Single-Particle Excitations Generated by Voltage Pulses

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Abstract. We analyze properties of excitations due to voltage pulses applied to a 1D noninteracting electron gas, assuming that the integral of the voltage over time is equal to the unit of flux. We show that the average charge transfer due to such pulses does not depend on the pulse shape. For pulses with a Lorentzian profile, we prove the single-particle nature of the electron and the hole excitations.

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1. INTRODUCTION

Recently, much interest has concentrated on single-particle sources feeding devices with individual electrons [1–8]. The action of a voltage pulse can be seen as a unitary transformation $\hat{U}$ (a scattering operator) acting on the degenerate Fermi sea $|\Phi_\mathrm{F}\rangle$ of the one-dimensional system. In this work, we will show that for properly designed pulses this state is free of entangled particle-hole pairs but instead involves a pure one-particle excitation on top of a complete Fermi sea. This result was first derived by Keeling *et al.* who describe the action of the voltage pulse on the single particles states and argue that many-particle excitations are absent [5]. Here, we present an alternative proof which describes the application of the voltage pulse as a (second-quantized) scattering matrix $\hat{U}$ such that the nature of the excitation can be shown rigorously within the many-body setup. First experiments involving single-electron sources were performed recently by Fève *et al.* [6] (for a theoretical description see Ref. [7]), using a quantum dot in a quantum Hall sample at integer filling.

The calculations are performed using a linear dispersion relation $E_{\mathrm{R/L}} = \pm v_\mathrm{F} p$ valid close to the Fermi points $\pm p_\mathrm{F}$ [cf. Fig. 1], where v_F (p_F) denotes the Fermi velocity (momentum). In one dimension, the linear-spectrum approximation disconnects the dispersion relation of the electrons into right (R) and left (L) moving branches. Here, we are interested in generating, via a voltage pulse, a particle in the right-moving branch together with a hole in the left-moving branch which leads to a net current in the device. The voltage pulse (together with the linear-spectrum approximation) conserves the number of particles in each branch individually. In order to be able to describe the above process, we need to introduce an infinitely deep Fermi sea (by setting the lower bound of the momentum in each branch to $-\infty$) [9]. We then proceed as follows: First, we prove that the average charge transmitted through the wire is related to the voltage integrated over time, i.e., each flux-quantum produces one electron on average (Secs. 2 and 3). Subsequently, we concentrate on the right-moving branch and show how a voltage pulse can be described via a (unitary) scattering operator $\hat{U}$ and that for a unit-flux Lorentzian pulse the excitation is of a single-particle nature (Sec. 4). Finally, to complete the picture, we turn our attention to the left-moving branch and demonstrate that the single-particle excitation in the right-moving branch is accompanied by a single hole in the left-moving branch (Sec. 5).

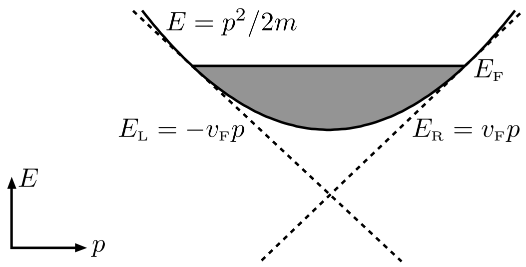

FIGURE 1. For low temperatures and excitation energies, the quadratic dispersion relation $E = p^2/2m$ can be expanded around the Fermi surface. In one dimension, this leads to two independent branches $E_{\mathrm{R/L}} = \pm v_\mathrm{F} p$ with $v_\mathrm{F} = p_\mathrm{F}/m$ (up to an irrelevant energy shift). Here, the subscript R (L) refers to a particle moving to the right (left).

