound pairs in optics, to compound charge-spin solitons. In charge density waves, the individual phase solitons have been visually captured in recent STM experiments. The resolved subgap tunneling spectra recover these solitons (in aggregated form of dislocations in statics and as instantons - the phase slips in dynamics), as well as the amplitude kinks - the spinons.

The theory relies upon the regime of quantum dissipation provided by soft mode emittance in the course of the soliton creation, and on effects of dimensional crossover. With onset of a 2D or a 3D long range order, the topologically nontrivial solitons experience the confinement resulting in the spin-charge recombination. It originates the symmetry broken spin- or charge- roton configurations with charge- or spin- kinks localized in the core, correspondingly for cases of repulsion and attraction. These complex excitations can be viewed as nucleuses of the melted stripe phases, which appears in doped antiferromagnetic - Mott insulators or in spin-polarized superconductors and charge density waves.

Keywords: topological defect, soliton, vortex, stripe, tunneling, superconductivity, CDW, SDW, AFM, FFLO, charge ordering.
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INTRODUCTION.

Possibility of constructing elementary particles from continuous matter - fields, is the old paradigm in physics [1]. Such hypothetical objects, known as solitons (or instantons for related transient processes) have been discussed in field theory, high energy physics, cosmology, in conjugated polymers and other quasi one-dimensional (1D) conductors, in biological macro-molecules, etc.

Most of strongly correlated electronic systems show various types of symmetry breaking giving rise to degenerate ground states. The degeneracy allows for topologically nontrivial perturbations exploring the possibility of traveling through different allowed ground states. Their most known forms are plain domain walls, vortex lines or dislocations, which are still macroscopic objects extending in two or three dimensions (see [2, 3] for general views and sophisticated applications). Of our special interest, however, are totally localized and truly microscopic objects which energies and quantum numbers are on the one-electron scale. These subgap particles would determine the observable properties, which are usually ascribed to conventional electronic excitations.

Role of solitons in electronic properties was appreciated in theories since mid 70's (see reviews [4, 5], and also [6, 7] for a more recent development). The solitons were firstly accessed in experiments on conducting polymers of early 80's [8]. New motivations came in early 2000's from discoveries of the ferroelectric charge ordering ([10, 11], and a review [12]) in organic conductors [9] , from new accesses via nano-scale experiments [13, 14] in materials with Charge Density Waves (CDW) [15], from optics of new conducting polymers [16]. Today, various solitons show up in conductivity, tunneling spectroscopy, optical absorption. Instantons - the corresponding dynamical processes - are responsible for subgap transitions leading to a pseudogap formation [17, 18, 19].

The content of this review will be mostly endorsed by events in Electronic Crystals, see [20, 21], especially cases of Charge/Spin Density Waves, Charge Ordering/Disproportionation, Wigner crystals, and stripes. We shall review the last decade experiments confirming the solitons in organic conductors with the ferroelectric charge ordering. Three different types of solitons show up in experiments: a single-charged amplitude king - the "holon" in a general terminology of strongly repulsive electronic systems, the polarization kink carrying a fractional charge, and the topologically bound charge-spin soliton. We shall interpret new experiments on direct observations of microscopic solitons in processes of the coherent internal tunneling in CDWs [13, 14]. There are amplitude solitons corresponding to the long sought special quasi-particle - the spinon. The same experiment also gives access to the junction reconstruction via spontaneous creation of the grid of phase solitons. The low energy tunneling recovers the quantum phase slips at adjacent chains.

We shall recall the theory [19] of internal coherent tunneling in the pseudogap region. The instanton approach allows to calculate the interchain tunneling current both in single electron (amplitude solitons) and bi-electron (phase solitons) channels.

OBSERVATIONS OF SOLITONS IN CHARGE-GAP SYSTEMS

Ferroelectricity and charge ordering in quasi 1D organic conductors.

This decade, a true workshop on solitons was opened in organic conductors from the family $(TMTCF)_2X$, see [9] - chapter [12]. The facility is provided by the discovery of the ferroelectricity endorsed by the charge ordering [10, 11, 22]. The zoo of solitons is largely accessed thanks to possibility of switching on/off of the Mott state by means of the charge ordering.

