# Effect of magnetic field on transport in granular materials.

K.B. Efetov* and M.Yu. Kharitonov†

*Theoretische Physik III, Ruhr-Universität Bochum, Germany
†Material Science Division, Argonne National Laboratory, Argonne, IL, 60439, USA

**Abstract.** We review properties of electron transport in granular materials in the presence of a weak magnetic field. In addition to longitudinal responses we present results for the Hall conductivity and resistivity. We demonstrate that, at sufficiently high temperatures when Coulomb blockade effects can be neglected, the Hall resistivity does not depend on the tunnelling amplitude between the grains and gives information about the interior of the grains. At lower temperatures this quantity acquires a logarithmic in temperature contribution in all dimensions of the array of the grains. In the limit of very low temperatures the dependence of the Hall resistivity on temperature is similar to the one for homogeneously disordered metals.

**Keywords:** grains, intergrain tunnelling, Coulomb blockade, Hall resistivity
**PACS:** 73.63.-b, 73.23.Hk, 61.46.Df

## INTRODUCTION

Hall resistivity (HR) of metals and semiconductors gives a very important information about their properties. According to the classical Drude-Boltzmann theory HR

$$\rho_{xy} = H/(nec) \tag{1}$$

does not depend on the mean free path and allows one to experimentally determine the carrier concentration $n$. At sufficiently low temperatures quantum effects (e.g. Coulomb interaction and weak localization) set in (see, e.g. [1, 2]), giving corrections to Eq. (1).

Recently, much attention from both experimental and theoretical sides has been paid to granular systems (see a review Ref.[3] and references therein). Although various physical quantities have been calculated in different regimes [4], Hall transport in granular matter has not been addressed theoretically in all these works. The absence of a theoretical description is apparently one of the reasons, why measurements of the Hall resistivity have not become a standard tool for characterization of the granular metals, although they do not seem to be very difficult.

Trying to apply the conventional theory of disordered metals to the granular systems, the following questions can be asked:

To what extent is the formula (1) applicable for granular metals? How is the carrier concentration extracted from Eq. (1) related to the actual carrier concentration inside the grains? How can quantum effects change HR of the granular system?

In this talk, theory of the Hall effect in granular system in the metallic regime is presented and these questions are answered.

In the metallic regime, when the intergrain tunnelling conductance $G_T = (2e^2/\hbar)g_T$ is large, $g_T \gg 1$ (further we set $\hbar = 1$), the granular system as a whole is roughly speaking a good conductor and its properties are quite similar to those of ordinary homogeneously disordered metals (HDMs). At the same time, the granularity of the system brings a new physical aspect, namely, confinement of electrons inside the grains. In a system with "well-pronounced" granularity electron traverses each grain many times before it escapes to a neighboring grain due to the tunnelling. This is ensured by the condition that the tunnelling escape rate $\Gamma$ is much smaller than the Thouless energy $E_{Th}$:

$$\Gamma \ll E_{Th}, \tag{2}$$

or, equivalently, the conductance $G_0 = (2e^2/\hbar)g_0$ of the grain is much larger than the tunnelling conductance $G_T$:

$$g_0 \gg g_T, \tag{3}$$

since $\Gamma = g_T\delta$ and $E_{Th} \propto g_0\delta$ ($\delta$ is the mean level spacing of the grain).

At the same time, the conditions (2, 3), leading to new physics absent in HDMs, simplify calculations. For example, in the limit $g_T \ll g_0$ the main contribution to the classical resistivity $\rho_{xx}^{(0)} = \left(\sigma_{xx}^{(0)}\right)^{-1}$ comes from the tunnel barriers between the grains rather than from scattering on impurities inside the grains and the longitudinal conductivity (LC) equals

$$\sigma_{xx}^{(0)} = G_T a^{2-d}, \tag{4}$$

where $a$ is the size of the grains and $d$ is the dimensionality of the system.

Formally, the conditions (2, 3) enable one to consider only the zero space harmonics for phases and potentials inside the grains [4, 3]. Therefore when studying the longitudinal transport one may neglect electron dynamics inside the grains, which is a significant simplification.