2. VOLTAGE PULSES

Consider a quantum wire where voltage pulses $V(t)$ can be applied over a small region around $x = x_V$. Such voltage pulses act on the right-moving electrons with

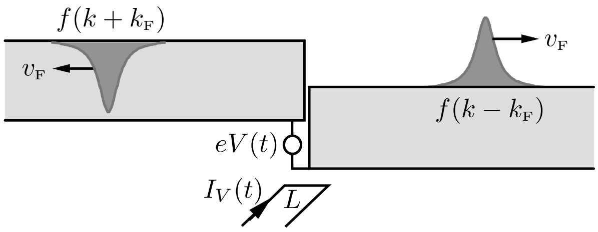

FIGURE 2. Due to the application of a time-dependent, external current $I_V(t)$, a time-dependent voltage $V(t)$ acts on the electrons in the quantum wire. Assuming that the voltage pulse has unit-flux and a Lorentzian shape, the shakeup of the Fermi sea creates a single-particle excitation with wave function $f(k-k_{\rm F})$ which moves to the right and simultaneously a hole excitation of the same type which moves to the left. Note that the electron-hole pair is entangled [10].

dispersion $E_{\rm R} = v_{\rm F}p$ and produce electronic excitations in the quantum wire [cf. Fig. 2].[1] If the time dependence of the voltage is slow compared to the transition time of an electron through the region with the voltage drop, the potential can be considered as quasistatic. In this case, the effect of the voltage pulse can be incorporated in a phase factor $\exp[i\phi(t-x/v_{\rm F})\Theta(x-x_V)]$ multiplying the wave function, where the phase

$$\phi(t) = \frac{e\Phi(t)}{\hbar c} = \frac{e}{\hbar}\int_{-\infty}^{t} dt'\,V(t') \qquad (1)$$

is proportional to the flux $\Phi(t)$, $\Theta(x)$ denotes the unit-step function, and $e>0$ is the unit of charge. The right-moving scattering state $\psi_{{\rm R},k}(x;t) = \exp[ik(x-v_{\rm F}t) + i\phi(t-x/v_{\rm F})\Theta(x-x_V)]$ is a solution of the linearized time-dependent Schrödinger equation

$$\Big[i\hbar(\partial_t + v_{\rm F}\partial_x) + \hbar v_{\rm F}\phi(t)\delta(x-x_V)\Big]\psi_{{\rm R},k}(x;t) = 0 \qquad (2)$$

including the time-dependent voltage $V(t)$ via $\phi(t)$ [5, 12]. Left of the position of the voltage pulse, $x<x_V$, $\psi_{{\rm R},k}$ is a plane wave with well-defined energy $\hbar v_{\rm F}k$. On the right of the position of the voltage pulse, $x>x_V$, the wave function $\psi_{{\rm R},k}(x;t)$ can be decomposed into energy eigenmodes $\exp[ik(x-v_{\rm F}t)]$ of the free Hamiltonian

$$\psi_{{\rm R},k}(x>x_V;t) = \int \frac{dk'}{2\pi} U(k'-k) e^{ik'(x-v_{\rm F}t)} \qquad (3)$$

where the transformation kernel

$$U(q) = v_{\rm F}\int dt\, e^{i\phi(t)+iqv_{\rm F}t} \qquad (4)$$

is the Fourier transform of the phase factor $\exp[i\phi(t)]$. Here, we are interested in applying integer flux pulses such that $\phi(t\to\infty)\in 2\pi\mathbb{N}$, as noninteger flux pulses do not produce clean single-particle excitations and lead to logarithmic divergences in the noise of the transmitted charge [2, 13]. In the case of integer flux pulses, the long time asymptotics $\exp[i\phi(t\to\pm\infty)]\to 1$ generates a Dirac delta function $2\pi\delta(q)$ in $U(q)$. The remaining part

$$U^{\rm reg}(q) = U(q) - 2\pi\delta(q) = v_{\rm F}\int dt\left[e^{i\phi(t)}-1\right]e^{iqv_{\rm F}t} \qquad (5)$$

is finite for a localized (in time) voltage pulse. The transformation $U(k'-k)$ describes the scattering amplitude for the transition from a momentum state k (for $x<x_V$) to the state k' (for $x>x_V$) due to the application of the voltage pulse. The statement that the wave function is in the momentum state k' holds in the asymptotic region $x-x_V \gg v_{\rm F}\tau$ with τ the typical timescale associated with the dynamics of the voltage pulse.