This is the state with no magnetic long range order: spin degrees of freedom are gapless and split-off, charge degrees of freedom are described by the chiral phase $\varphi(x,t)$ which we define as for a conventional $2K_F$ CDW $\sim \cos(\varphi + 2K_F x)$, and the phase Hamiltonian becomes

$$H = (\hbar/4\pi\gamma)[v(\partial_x\varphi)^2 + (\partial_t\varphi)^2/v] - U\cos(2\varphi - 2\alpha)$$

Here v is a phase velocity, and the spontaneous phase center shift α is the signature of the ferroelectricity, U is the Umklapp scattering amplitude [23] which is responsible for the charge gap formation. The constant γ (usually called the Luttinger liquid parameter - K_ρ) determines several regimes: $\gamma < 1$ at any repulsion, then U is preserved if it is already build-in, - this is the generic Mott state case for a half-filled band; $\gamma < 1/2$ at a stronger repulsion, then U is spontaneously generated even away from the bare half-filling, - this is our Charge Ordering case.

The multiple ground state degeneracy gives rise to a variety of solitons. Even at the given charge ordering amplitude, the ground state energy $H_U = -U\cos(2\varphi - 2\alpha)$ is doubly degenerate between $\varphi = \alpha$ and $\varphi = \alpha \pm \pi$ (we assume $U > 0$). It allows for phase $\mp\pi$ solitons - "holons" with the charge e and no spin. The quantum Sine-Gordon model is exactly solvable; the excitation spectrum and even transitions intensities are known, see [24, 25]. Just at the border line $\gamma = 1/2$ of the Charge Ordering regime, the spectrum changes giving rise to kink's bound states - breathers.

Indirect evidences on solitons come already from conductivity and thermodynamics: separation of charge from spin; enhanced activation energy for interchain transport. But particularly intriguing is the observation of solitons in dynamics - via optics for organics (and conducting polymers) and via tunneling (by now only in CDWs, see the next section).

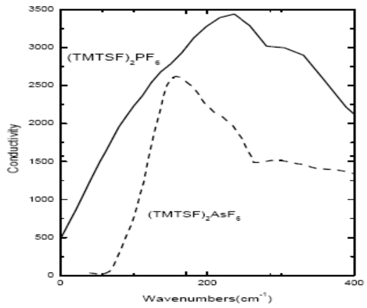

FIGURE 1. Optical absorption in two organic conductors from the family $(TMTSF)_2X$ showing a coexistence of the high frequency features inherent to the 1D Mott insulator state, and the low energy metallic rise. We interpret the major peak as the excitation of the two-kink bound state, while the shoulder above it should be the threshold for creating unbound pairs. The lower curve, for $X = AsF_6$, shows an almost ideal example, while the upper, for $X = PF_6$, curve is complicated by an additionally symmetry breaking (the spin-Peierls transition) at lower energies; it signifies the processes of spin-charge topological reconfinement described in the next subsection. Optical conductivity is replotted from data of M. Dressel group (unpublished, see [26, 27] for complementary information).

The interpretation of the Fig.1 corresponds to detailed theory for optics of solitons [24, 25] based upon the exact solution of the quantum sine-Gordon model. A general feature of the whole gapful regime $\gamma < 1$ is the two-particle gap $E_g = 2\Delta$ for creation of a pair of $\pm\pi$ solitons. But contrary to the common sense intuition, and to the elementary theory of semiconductors, the absorption intensity $I(\Omega)$ has no singularity at the threshold E_g. Optical density of states law $DOS \sim (\Omega - E_g)^{-1/2}$ is compensated by vanishing of the matrix element, then the absorption starts smoothly as $I \sim (\Omega - E_g)^{1/2}$. Moreover, for the generic Mott state without the charge ordering, $1/2 < \gamma < 1$, there would be no bound states, hence no absorption below E_g. But our case of the Mott state due to the spontaneous charge ordering requires $\gamma < 1/2$, hence a number of sharp peaks in absorption even below the two-particle gap $\Omega < 2\Delta$ - at a sequence of quantum breathers which are bound states of two solitons. This is what seems to be observed in experiment, Fig.1.

Further symmetry breaking: spin-charge reconfinement and compound solitons.

Some compounds from the $(TMTTF)_2X$ family show a subsequent, at lower temperature T_2 phase transition of the tetramerization which can be identified as the spin-

Peierls state; it gains the energy from opening of the spin gap Δ_s. We argue that this transition brings to life a special state of two topologically coupled solitons which explore both the charge and the spin sectors, thus realizing the spin-charge reconfinement. Within the reduced symmetry, the potential part of the Hamiltonian becomes