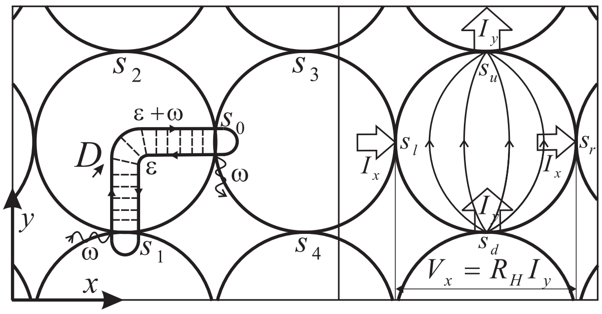


**FIGURE 1.** Granular system and classical picture of Hall conductivity. The Ohmic current $I_y = G_T V_y$ running through the grain in the $y$ direction causes the Hall voltage drop $V_H = R_H I_y$ between its opposite banks in the $x$ direction. In the left part, a diffuson giving the main contribution into the Hall conductivity is represented

For the Hall transport, however, the situation appears to be more complicated. The Hall current originates from the transversal drift in crossed magnetic and electric fields *inside* the grains. From simple classical considerations (Fig. 1) one obtains, that the Hall conductivity(HC) $\sigma_{xy}^{(0)}$ in the leading in $g_T/g_0$ order is

$$\sigma_{xy}^{(0)} = G_T^2 R_H a^{2-d}. \tag{5}$$

where $R_H$ is the Hall resistance of the grain. The Hall resistance $R_H$ should be obtained from the solution of a classical electrodynamics problem for the distribution of the electric potential inside the grain. We come to the situation when one is forced to take the intragrain electron dynamics into account, no matter how well the condition Eq. (3) is satisfied. In other words, the zero space harmonics approximation is no longer suitable for description of the Hall transport and one should take into account higher harmonics.

However, purely classical approach to the problem based on the classical electrodynamics, giving a quick answer Eq. (5), does not allow to include into considerations quantum effects (such as the screening of the Coulomb interaction and the weak localization) that come into play at sufficiently low temperatures and can significantly affect transport properties.

In this paper we review results obtained by a method based on diagrammatic technique that allowed us to take the intragrain electron dynamics into account by considering non-zero modes of standard two-particle propagators ("diffusons") inside the grain. The suggested procedure accounts for the finiteness of the ratio $g_T/g_0$ and reproduces the solution of the classical electrodynamics problem for the conductivity of a granular medium. The generality of our approach allows us, in principle, to study both LC and HC of the granular system for arbitrary ratio $g_T/g_0$ and for arbitrary type of the intragrain electron dynamics, either ballistic or diffusive. Non-zero modes of two-particle propagators are eventually related to the longitudinal $G_0^{-1}$ and Hall $R_H$ resistances of the grain. We apply our method to the problem of Hall transport for which considering intragrain dynamics is inevitable. Neglecting quantum effects, we do recover the classical formula Eq. (5). Diagrammatic approach allows us to include quantum effects of the Coulomb interaction and weak localization straightforwardly into the developed scheme. We study the influence of the Coulomb interaction on HC and HR by calculating first order corrections. A more detailed description of our work can be found in the publications [5].

## MODEL

We consider a quadratic ($d = 2$) or cubic ($d = 3$) lattice of equal in form and size metallic grains coupled to each other by tunnel contacts (Fig. 1). At the same time, they may be different microscopically, which means that the grains may have different impurities on the surface and inside them.

To provide more explicit results and simplify the calculations we assume that the intragrain electron dynamics is diffusive, i.e, the bulk mean free path $l$ in the grains is much smaller than the size $a$ of the grain, $l \ll a$. In this case details of electron scattering off the grain boundary are irrelevant. However, our approach is also perfectly applicable to the case of ballistic ($l \sim a$) intragrain disorder, when the surface scattering becomes important. The main results are valid for both diffusive and ballistic grains.

In the metallic regime, $g_T \gg 1$, quantum effects of the Coulomb interaction can be considered as a perturbation with the small parameter $1/g_T$.