As we are interested in the region with $x>x_V$, the scattering states originating from the right do not enter the region where the voltage pulse is applied and therefore are unperturbed

$$\psi_{{\rm L},k}(x>x_V;t) = e^{-ik(x+v_{\rm F}t)} \qquad (6)$$

without any shift in energy.[2] The time-dependent field operator in the region $x>x_V$ behind the voltage pulse is given by

$$\Psi(x>x_V;t) = \int \frac{dk}{2\pi}\left[\psi_{{\rm R},k}(x;t)a_{{\rm R},k} + \psi_{{\rm L},k}(x;t)a_{{\rm L},k}\right];$$

here, $a_{{\rm R},k}$ ($a_{{\rm L},k}$) denote fermionic annihilation operators for states moving to the right (left), i.e., incoming from the left (right) reservoir. Averaging the current operator

$$I(x>x_V;t) = \frac{ie\hbar}{2m}\left[\Psi^\dagger(x;t)\partial_x\Psi(x;t) - {\rm H.c.}\right]$$

over the Fermi reservoirs assuming the same Fermi distribution $n(k) = n_{\rm R}(k) = n_{\rm L}(-k)$ initially at the far left and right of the interaction region and integrating the ensemble averaged current over time, we obtain the transmitted charge

$$\langle -Q/e\rangle = \int \frac{dk'\,dk}{(2\pi)^2} K(k'-k)n(k) \qquad (7)$$

with the kernel

$$K(q) = |U^{\rm reg}(q)|^2 + 4\pi\delta(q){\rm Re}\left[U^{\rm reg}(0)\right]; \qquad (8)$$

[1] Note that in the experiment [11] on photon-assisted noise [12] instead of a voltage pulse localized at a specific position a bias voltage over the whole sample was used. This procedure might not work in our case as our effect depends on the details of the voltage drop across the sample.

[2] In the linear-spectrum approximation particles are not back-reflected due to the voltage-pulse. The hole appears for $x<x_V$, see later Sec. 5.

a more careful calculation shows that $\mathrm{Re}[U^{\mathrm{reg}}(0)]$ has to be replaced by the symmetric limit $\mathrm{Re}[U^{\mathrm{reg}}(0^+) + U^{\mathrm{reg}}(0^-)]/2$ in those cases where $U^{\mathrm{reg}}(q)$ has a discontinuous real part near $q = 0$.[3] Equivalently, the kernel $K(q)$ can be defined as

$$K(q) = v_{\mathrm{F}}^2 \int dt dt' \left[e^{i\phi(t)-i\phi(t')} - 1\right] e^{iqv_{\mathrm{F}}(t-t')}. \quad (9)$$

Note that $\int dq K(q) = 0$, which can be obtained from the fact that integrating over q in Eq. (9) leads to a δ-function imposing $t = t'$, such that the term in the rectangular bracket vanishes.

3. TOTAL TRANSFERRED CHARGE

In this section, we will show that charge transport caused by an integer-flux pulse at zero temperature depends only on the number of flux units it contains, independent of the pulse shape. In the last section, we have derived the formula [cf. Eq. (7)]

$$\langle -Q/e \rangle = \int \frac{dk' dk}{(2\pi)^2} K(k'-k) n(k) \quad (10)$$

providing the charge $\langle -Q/e \rangle$ transferred in a process involving an integer-flux voltage-pulse with arbitrary shape.[4] To proceed, we assume the thermal energy $k_{\mathrm{B}}\vartheta$ to be smaller than any other energy scale, so we can employ the zero temperature distribution $n(k) = \Theta(k_{\mathrm{F}} - k)$. Inserting the distribution function into (10) and changing the integration variable k to $q = k' - k$ yields

$$\langle -Q/e \rangle = \int \frac{dk'}{2\pi} \int \frac{dq}{2\pi} K(q) \Theta(k_{\mathrm{F}} - k' + q).$$

We want to obtain a form for the transmitted charge, which only depends on the first term of Eq. (8) for $K(q)$. We do that by analyzing the q integration at fixed values of k'. If $k' > k_{\mathrm{F}}$, the δ-function in $K(q)$ lies outside of the integration range and the second term in Eq. (8) does not contribute. In the other case $k' < k_{\mathrm{F}}$, we first use the identity $\int dq K(q)\Theta(k_{\mathrm{F}} - k' + q) = -\int dq K(q)\Theta(k' - k_{\mathrm{F}} - q)$ which follows from $\int dq K(q) = 0$, and again the δ-function lies outside the integration range. In summary, we obtain

$$\langle -Q/e \rangle = \int \frac{dk' dq}{(2\pi)^2} |U^{\mathrm{reg}}(q)|^2 \Big[\Theta(k' - k_{\mathrm{F}})\Theta(k_{\mathrm{F}} - k' + q) - \Theta(k_{\mathrm{F}} - k')\Theta(k' - k_{\mathrm{F}} - q)\Big].$$