$$H_U = -U cos(2\varphi - 2\alpha) - V cos(\varphi - \beta)\cos\theta$$

Here θ is the spin chiral phase, such that θ'/π is the smooth spin density. The new, at $T < T_2$, V - term is the amplitude of the mixed ($\beta \neq 0$ due to inversion symmetry breaking of the ferroelectricity already at $T_{co} > T_2$). Its formation destroys the spin liquid which existed at $T > T_2$ on top of the charge ordering. Major effects of the V-term are a) to open the spin gap $2\Delta_s$ corresponding to creation of the triplet excitation as a new $\{\delta\theta = 2\pi,\ \delta\varphi = 0\}$ purely spin solitons; b) to prohibit former $\delta\varphi = \pi$ charged solitons - the holons, now they are confined in pairs bound by spin strings; c) to allow for compound spin-charge topologically bound solitons $\{\delta\varphi = \pi,\ \delta\theta = \pi\}$ which leave the Hamiltonian invariant. This particle bears the normal electron's quantum numbers: the charge e and the spin $1/2$, but their localization is different: sharply within $\hbar v/\Delta$ for the charge e, while loosely within $\hbar v/\Delta_s$ for the spin $1/2$.

SPIN-GAP CASES WITH CONTINUOUS SYMMETRY: INCOMMENSURATE CDW AND SUPERCONDUCTORS.

Common problems of excitations and stripes in CDWs and superconductors

Both CDWs and superconductors (SC) are the spin-singlet states, which are subject to deparing under the Zeeman splitting effect of the magnetic field [28]. Above a certain critical field, the ground state develops a periodic superstructure (see a short review and references in [6, 7]). In superconductors it is known as the FFLO state named after [29, 30], which has attracted a great deal of attention recently (see e.g. [31, 32]) because of events in organic and heavy-fermion superconductors, in cold atoms. Its formation is expected to be a very week effect in conventional superconductors, unless it is endorsed by strong coupling $\Delta \sim E_f$, or by flattened Fermi surfaces - particularly in quasi-1D case. Then the theories of early 80's (see [44] and the review [4] for CDWs, and [33] for superconductors), predict formation of solitonic lattices with unpaired spins localized at midgap states near the order parameter nodes. For CDWs there are convincing theoretical and experimental evidences, that beyond the coherent walls, also the separate amplitude solitons (the walls building blocks) exist as quasi particles - the spinons. Based upon the CDW notion, in superconductors we expect to find a tightly bound pair of half-integer vortices sharing one unpaired spin, as we shall describe below.

We start to show most typical and convincing evidences for existence of the gap in the excitation spectrum of singlet-ground-state electronic systems: superconductors and CDWs, Fig.2. Within the standard BCS - Bogolubov view, the excited states are the linear combinations of: electrons and holes at $\pm p$ for a SC, or of electrons or holes at $-p$ and $p + 2p_f$ for CDWs. Corresponding spectra are $E(k) = \pm(\Delta^2 + (v_f k)^2)^{1/2}$, $k = p - p_f$. But is it always true? It is proved for typical SCs - Fig.2a; left questionable for strong coupling SC cases - Fig.2b: high-T_c, real space pairs, cold atoms, bi-polarons; stays clearly incomplete for CDWs - Fig.2c, as proved by modern experiments ; is certainly inconsistent for 1D and even quasi 1D systems as proved theoretically - Fig.2d, [6, 7]. There are the instantons, the solitons and their arrays which are responsible for these confusions. A related complex of well established facts for incommensurate CDWs (ICDW) which are symmetrically equivalent to SCs is summarized in [34].

Solitons in incommensurate CDWs

While the charge ordering was a crystal of electrons, the ICDW is a crystal of electron pairs. Its lowest energy current carrier may be the charge-$2e$ defect of adding/missing one period at the defected chain. It is the $\pm 2\pi$ soliton of the ICDW order parameter $\mathcal{O}_{ICDW} = A\cos(2K_f x + \varphi)$, which has been recently captured and visualized in STM experiments [35], Fig.3.

The singlet pair can be broken into spin $1/2$ components, but it will not be an expectedly liberated electron-hole pair at $\pm\Delta_0$. Rather, there will be two spin carrying "amplitude solitons" (AS) - zeros of the order parameter distributed over the length $\xi_0 = \hbar v_F/\Delta_0$. ([37], see the Appendix). The unpaired electron is trapped at the midgap state associated to the amplitude soliton, with the energy $\approx 2\Delta_0/3$, the total charge 0, and the spin $1/2$, so this is the CDW realization of the spinon. The AS has been visualized, see Fig.3b, by the STM as a half-period defect [36], similarly to the full-period phase soliton of Fig.3a.