We write the Hamiltonian describing the system as

$$\hat{H} = \hat{H}_0 + \hat{H}_t + \hat{H}_c. \tag{6}$$

In Eq. (6), the first term $\hat{H}_0$,

$$\hat{H}_0 = \sum_{\mathbf{i}} \int d\mathbf{r_i} \psi^\dagger(\mathbf{r_i}) \left[\xi\left(\mathbf{p_i} - \frac{e}{c}\mathbf{A}(\mathbf{r_i})\right) + U(\mathbf{r_i})\right] \psi(\mathbf{r_i}), \tag{7}$$

is the Hamiltonian of isolated grains, $\xi(\mathbf{p}) = \mathbf{p}^2/(2m) - \varepsilon_F$, $\mathbf{A}(\mathbf{r_i})$ is the vector potential describing uniform magnetic field $\mathbf{H} = H\mathbf{e}_z$ directed along the $z$ axis, $U(\mathbf{r_i})$ is the random disorder potential of the grains, $\mathbf{i} = (\mathbf{i_1}, \ldots, \mathbf{i_d}) \in \mathbf{Z^d}$ is an integer vector numerating the grains. The integration over $\mathbf{r_i}$ is performed over the volume of the grain $\mathbf{i}$. Since we do not deal with spin-related phenomena in this paper, we omit spin indices of the operators $\psi(\mathbf{r_i})$. Accounting for spin degeneracy in calculations is simple: each electron loop comes with the factor

2. We consider white-noise disorder and perform averaging using Gaussian distribution with the variance

$$\langle U(\mathbf{r_i})U(\mathbf{r'_i})\rangle_U = \frac{1}{2\pi\nu\tau_0}\delta(\mathbf{r_i}-\mathbf{r'_i}), \tag{8}$$

where $\nu$ is the density of states in the grain at the Fermi level per one spin projection.

The tunnelling Hamiltonian $\hat{H}_t$ in Eq. (6) is given by

$$\hat{H}_t = \sum_{\langle \mathbf{i},\mathbf{j}\rangle}(X_{\mathbf{i},\mathbf{j}} + X_{\mathbf{j},\mathbf{i}}) \tag{9}$$

where $X_{\mathbf{i},\mathbf{j}}$ is the operator describing tunnelling from the grain $\mathbf{j}$ to the grain $\mathbf{i}$, the summation is taken over the neighboring grains connected by a tunnel contact, such that each contact is counted only once.

For studying Hall effect the geometry of the grains and contacts is essential, therefore we write the tunnelling operators $X_{\mathbf{ij}}$ in the coordinate representation:

$$X_{\mathbf{i},\mathbf{j}} = \int d\mathbf{s_i}d\mathbf{s_j}\, t(\mathbf{s_i},\mathbf{s_j})\psi^\dagger(\mathbf{s_i})\psi(\mathbf{s_j}), \tag{10}$$

where the integration is carried out over two surfaces of the contact: one of them ($\mathbf{s_i}$) is in the $\mathbf{i}$-th grain, whereas the other ($\mathbf{s_j}$) in the $\mathbf{j}$-th grain. Such a form implies that the tunnelling occurs from a close vicinity of the contact but not from the bulk of the grain. This is a natural assumption, because we consider the limit of a good metal in the grains, such that the electron wave length is short. Fast oscillations of the wave functions in the grains result in a fast decay of the overlap of the wave functions of different grains outside the contacts. Since $\hat{H}_t^\dagger = \hat{H}_t$, we have $X^\dagger_{\mathbf{i},\mathbf{j}} = X_{\mathbf{j},\mathbf{i}}$ and $t^*(\mathbf{s_i},\mathbf{s_j}) = t(\mathbf{s_j},\mathbf{s_i})$.

The amplitudes $t(s_\mathbf{i},s_\mathbf{j})$ in Eq. (10) describe probability of the electron tunnelling from a point $s_\mathbf{j}$ to another point $\mathbf{s_i}$ on the other side of the contact. It is natural to assume that the electrons effectively tunnel from the point $\mathbf{s_j}$ to the points $\mathbf{s_i}$ in the vicinity of $\mathbf{s_j}$ of atomic size only.

Therefore $t(\mathbf{s_i},\mathbf{s_j})$ should decay rapidly on atomic scale as a function of $\mathbf{s_i}-\mathbf{s_j}$. The tunnelling amplitude $t(s_\mathbf{i},s_\mathbf{j})$ can fluctuate as a function of $\mathbf{s_i}$ for fixed $\mathbf{s_i}-\mathbf{s_j}$ due to irregularities of the contact on atomic scale.