The integration of the rectangular bracket over k' yields a factor q and we arrive at the formula

$$\langle -Q/e \rangle = \int \frac{dq}{(2\pi)^2} q |U^{\mathrm{reg}}(q)|^2 \quad (11)$$

for the average transmitted charge. Here, the factor q is proportional to the number of particles which can be excited above the Fermi edge by providing them the momentum q and $|U^{\mathrm{reg}}(q)|^2$ denotes the probability for such an event to happen. Equation (11) has a number of advantages over the expression in Eq. (10). The integration over the Fermi sea is already performed and only one integration over the momentum q of the excitations remains. Furthermore, it involves the kernel $U^{\mathrm{reg}}(q)$ [instead of $K(q)$] which is directly related to the Fourier transform of the phase acquired by the voltage pulse $V(t)$ [see Eq. (5)]. Inserting the definition (5) into Eq. (11) and performing the momentum integration yields the Fourier-transformed expression

$$\begin{aligned}\langle -Q/e \rangle &= \int \frac{dt}{2\pi} \left[e^{-i\phi(t)} - 1\right] \frac{d}{idt} \left[e^{i\phi(t)} - 1\right] \\ &= \int \frac{dt}{2\pi} \dot{\phi}(t) \left[e^{-i\phi(t)} - 1\right] e^{i\phi(t)},\end{aligned}$$

where the boundary terms vanish because the voltage pulse carries an integer flux with $\exp[i\phi(\pm\infty)] = 1$. Changing the integration variable from t to ϕ and performing the integration yields

$$\langle -Q/e \rangle = -\frac{\phi(+\infty) - \phi(-\infty)}{2\pi} \quad (12)$$

In this form, it is visible that the average transmitted charge $\langle -Q/e \rangle$ does not depend on the pulse shape but only on the winding number of $e^{i\phi(t)}$. For example, a negative unit-flux pulse produces a phase which is decreasing in time with $\phi(+\infty) - \phi(-\infty) = -2\pi$. Therefore, on average one electron with charge $-e$ transferred.

4. SINGLE-PARTICLE EXCITATION

In a time-dependent basis of right-moving position eigenstates $\langle x; t | x_{\mathrm{R}} \rangle = \delta(x - v_{\mathrm{F}} t)$, labeled by the retarded position $x_{\mathrm{R}} = x - v_{\mathrm{F}} t$, the effect of the voltage pulse is easy to incorporate. In a first step, we will concentrate on these right-moving states (in this section, we drop the subscript R). The states pick up a scattering phase

[3] This is the case for the unit-flux Lorentzian voltage pulses which produce the single-particle excitations.

[4] Note that the integrals in (7) cannot be performed in arbitrary order. Calculating first the integral over k', a vanishing transferred charge is obtained. Similarly, the charge vanishes in the case of a finite Fermi sea. Only in the case of an infinitely deep Fermi sea a finite answer is obtained. This reflects the fact that the number of particles in right-moving states is conserved in a system with a finite Fermi sea. In reality, the quadratic spectrum connects the left- with the right-going branch and effectively promotes one left-going electron to a right-going state.

$\exp[i\phi(-x_R/v_F)]$ when they pass through $x = x_V$, with $\phi(t) = (e/\hbar)\int_{-\infty}^{t} V(t')dt'$. The operator $\hat{U}$, which transforms the initial states $|x_R\rangle$ at $x < x_V$ into the corresponding voltage-driven states at $x > x_V$, can then be written as the unitary operator

$$\hat{U} = \exp\left[i\int dx_R\, \phi(-x_R/v_F)\Psi^\dagger(x_R)\Psi(x_R)\right], \qquad (13)$$

where $\Psi(x_R)$ is a second-quantized fermion operator which annihilates a particle in the state $|x_R\rangle$. Applying a unit-flux Lorentzian voltage pulse

$$V(t) = -\frac{\hbar}{e}\frac{2v_F\xi}{(v_F t)^2+\xi^2} \qquad (14)$$

with width ξ, adds a phase

$$\phi(t) = -[\pi + 2\arctan(v_F t/\xi)] \qquad (15)$$

to the right going scattering states and thereby excites the Fermi sea. Here, we want to prove that