A regular lattice of ASs should appear in high magnetic field [38] as it might have been observed as the CDW superstructure [28] and in spin-Peierls systems (it was clearly seen by the NMR experiments [39]). Generalization of the spin-soliton lattice from the ICDW to the superconductor is the FFLO [29, 30] phase in spin-polarized superconductors. Then the same AS becomes

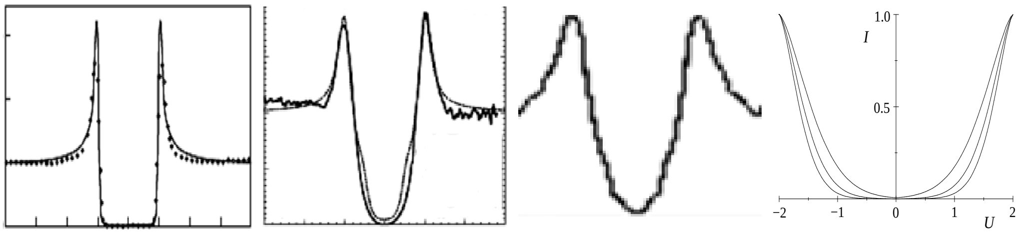

FIGURE 2. Deparing gaps from tunneling experiments - plots of the tunneling conductance dI/dU versus voltage U. Superconductors: Nb (a) and CaC_6 (b); CDW in $NbSe_3$ (c). The panel (d) - gives the theoretical prediction [19] for the instanton-mediated tunneling current $I(U)$ in the subgap region $|U| < 2\Delta$ (U is shown in units of Δ; the plots correspond to temperatures $T/\Delta = 1/4, 1/6, 1/8$).

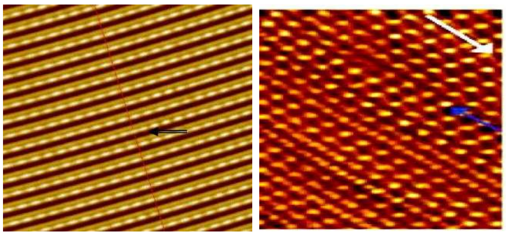

FIGURE 3. Left: Visualization of the 2π soliton - the prefabricated pair of electrons, by the STM on $NbSe_3$ [35]. At the (red) front line the defected chain is displaced by half of the period. Along the defected chain the whole period $\pm 2\pi$ is missed or gained. Right: STM visualization of the half-period soliton (the amplitude kink) in a quasi1D complex $Ni_{0.05}Pd_{0.95}Br$ [36]. White arrow: 1D chains direction, blue arrow: the defected chain.

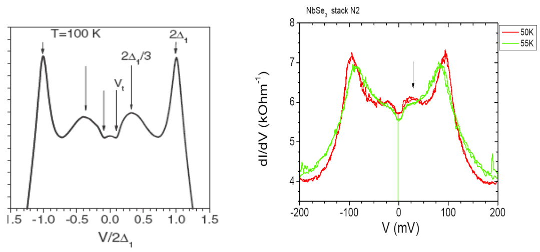

FIGURE 4. Tunneling in mesa-junctions of $NbSe_3$ [13, 14] - the material with two CDWs, shows a coexistence of several spectral feature: the peak at $2\Delta_0$ for the inter-gap creation of free e-h pairs, creation of the amplitude soliton at $E_{as} \approx 2\Delta_0/3$, the bi-particle spinless charge injection threshold at $V_t \ll \Delta_0$. a) the high T CDW1; b) the low T CDW2 [40].

an elementary stripe fragment in both cases of the CDW and FFLO, with a similarity [7] to holons versus stripes in doped AFM insulator.

Fig.4 quotes some results from a new type of internal interlayer tunneling experiments (see [13, 14, 40] for the CDW applications). Together with the expected sharp feature at the threshold 2Δ for free electron-hole pairs, we see the lower energy peak positioned close to the theoretical value $E_{AS} = 2/\pi\Delta$ for the amplitude soliton energy [13]. Moreover, at even lower energies forbidden for pair-breaking processes, we see the threshold V_t which terminates the bi-electron tunneling processes leading to formation of solitons captured in the Fig.3. Even the oscillating fine structure within the gap is not a noise - it records events of sequential entering into the junction of dislocation lines which are aggregates of 2π solitons [14].

SOLITONS CONFINEMENT AND THE LONG RANGE ORDER.

A major puzzle, as well as the inspiration, coming from the described experiments is that the amplitude solitons were observed within the low temperature ($T < T_c$) phase with the long range 3D order. The hidden obstacle is the effect of the confinement appearing in higher dimensions D>1 [7, 41]. Commuting between degenerate minima on only one chain would lead to a loss of the interchain ordering energy proportional to $L_\|$ - the length along the chain till the next defect. In case of discrete symmetries (only the amplitude kinks as in the above Z_2 case of charge ordering) the solitons are bound in topologically trivial pairs with an option for a subsequent phase transition to form cross-sample domain walls [42, 43]. But for a continuous symmetry, the gapless mode can cure the interruption from the amplitude kink.