To effectively model this behavior of the tunnelling amplitudes we will consider $t(\mathbf{s_i},\mathbf{s_j})$ as Gaussian random variables and average over them with the variance

$$\langle t(\mathbf{s_i},\mathbf{s_j})t(\mathbf{s_j},\mathbf{s_i})\rangle_t = t_0^2\delta(\mathbf{s_i}-\mathbf{s_j}), \tag{11}$$

where $\delta(\mathbf{s_i}-\mathbf{s_j})$ is an atomic scale $\delta$-function on the contact surface, $t_0^2$ has the meaning of the tunnelling probability per unit area of the contact.

The assumption of the short electron wave length enables us to neglect contributions coming from the regular parts $\langle t(\mathbf{s_i},\mathbf{s_j})\rangle_t$ of the tunnelling amplitudes.

The third term in Eq. (6) stands for the Coulomb interaction between electrons. In principle, one has to start with the bare Coulomb interaction between the electrons

$$\hat{H}_c = \frac{1}{2}\sum_{\mathbf{i},\mathbf{j}}\int d\mathbf{r_i}d\mathbf{r_j}\,\psi^\dagger(\mathbf{r_i})\psi^\dagger(\mathbf{r_j})\frac{e^2}{|\mathbf{r_i}-\mathbf{r_j}|}\psi(\mathbf{r_j})\psi(\mathbf{r_i}). \tag{12}$$

Proceeding with the calculations one has to take into account the screening of the Coulomb interaction by electron motion. One should distinguish between the *intragrain* and *intergrain* electron motion. In the static limit, one may model the Coulomb interaction, Eq. (12), by an effective charging energy $E_{\mathbf{ij}}$. In this approximation, one considers the interaction of the total charges of the grains. Accounting for the tunnelling leads to a screened form of the charging energy interaction [6], which is sufficient for studying *intergrain* transport. However, *coordinate-dependent* interaction modes inside each grain arising from the *intragrain* motion are necessary to get the correct classical expression for the "bare" (without interactions) Hall resistance $R_H$ of a single grain.

## RESULTS

The model introduced in the previous section was studied using a diagrammatic technique developed for the granular systems. The details of this method can be found in Refs. [5]. In principle, the diagrams are analogous to those used for description of homogeneously disordered metals [1, 2], although specifics of the granularity is definitely important. One should distinguish between diagrams containing expressions oscillating at the Fermi length in the space and those that vary in space smoothly. Only the latter give an essential contribution into the conductivity. Several examples of such diagrams are represented in Fig. 2

We perform calculations for magnetic fields $H$ such that $\omega_H\tau_0 \ll 1$, where $\omega_H = eH/mc$ is the cyclotron frequency and $\tau_0$ is the electron scattering time inside the grain. Since the effective mean free path $l = v_F\tau_0$ does not exceed the grain size $a$, and typically $a \approx 10 - 100nm$, the condition $\omega_H\tau_0 \ll 1$ is well fulfilled even for experimentally very high fields $H$. We also assume that the granularity of the system is "well-pronounced", i.e. the conditions, Eq. (2, 3), are satisfied.

First, we neglect quantum effects of the Coulomb interaction and obtain Eq. (5) for Hall conductivity $\sigma_{xy}^{(0)}$ in the lowest non-vanishing order in $g_T/g_0$. This result obtained by diagrammatic methods is of completely classical origin provided the tunnelling contact is viewed as a surface resistor with conductance $G_T$. The HR of the

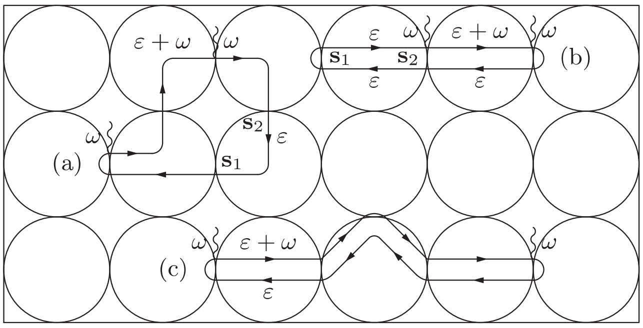


**FIGURE 2.** Diagrams for the current-current correlation function. "Prohibited" diagrams (a) and (b) contain oscillating at Fermi length $\lambda_F$ functions, which after the integration of the contacts surfaces give 0. (c) The only type of "allowed" diagram, that does not contain oscillating functions and gives nonvanishing contribution.