$$\hat{U}|\Phi_F\rangle = A^\dagger|\Phi_F\rangle \qquad (16)$$

where the single-particle creation operator $A^\dagger$ is defined via

$$A^\dagger = \int \frac{dk}{2\pi} f(k-k_F)a_k^\dagger \qquad (17)$$

and the creation operators $a_k^\dagger$ are associated with right-moving plane waves $\exp(ikx_R)$; the amplitude

$$f(\varkappa) = \sqrt{4\pi\xi}\, e^{-\xi\varkappa}\Theta(\varkappa) \qquad (18)$$

is (up to normalization N) given by $f(\varkappa) = NU^{\mathrm{reg}}(\varkappa)$ [cf. Eq. (21)]. Equation (16) tells us that a unit-flux Lorentzian pulse creates a clean single-particle excitation, i.e., a product state of the filled Fermi sea $|\Phi_F\rangle$ and a particle with wave function $f(k-k_F)$ propagating independently of $|\Phi_F\rangle$. This excitation can then be described as a Lorentzian wave packet in the first-quantized language.

We prove Eq. (16) by calculating general correlators of the form

$$\langle\Phi|:a_{k_1}^\dagger a_{k_1}\cdots a_{k_n}^\dagger a_{k_n}:|\Phi\rangle \qquad (19)$$

and show that they are equal for the states $|\Phi_A\rangle = A^\dagger|\Phi_F\rangle$ and $|\Phi_U\rangle = \hat{U}|\Phi_F\rangle$. Here and below $:\mathscr{O}:$ denotes the normal ordering of the operator $\mathscr{O}$ defined via $:\mathscr{O}: = \mathscr{O} - \langle\Phi_F|\mathscr{O}|\Phi_F\rangle$ which has to be applied to render the correlators finite.[5] Since the plane waves associated with the creation operators $a_k^\dagger$ form a complete basis of the one-particle Hilbert space of right-moving electrons, an arbitrary many-particle state $|\Phi\rangle$ (assuming a fixed number of particles) can be fully specified by the set of correlators (19). While the calculation of the correlators for the state $|\Phi_A\rangle = A^\dagger|\Phi_F\rangle$ is simple, it is not at all straightforward for $|\Phi_U\rangle$. The key part in our derivation is to show that the correlators are zero for $n \geq 2$ which proves the single-particle nature of the excitation.

Using the explicit form (13) of the unitary scattering operator, the transformation

$$\begin{aligned}\hat{U}^\dagger a_k \hat{U} &= a_k + \int \frac{dk'}{2\pi} U^{\mathrm{reg}}(k-k')a_{k'} \\ &= \int \frac{dk'}{2\pi} U(k-k')a_{k'}\end{aligned} \qquad (20)$$

of the annihilation operator a_k can be derived straightforwardly, where the kernel

$$U^{\mathrm{reg}}(q) = -2\pi(2\xi)e^{-\xi q}\Theta(q) \qquad (21)$$

has been introduced before [cf. Eq. (5)]. Using this relation, we calculate the one-particle matrix element[6]

$$\begin{aligned}\langle\Phi_U|:a_{k'}^\dagger a_k:|\Phi_U\rangle = \int_{-\infty}^{k_F} \frac{dk''}{2\pi}[U^*(k'-k'')U(k-k'') \\ - (2\pi)^2\delta(k'-k'')\delta(k-k'')].\end{aligned} \qquad (22)$$

Inserting the particular form (21) of $U(q)$ for a Lorentzian pulse into Eq. (22), the matrix element yields

$$\langle\Phi_U|:a_{k'}^\dagger a_k:|\Phi_U\rangle = f^*(k'-k_F)f(k-k_F) \qquad (23)$$

with $f(k-k_F)$ defined in Eq. (18). The one-particle correlator (23) is the same as the one generated by the state $|\Phi_A\rangle$.