At the 1D level, for systems with a complex order parameter like ICDW or SC, the amplitude soliton $A(x = -\infty) \Rightarrow -A(x = +\infty)$ performs the sign change of the order parameter $\mathcal{O}$ at an arbitrary $\varphi = cnst$. Even being favorable in energy in comparison with an electron, it cannot be created dynamically even in 1D, and is prohibited to exist even stationary at D>1. The resolution is to invoke the combined symmetry: the amplitude kink $A \Rightarrow -A$ coupled with the half-integer $\varphi \Rightarrow \varphi \pm \pi$ vortex of the phase rotation which compensates for the amplitude sign change. The resulting Spin-Roton complex allows for several interpretations: 2D view is a pair of π-vortices sharing the common core which bears one unpaired spin which stabilizes the state; 3D view is a ring of a half-flux vortex line, its center confines the spin; at any D>1 this is a nucleus of the melted FFLO phase in the spin-polarized superconductors.

FFLO phase in superconductors.

FFLO refers to an undulating phase in superconductors with an imbalanced spin population. In this abbreviation, FF and LO stand for articles [29] and [30]. The conventional homogeneous phase implies filling the excess spins to quasi-particle states above the gap, according to the Fig.5a copied from [29].

Modulated phases of the complex order parameter $\mathscr{O}$ with a wave number $q \neq 0$ have been suggested: FF - $\mathscr{O} \sim \exp(iqx)$, LO - $\mathscr{O} \sim \cos(qx)$. The appropriately chosen vector q (Fig.5b) erases mismatching at some (at all in a quasi-1D case) parts of the FS, hence extending existence of the superconducting phase against the Zeeman splitting in the magnetic field. This interpretation is valid for both suggestions FF and LO, and within the Ginzburg-Landau phenomenology their distinction seems to be an accidental matter of combinatorics. But for our goals, there is a particular insight to the LO case relevant to solitonic lattices. Planes of the order parameter zeros concentrate the excess spins providing the split intragap states which are able to accommodate unpaired electrons. This scenario is directly linked to the solitonic lattices in quasi-1D case. The exact solution in 1D [33] for the FFLO, equivalent to the one for the CDW [44], is shown in Fig.6.

Inverse rout: from stripes to solitons and fractional vortices.

If the solitonic lattice melts, then in 1D each element becomes a particle - the amplitude soliton i.e. the spinon. In 2D, the amplitude defect should be complemented by the pair of π-vortices, Fig.7. This quasi 1D picture is a secure generalization of the rigorous 1D picture. In general, the cost of creating a pair of vortices is $\sim E_\varphi \ln(L_\perp)$, where E_φ is a characteristic energy of phase deformations and the string length $L_\perp$ is the distance (in units of the interchain spacing) between the opposite π-vortices. This loss must be equilibrated by the gain $-\Delta^* L_\perp$ for the string formation, where $\Delta^* = \Delta - E_{AS} \sim \Delta$ is the energy yielded from accommodating unpaired electrons to the midgap states of the string. In the quasi 1D case, $E_\varphi \sim T_c < \Delta$ is given by the low phase-ordering temperature $T_c \ll \Delta$, then the total energy $E_\varphi \ln(L) - \Delta^* L_\perp$ keeps to be negative down to smallest elementary length - this is why the combined kink-roton complex is certainly a stable elementary quasi-particle. But for isotropic superconductors $E_\varphi \sim E_F$ which allows for only a large scale complex, at $L_\perp > E_F/\Delta^*$. The strong coupling limit $\Delta \sim E_F$ is necessary, which leaves this scenario for a bi-polaronic (real space pairing) superconductor or for a condensate of paired cold Fermi atoms.

In absence of a microscopic theory for strong coupling vortices (i.e. with only a single pair of intra-gap states - their number is $\sim E_F/\Delta$ [45]), we can rely upon existing [47] numeric modeling (still performed within the weak coupling BCS scheme). It confirms indeed, that at presence of unpaired spins the usual integer 2π vortex, created by rotation (magnetic field), splits into two π-vortices, see Fig.8.

The energetics behind the vortex splitting is clear. A $2\pi N$ vortex energy $\sim N^2$, and it can be decreased by fragmentation, normally down to N of 2π vortices. But even a further splitting into two half-integer vortices becomes allowed if the amplitude domain wall opens between the split cores. At the macroscopic level, one can find analogies among richness of topological properties of the superfluid ^3He [3, 46]

A hole in the AFM environment.