system, following from Eq. (4, 5),

$$\rho_{xy}^{(0)} = \frac{\sigma_{xy}^{(0)}}{(\sigma_{xx}^{(0)})^2} = R_H a^{d-2} = \frac{H}{n_d^* ec} \tag{13}$$

is given by the Hall resistance of a single grain $R_H$ that depends on *the geometry of the grain* but *not on the intragrain disorder*.

Eq. (13) defines the effective carrier density $n_d^*$ of the granular medium. For a three-dimensional (3D) ($d = 3$, many granular layers) array $n_3^* = An$ differs from the electron density $n$ in the grain by a numerical factor $A$, $0 < A \leq 1$, determined by the grain geometry. For grains of a simple geometry (e.g. having reflectional symmetry in all three dimensions) this factor is given by the ratio of the largest cross section area $S$ to the cross section area of the lattice cell $a^2$: $A = S/a^2$. So, $A = 1$ for cubic grains ($S = a^2$), and $A = \pi/4$ for spherical grains ($S = \pi a^2/4$). For a two-dimensional(2D) ($d = 2$, granular monolayer) array the 3D result must be multiplied by the thickness of the layer $a$: $n_2^* = aAn$.

The result, Eq. (13), for the Hall resistivity $\rho_{xy}^{(0)}$ is quite *universal*. It is valid even if 1) the tunnelling conductances $G_T$ fluctuate from contact to contact: HR is simply independent of the distribution of $G_T$. 2) the mean free path $l$ fluctuates from contact to contact. Therefore Eq. (13) is applicable to real granular arrays where such fluctuations are always present.

Next, we calculate the first-order correction to HC $\sigma_{xy}^{(0)}$, Eq. (5), due to Coulomb interaction. We find significant corrections for temperatures $T < g_T E_c$ not exceeding the inverse $RC$ time $g_T E_c$ of the system ($E_c = e^2/a$ is the charging energy of the grain), whereas for $T > g_T E_c$ the relative corrections are of the order of $1/g_T$ or smaller.

Three types of corrections to HC can be identified:

$$\sigma_{xy} = \sigma_{xy}^{(0)} + \delta\sigma_{xy}^{TA} + \delta\sigma_{xy}^{EC} + \delta\sigma_{xy}^{AA} \tag{14}$$

The first one, $\delta\sigma_{xy}^{TA}$, can be attributed to the renormalization of the individual tunnelling conductances $G_T$ (*tunnelling anomaly* (TA) [1, 8, 9]) in the granular medium and has the form:

$$\frac{\delta\sigma_{xy}^{TA}}{\sigma_{xy}^{(0)}} = -\frac{1}{\pi g_T d} \ln\left[\frac{g_T E_c}{\max(T,\Gamma)}\right]. \tag{15}$$

This correction renormalizes the tunnelling conductances $G_T$ in Eq. (5) but does not affect the Hall resistance of the grain $R_H$.

The second correction $\delta\sigma_{xy}^{EC}$ corresponds to the process of virtual electron diffusion through the grain:

$$\frac{\delta\sigma_{xy}^{EC}}{\sigma_{xy}^{(0)}} = \frac{c_d}{4\pi g_T} \ln\left[\frac{\min(g_T E_c, E_{Th})}{\max(T,\Gamma)}\right]. \tag{16}$$

where $c_d$ is a numerical factor. Contrary to $\delta\sigma_{xy}^{TA}$, the correction $\delta\sigma_{xy}^{EC}$ is suppressed at temperatures greater than the Thouless energy of the grain $E_{Th}$. Physically, it is analogous to the *elastic cotunneling* (EC)[10] process for weakly coupled grains.