Having shown the equivalence of the states $|\Phi_A\rangle$ and $|\Phi_U\rangle$ on the one-particle level, we want to proceed showing that this relation also holds for higher-order correlators. In a first step, we show that (19) vanishes, whenever one of the k's is smaller than the Fermi wave-vector k_F, thereby we use the property that $U(q) \propto \Theta(q)$ (I). Physically, this means that there is no hole excitation in the system. Then, we show that the correlator (19) also vanishes whenever there are two entries k which are larger than the Fermi wave-vector. To this end, we use the fact that $U(q) \propto e^{-q}$ (II). Properties (I) and (II) uniquely restrict the corresponding voltage pulse to the unit-flux

[5] For fermionic operators a_k, the normal ordering $:a_k^\dagger a_k:$ is given by $a_k^\dagger a_k$ for $k > k_F$ and $-a_k a_k^\dagger$ for $k < k_F$.

[6] We have evaluated the creation and annihilation operator at different momenta in order to avoid a subtraction of two divergences.

Lorentzian form. Thus, this is the only pulse shape which may generate a single-particle excitation.[7]

First, we show that the correlator (19) vanishes if one $k_j < k_{\mathrm{F}}$. Due to the normal ordering, the creation operator $a^\dagger_{k_j}$ can be moved to the very right where it acts on $|\Phi_U\rangle$ and vanishes,

$$\begin{aligned} a^\dagger_{k_j}|\Phi_U\rangle &= \hat{U}\hat{U}^\dagger a^\dagger_{k_j}\hat{U}|\Phi_{\mathrm{F}}\rangle \\ &= \hat{U}\int \frac{dk'}{2\pi} U^*(k_j - k') a^\dagger_{k'}|\Phi_{\mathrm{F}}\rangle = 0; \end{aligned} \tag{24}$$

here, we have used the adjoint of Eq. (20). The last equality follows from the fact that $k' > k_{\mathrm{F}}$ and $k_j < k_{\mathrm{F}}$, together with the property that the kernel $U(q)$ only increases the energy [as it is proportional to $\Theta(q)$].

Having found that there are no hole excitations present, we have to check the single-particle nature of the state $|\Phi_U\rangle$. For this, we assume that there are two k's, k_j and k_l, which are larger than the Fermi wave-vector k_{F}. Here, we show that $a_{k_j}a_{k_l}\hat{U}|\Phi_{\mathrm{F}}\rangle = 0$ which makes all correlators with more than one particle above the Fermi sea vanish, effectively proving that the excitation above the Fermi sea is of single-particle nature. Applying the annihilation operators on the excited state $|\Phi_U\rangle$ yields

$$\begin{aligned} a_{k_j}a_{k_l}|\Phi_U\rangle &= \hat{U}\hat{U}^\dagger a_{k_j}\hat{U}\hat{U}^\dagger a_{k_l}\hat{U}|\Phi_{\mathrm{F}}\rangle \\ &= \hat{U}\left(a_{k_j} + \int \frac{dk}{2\pi} U^{\mathrm{reg}}(k_j - k) a_k\right) \\ &\quad \times \left(a_{k_l} + \int \frac{dk}{2\pi} U^{\mathrm{reg}}(k_l - k) a_k\right)|\Phi_{\mathrm{F}}\rangle. \end{aligned}$$

Next, we use the fact that the contribution from terms containing an annihilation operator a_k with $k > k_{\mathrm{F}}$ vanishes. This includes both the isolated a_{k_j} and a_{k_l}, as well as the parts of the integrals with $k > k_{\mathrm{F}}$. Moreover, we insert the explicit form Eq. (21) of $U^{\mathrm{reg}}(q)$ and obtain

$$\begin{aligned} a_{k_j}a_{k_l}\hat{U}|\Phi_{\mathrm{F}}\rangle =& \hat{U}\left(2\xi e^{-\xi(k_j - k_{\mathrm{F}})}\int_{-\infty}^{k_{\mathrm{F}}} dk\, e^{-\xi(k_{\mathrm{F}} - k)} a_k\right) \\ &\times \left(2\xi e^{-\xi(k_l - k_{\mathrm{F}})}\int_{-\infty}^{k_{\mathrm{F}}} dk\, e^{-\xi(k_{\mathrm{F}} - k)} a_k\right)|\Phi_{\mathrm{F}}\rangle \\ =& 0. \end{aligned} \tag{25}$$

The last equality follows from the fact that both factors are proportional to the same fermionic operator $\int_{-\infty}^{k_{\mathrm{F}}} dk\, \exp[-\xi(k_{\mathrm{F}} - k)]a_k$. Due to Fermi statistics, the corresponding state can be occupied by only one particle.[8]