Consider the quasi-1D system with repulsion at a nearly half filled band, which is the SDW rout to a general doped antiferromagnetic Mott-Hubbard insulator [48]. The 1D bosonized Hamiltonian can be written as () augmented by part for the gapless spin mode $\sim \partial\theta)^2$. In 1D, the excitations are the *(anti)holon* as the $\pm\pi$ soliton in φ, and the spin sound in θ, which are decoupled. At D>1, below the SDW ordering transition, the order parameter (the staggered magnetization $< S_x + iS_y >$) is $\mathscr{O}_{sdw} \sim \cos\varphi \exp(i\theta)$. Hence, the $\pi-$ soliton in φ becomes the amplitude kink ($\cos\varphi \to -\cos\varphi$), and to survive in D>1 it should enforce the π rotation in θ, then the sign changes in both of the two factors composing $\mathscr{O}_{sdw}$, compensate and the configuration becomes allowed. Such a combined semi-vortex - Fig.9 might be significant also in sliding incommensurate SDWs which processes an even richer order parameter [49].

CONCLUSIONS.

Existence of solitons is proved experimentally in single- and bi-electronic processes of CDWs in several quasi 1D materials. Solitons take over band electrons in the role of primary excitations - charge or spin carriers. They feature self-trapping of electrons into mid-gap states and

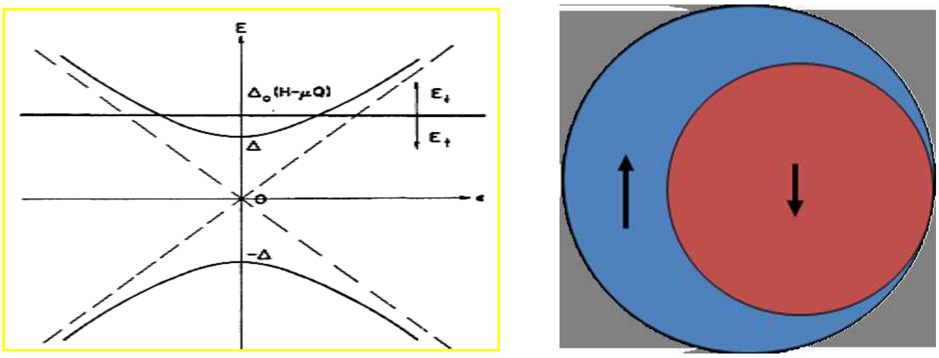

FIGURE 5. Spin-imbalanced superconductor. a) Filling of the bare spectrum of the homogeneous phase. b) Modulated phase improves the matching at some parts of the split up/down Fermi surfaces.

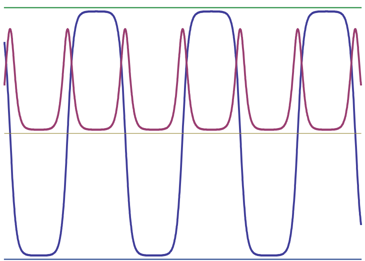

FIGURE 6. Solitonic lattice in the CDW or the superconductor under a slightly supercritical Zeeman splitting. The plots show distributions of the order parameter, and of the density of unpaired spins - mid-gap states concentrated near the zeros of the order parameter.

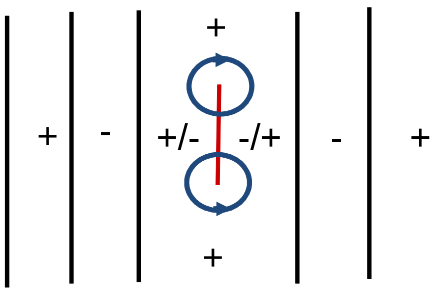

FIGURE 7. Kink-roton complexes as nucleuses of melting of the FFLO lattice, importantly in the LO version. The defect is embedded into the regular stripe structure (black lines for the amplitude zeros); +/- are the alternating signs of the order parameter $\mathscr{O}$ amplitude. Termination points of a finite segment $L_\perp$ (red color) of the $\mathscr{O}=0$ line must be encircled by semi-vortices of the phase rotation (blue circles) to resolve the signs mismatch. The minimal segment corresponds to the elementary kink carrying spin 1/2.