For $T\,\Gamma$ both the corrections are $\ln T$-dependent. This dependence saturates at temperature $T \sim \Gamma$, such that both $\delta\sigma_{xy}^{TA}$ and $\delta\sigma_{xy}^{EC}$ remain logarithmically large constants at $T < \Gamma$.

These two corrections are specific for granular systems. They arise from spatial scales of the order of the grain size $a$ and are absent in homogeneously disordered metals (HDMs). The logarithmic form of the corrections is due to the screened Coulomb interaction in granular systems obtained in [6]. They have the same functional form in 2D and 3D but the coefficients are not universal, being lattice-dependent: $1/d$ and $c_d$ are the results for the cubic (3D) or quadratic (2D) lattice, respectively, and we assume these lattice symmetries in our calculations.

The third correction $\delta\sigma_{xy}^{AA}$ is analogous to the one present in HDMs. It might be significant at $T \ll \Gamma$ only, when the thermal length $L_T = a\sqrt{\Gamma/T} \gg a$ exceeds the size of the grain ($\Gamma a^2$ is the effective diffusion coefficient at scales $\gg a$). However, we find that this correction *vanishes identically* due to the symmetry in quasimomentum space both in 2D and 3D:

$$\delta\sigma_{xy}^{AA} = 0 \tag{17}$$

It can be instructive to compare the results for a granular system with those for a HDM. For the quantities arising from spatial scales exceeding the size of the grain $a$ one expects the correspondence, because at such scales the microscopic structure of the system becomes irrelevant.

Indeed, the result Eq. (17) for $\delta\sigma_{xy}^{AA}$ agrees with the one obtained for HDMs in Refs. [7].

The quantity directly measured in experiments is the Hall resistivity

$$\rho_{xy} = \frac{\sigma_{xy}}{\sigma_{xx}^2} = \rho_{xy}^{(0)} + \delta\rho_{xy}, \tag{18}$$

where $\rho_{xy}^{(0)}$ is the bare HR, Eq. (13), and $\delta\rho_{xy}$ is the Coulomb interaction correction,

$$\frac{\delta\rho_{xy}}{\rho_{xy}^{(0)}} = \frac{\delta\sigma_{xy}}{\sigma_{xy}^{(0)}} - 2\frac{\delta\sigma_{xx}}{\sigma_{xx}^{(0)}} \tag{19}$$

The interaction corrections to LC were studied in Refs. [4, 3] and the following result has been obtained:

$$\sigma_{xx} = \sigma_{xx}^{(0)} + \delta\sigma_{xx}^{TA} + \delta\sigma_{xx}^{AA}. \tag{20}$$

($\delta\sigma_{xx}^{TA}$ and $\delta\sigma_{xx}^{AA}$ correspond to $\delta\sigma_1$ Eq. (2b) and $\delta\sigma_2$ Eq.(2c) in Ref. [3], respectively). The correction $\delta\sigma_{xx}^{TA}$ is due to the tunnelling anomaly and renormalizes the tunnelling conductance $G_T$ in Eq. (4). Its Hall counterpart is $\delta\sigma_{xy}^{TA}$. The correction $\delta\sigma_{xx}^{AA}$ is analogous to that for HDM first obtained by Altshuler and Aronov(AA) [1]. Its Hall counterpart is $\delta\sigma_{xy}^{AA}$. The AA correction does not diverge at large scales in 3D, and is significant in 2D at temperatures $T \ll \Gamma$ [3]:

$$\frac{\delta\sigma_{xx}^{AA}}{\sigma_{xx}^{(0)}} = -\frac{1}{4\pi^2 g_T}\ln\frac{\Gamma}{T},\ T \ll \Gamma, d = 2 \tag{21}$$

Since the TA effects lead to the renormalization of the tunnelling conductance $G_T$ only, it cannot affect the HR $\rho_{xy}^{(0)}$, Eq. (13), that does not contain $G_T$.

Indeed, we have

$$\frac{\delta\sigma_{xy}^{TA}}{\sigma_{xy}^{(0)}} = 2\frac{\delta\sigma_{xx}^{TA}}{\sigma_{xx}^{(0)}} \tag{22}$$

and the correction to HR from TA effect equals zero. Therefore, the total correction $\delta\rho_{xy}$ to HR is

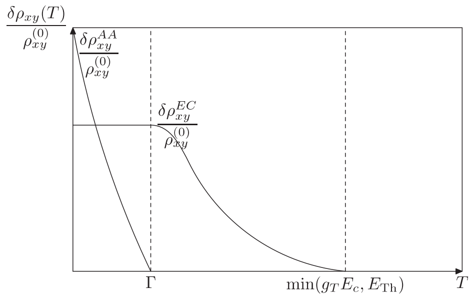


**FIGURE 3.** Temperature dependence of the total correction to Hall resistivity $\delta\rho_{xy}(T) = \delta\rho_{xy}^{EC}(T) + \delta\rho_{xy}^{AA}(T)$, Eq. (23).

$$\frac{\delta\rho_{xy}}{\rho_{xy}^{(0)}} = \frac{\delta\sigma_{xy}^{EC}}{\sigma_{xy}^{(0)}} - 2\frac{\delta\sigma_{xx}^{AA}}{\sigma_{xx}^{(0)}}, \tag{23}$$

where $\delta\sigma_{xy}^{EC}$ is given by Eq. (16), and $\delta\sigma_{xx}^{AA}$, Eq. (21), is significant in 2D at $T \ll \Gamma$. In a wide temperature range $\Gamma < T < \min(g_T E_c, E_{Th})$, the essential $T$-dependent correction comes both in 2D and 3D from EC effect, Eq. (16), only:

$$\frac{\delta\rho_{xy}}{\rho_{xy}^{(0)}} = \frac{\delta\sigma_{xy}^{EC}}{\sigma_{xy}^{(0)}} = \frac{c_d}{4\pi g_T}\ln\left[\frac{\min(g_T E_c, E_{Th})}{T}\right]. \tag{24}$$

The temperature behavior of $\delta\rho_{xy}$, Eq. (23), is shown in Fig. 3.

Another effect occurring at similar temperatures is weak localization (WL). The WL corrections to LC were studied in Refs. [11, 12, 13]. In 2D, the essential logarithmic contribution arises from spatial scales greater than the grain size $a$, when the inverse dephasing time is small, $1/\tau_\phi < \Gamma$ (if $1/\tau_\phi \propto T/g_T$ [11, 12], this corresponds to $T < T_{WL} \equiv g_T\Gamma$). However, we find [5] that the first-order in $1/g_T$ WL correction to HR *vanishes identically* both in 2D and 3D in correspondence with the result for HDMs [14, 7, 15]:

$$\delta\rho_{xy}^{WL} = 0. \tag{25}$$

Therefore weak localization effects do not change our results, Eqs. (13, 23), for HR.

Now we shortly summarize our findings. At temperatures $T > \min(g_T E_c, E_{Th})$ the Hall resistivity $\rho_{xy} = \rho_{xy}^{(0)}$ is given by Eq. (13) and is independent of both the intragrain and tunnel contact disorder. Measuring $\rho_{xy}$ at such $T$ and using Eq. (13) one can extract its effective carrier density $n_d^*$ that is an important characteristics of the granular system.

At temperatures $\Gamma < T < \min(g_T E_c, E_{Th})$, the Coulomb interaction leads to $\ln T$-dependent corrections to the Hall resistivity $\rho_{xy}$. Comparison of Eqs. (13, 23) with experimental data may serve as a good check of the theory developed here. The temperature dependence of the total correction to the the Hall conductivity is represented in Fig. 3

## DISCUSSION

In conclusion, we presented theory of the Hall conductivity of granular metals. In spite of its importance this question has not been addressed before. It turned out that considering only zero intragrain space harmonics that was very successful in describing the longitudinal conductivity [4, 3] is not sufficient for computation of the Hall conductivity and we considered also higher harmonics. Proceeding in this way we have shown that at high enough temperatures the Hall resistivity is given by the classical expression, from which one can extract the effective carrier density of the system. At lower temperatures, charging effects give a logarithmic temperature dependent contribution to the Hall resistivity that has the form of Eq. (24).