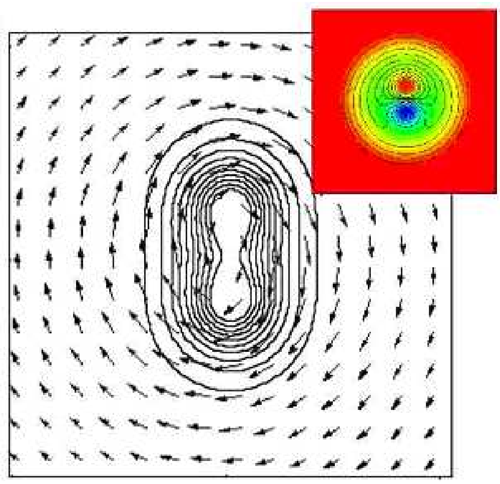

FIGURE 8. Splitting of the conventional integer orbital vortex into two counterparts in presence of a population of unpaired spins [47]. For our goals, we just reformulate these results inversely – unpaired spins creates the vortex pair even at no orbital magnetic field.

separation of spin and charge into spinons and holons, sometimes with their reconfinement at essentially different scales. Continuously broken symmetries allow for individual solitons entering the low temperature phases with long range ordered states: SC, ICDW, SDW. Then the solitons take forms of amplitude kinks which are topologically bound to half-integer vortices of gapless modes - π-rotons. These combined particles substitute for electrons - certainly in quasi-1D systems, which is valid for both charge- and spin- gaped cases. The description may be extrapolated to strongly correlated isotropic cases. Here it meets the picture of fragmented FFLO or stripe phases.

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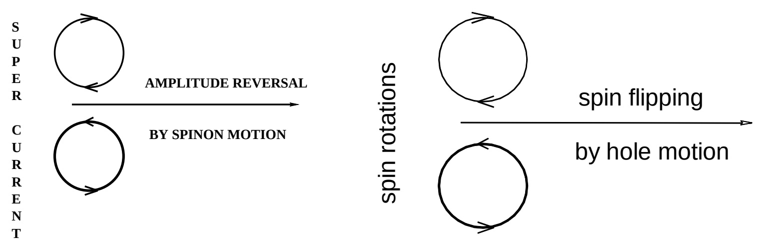

FIGURE 9. Motion of the kink-roton complexes. Left: For the ICDW or the superconductor, the amplitude kink is provided by the spinon. For the ICDW the curls are displacements contours for the half integer dislocation pair. For the superconductor, the curls are lines of electric currents circulating through the normal core carrying the unpaired spin. Right: For the SDW or the AFM, the string of the amplitude reversal of the order parameter created by the holon is cured by the semi-vortex pair of the staggered magnetization circulation.

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APPENDIX: MICROSCOPICS OF SOLITONS IN INCOMMENSURATE CDWS: .

Here we quote some details on microscopic origination of solitons via conversion of electrons in ICDW.

Solitons in CDWs

Recall firstly a universal, while restricted microscopic insight to excitations in the spin-gap cases - SC or CDW. The starting single chain level is well described by the bosonization language. The Hamiltonian (shown here schematically)

$$H_{1D} \sim (\partial\theta)^2 - V cos(2\theta) + (\partial\varphi)^2$$

is written in terms of phases for the spin - θ and the charge - φ. The energy $\sim V$ comes from the backward exchange scattering $V \sim g_1$ of electrons. The pair-breaking excitation - the $s = 1/2$ spinon, is the soliton connecting the degenerate minima of H_{1D}: $\theta \Rightarrow \theta + \pi$. The singlet order parameter, for either SC or CDW (depending on a definition of the charge phase φ) is $\mathscr{O}_{SC,CDW} \sim \cos\theta \exp(i\varphi)$. Its amplitude $A = \cos\theta$ changes the sign across the allowed θ- soliton, hence the spinon is an alternative description of the same amplitude soliton which appears in BCS-Peierls type models.

Within the quasiclassical or adiabatic BCS models, the topological solitons are characterized by a chiral angle $2\theta = \varphi(x = +\infty) - \varphi(x = -\infty)$. It defines a family of "chordus solitons" , Fig.10, which provides a continuous path in the configurational space for electron's selftrapping. The time evolution, $\theta(t)$, describes the selftrapping dynamics - the instanton [17].

With increasing of 2θ from 0 to $\pm 2\pi$, the level E_0 sweeps the gap from $\pm\Delta_0$ to $\mp\Delta_0$, providing the spectral flow across the gap. Being filled with two particles, this flow performs the conversion from the normal to the condensate density. Self-trapping of one electron at Δ_0 or one hole at $-\Delta_0$ will proceed gaining the energy until the configuration takes a stable form of the purely amplitude soliton. Now $E_0 = 0$ becomes a pure midgap state occupied by a single electron; thus the soliton carries the electronic spin s=1/2. Curiously, the electric charge is zero rather than e, being compensated by the dilatation of electronic wave functions of the filled band. That can be interpreted in a way that the AS is symmetric with respect to the charge conjugation: it is the adaptation of an electron added to the ground sate of $2M$ particles as well as of a hole upon the one of $2M + 2$ particles. Thus, in a 1D system, the AS is a realization of a spinon, the particle carrying the elementary spin $1/2$ but no charge. A theoretical value for the AS energy is $\Delta_{AS} = 2/\pi\Delta_0 \approx 0.65\Delta_0$; thus, the energy $\Delta^* \approx 0.35\Delta_0$ is gained by converting the electron into the soliton.