We emphasize, however, that Eq. (24 gives the first correction to the resistivity and the result is only valid when this correction is small. Therefore, the result of the calculation for the Hall conductivity (resistivity) is less accurate than the one obtained for the longitudinal conductivity in Ref. [4] using a renormalization group analysis. In the latter method, the logarithmic contribution could become very close in its value to the main part. In order to reach similar accuracy when calculating the Hall conductivity one should find a way to write proper renormalization group equations. This is not easy using the present diagrammatic approach and more sophisticated methods are needed.

The logarithmic dependence $\rho_{xx} = a + b\ln T$ of granular metals has been observed experimentally [16], and $\rho_{xy}$ can also be measured (see e.g. [17, 18]). The authors of Ref. [17] reported that HR $\rho_{xy}$ of their granular samples was *independent* of annealing temperature, although the latter did change the grain size $a$ and LR $\rho_{xx}$ (i.e. $G_T$, see Eq. (4)). This fact supports our result Eq. (13) for $\rho_{xy}^{(0)}$. Our theory may also be applied to indium tin oxide(ITO) materials (see e.g. [19]).

We hope that more experiments on this subject will be done in the nearest future and that the measurement of the Hall resistivity will evolve into a very important method of characterization of the granular materials.

## REFERENCES


1. B.L. Altshuler and A.G. Aronov, in *Electron-Electron Interactions in Disordered Systems*, edited by A.L. Efros and M. Pollak (Elsevier, Amsterdam, 1985).
2. P.A. Lee and T.V. Ramakrishnan, *Rev. Mod. Phys.* **57**, 287 (1985)
3. I.S. Beloborodov, V.M. Vinokur, A.V. Lopatin, and V.M. Vinokur, K.B. Efetov, *Rev. Mod. Phys.* **79**, 469 (2007)
4. K.B. Efetov and A. Tschersich, *Europhys. Lett.* **59**, 114, (2002); Phys. Rev. B **67**, 174205 (2003).
5. M. Yu. Kharitonov, K.B. Efetov, *Phys. Rev. Lett.* **99**, 056803 (2007); Phys. Rev. B **77**, 045116 (2008)
6. I.S. Beloborodov, K.B. Efetov, A. Altland, and F.W.J. Hekking, *Phys. Rev. B* **63**, 115109 (2001).
7. B.L. Altshuler, D.E. Khmelnitski, A.I. Larkin, and P.A. Lee, *Phys. Rev. B* **22**, 5142 (1980).
8. Yu.V. Nazarov, *Sov. Phys. JETP* **68**, 561 (1989).
9. L.S. Levitov, A. V. Shytov, *JETP Lett.* **66**, 214 (1997).
10. D.V. Averin and Yu. V. Nazarov, *Phys. Rev. Lett.* **65**, 2446 (1990).
11. I.S. Beloborodov, A.V. Lopatin, and V.M. Vinokur, *Phys. Rev. B* **70**, 205120 (2004).
12. Ya.M. Blanter, V.M. Vinokur, and L.I. Glazman, *Phys. Rev. B* **73**, 165322 (2006).
13. C. Biagini, T. Caneva, V. Tognetti, and A.A. Varlamov, *Phys. Rev. B* **72**, 041102(R) (2005).
14. H. Fukuyama, *J. Phys. Soc. Japan* **49**, 644 (1980).
15. M. Khodas, A. M. Finkel'stein, *Phys. Rev. B* **68**, 155114 (2003).
16. A. Gerber, A. Milner, G. Deutscher, M. Karpovsky, and A. Gladkikh, *Phys. Rev. Lett.* **78**, 4277 (1997); **R.V.** Simon, B.J. Dalrymple, D.V. Vechten, W.W. Fuller, and S.A. Wolf, *Phys. Rev. B* **36**, 1962 (1987)
17. Peng Xiong, Gang Xiao, J. Q. Wang, John Q. Xiao, J. Samuel Jiang, C. L. Chien *Phys. Rev. Lett.* **69**, 3220 (1992).
18. X.X. Zhang, Chuncheng Wan, H. Liu, Z.Q. Li, Ping Sheng, J.J. Lin, *Phys. Rev. Lett.* **86**, 5562 (2001);
19. J. Ederth *et al*, *Phys. Rev. B* **68**, 155410 (2003); J. Ederth *et al*, *Thin Solid Films.* **445**, 199 (2003).