In a quasi-stationary regime the problem can be solved within the Peierls-Froehlich or the similar chiral Gross-Neveu models. The spectra are related to the nonlinear Schroedinger equation, see [4, 37]). The process goes through the family of "chordus solitons" as illustrated at Figs.10,11.

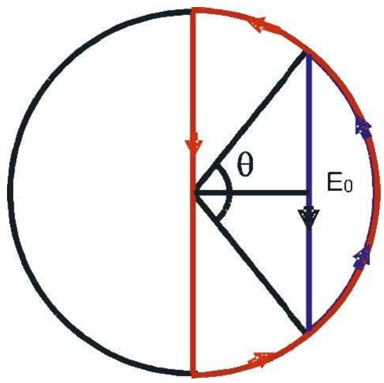

FIGURE 10. Soliton trajectories in the complex plane of the order parameter. Red line - the vertical diameter: the stable amplitude soliton. Blue line the vertical chord: an intermediate chordus soliton within a chiral angle θ (black radial lines). The arrowed arcs show the phase tails adapting to the chordus soliton evolution.

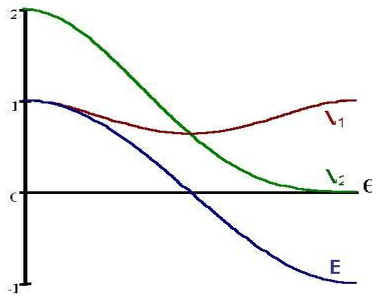

FIGURE 11. Selftrapping branches $V_n(\theta)$ for chordus solitons with the midgap sate fillings $n = 1$ and $n = 2$, and the energy $E_0(\theta)$ of the localized split-off state as functions of the chiral angle θ. The scale is $\Delta_0 = 1$

Instantons in ICDW.

Consider the solitons' creation rate which gives the tunneling current magnitude [19]. The interchain tunneling to the solitonic state takes place in moments when collective quantum fluctuations create an appropriate configuration with the necessary split-off intragap state. At first sight, one needs to prepare the AS in its full form, the probability of which is very low; but actually the spontaneous deformation is more shallow. Indeed, the energy E_{AS} is yielded from the voltage V by the interchain transfer of a single electron; hence the split-off energy level, prepared for this electron by the optimal fluctuations, must be at $E_0 = E_{AS}$. Therefore, the tunneling takes place when quantum fluctuations accumulate the chiral angle such that $\cos\theta/2 = 2/\pi$, which gives $\theta \approx 100^\circ$ rather than 180°.

Well below the free particle edge $2\Delta_0$, the subgap tunneling current is $J_1 \propto t_\perp^2\,((U/E_{AS}-2))^{v/2u}$, $u \ll v_F$ where u and v_F and are the phase and the Fermi velocities, and $t_\perp$ is the interchain overlap integral. This law gives the zero current at the threshold. The physical origin of this suppression comes from emittance of two phase fronts in the course of the chordus soliton development, which drives dynamics to the regime of quantum dissipation [17]. The phase tails, connecting the time-dependent chordus values $\varphi(t, x \to \pm 0) = \pm\theta(t)$ with the constant $\varphi(t, x \to \infty) \equiv 0$ at large distances, correspond to arcs shown in Fig10.

The 3D long range ordering, which energy scale is measured by the transition temperature $T_c \ll \Delta_0$, returns the tunneling to the normal dynamics. Then the peak of $J_1(U)$, instead of zero, will develop in a narrow vicinity $0 < U - 2E_{AS} < T_c$ of the threshold, in a qualitative agreement with experiment [13]. The factor $(T_c/E_{AS})^{v/2u}$ will give the overall reduction of the current in comparison with the one expected for free electrons.

The bi-electron channel stretches the tunneling spectrum down to the small energy $\sim T_c$, i.e. even to $U = 0$ in the 1D limit. The reason is that the particles with the charge $2e$ are the phase 2π solitons, see [50, 14] and Fig.3. The tunneling becomes a process of opposite, $\pm 2\pi$, phase slips taking place simultaneously at adjacent chains. The intensity is reduced as $J_2 \propto t_\perp^4\,(T_c/E_{AS})^{v/u}$.