Summing up, we have shown that the correlator (19) is given by (23) for $n = 1$. For $n \geq 2$, the correlator vanishes as there are either more than two k's larger than the Fermi momentum [using Eq. (25)] or at least one k is below k_{F} [using Eq. (24)]. We obtain

$$\begin{aligned} \langle\Phi_U|{:}a^\dagger_{k_1}a_{k_1}\cdots a^\dagger_{k_n}a_{k_n}{:}|\Phi_U\rangle &= |f(k_1 - k_{\mathrm{F}})|^2\delta_{n,1} \\ &= \langle\Phi_A|{:}a^\dagger_{k_1}a_{k_1}\cdots a^\dagger_{k_n}a_{k_n}{:}|\Phi_A\rangle, \end{aligned}$$

thus completing the proof of Eq. (16).

5. HOLE EXCITATION

The result (16) is yet incomplete, since it is not clear where the additional particle $A^\dagger$ comes from; the complete result must satisfy particle number conservation. To understand how this problem is solved, we have to recall that we described the electron system as *two independent* systems of basis states, right- and left-moving, respectively. This reflects the physical picture that there are two mutually-independent sets of states near the two Fermi points of the one-dimensional electron gas. Above, the action of the Lorentzian pulse on the set of *right-moving* states was determined.[9] The empty state is instead found in the set of *left-moving* states, i.e., the voltage pulse creates a hole near the point $-k_{\mathrm{F}}$, propagating in the direction opposite to that of the particle-like excitation. In the following, we will demonstrate how the problem of describing the hole excitation in the left-moving branch can be mapped back on the problem of generating an electron in the right-moving branch in two steps. First, a parity transformation is applied which changes the branch but also reverses the sign of the voltage pulse. To return back to the initial sign of the voltage pulse, we additionally employ a particle-hole transformation which effectively inverts the unit of charge.[10] Together, these transformations map the problem of the generation of the hole on the problem of the generation of the electron. In the following, we describe these transformations more formally.

Below, we want to prove that the voltage pulse $\hat{U}_{\mathrm{L}}$ acting on the ground state $|\Phi_{\mathrm{F}}\rangle_{\mathrm{L}}$ generates a single-hole,

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[7] This conclusion relies on the fact that $U(q)$ only depends on the transferred momentum q, which is valid in the present setup of a voltage driving the single-particle excitation. Allowing $U(k_1, k_2)$ to depend both on the incoming k_1 and on the outgoing k_2 momentum, other shapes become possible, see for example Ref. [7].

[8] Keeling *et al.* denote this decisive property as $\hat{U}$ being of rank 1.

[9] The fact that an operator $\hat{U}$ which evidently commutes with the particle number operator generates an additional particle is not a contradiction since the Fermi sea $|\Phi_{\mathrm{F}}\rangle$ contains an infinite number of particles. Similar ideas are also used in the context of bosonization of a 1D fermionic system, see Ref. [9].

[10] The 1D Fermi system with linearized momentum is particle-hole symmetric with the symmetry operation $\Psi^\dagger(x_{\mathrm{R}}) \to e^{-ip_0 x_{\mathrm{R}}}\Psi(x_{\mathrm{R}})$ in the branch of right-moving states. Note that the symmetry operation inverts the momentum $k \to p_0 - k$; here we choose $p = 2k_{\mathrm{F}}$.

i.e., defining the operator B via

$$\hat{U}_{\rm L}|\Phi_{\rm F}\rangle_{\rm L} = B|\Phi_{\rm F}\rangle_{\rm L}, \tag{26}$$

this operator generates a single-hole

$$B = \int \frac{dk}{2\pi} f(k+k_{\rm F}) a_{{\rm L},k} \tag{27}$$

in a specific state with amplitude $f(k+k_{\rm F})$. The scattering matrix for the left-moving states is given by

$$\hat{U}_{\rm L} = \exp\left[-i\int dx_{\rm R}\, \phi(x_{\rm L}/v_{\rm F}) \Psi_{\rm L}^\dagger(x_{\rm L})\Psi_{\rm L}(x_{\rm L})\right], \tag{28}$$

with $x_{\rm L} = x + v_{\rm F}t$ and the different sign of the phase with respect to Eq. (13) appears due to the fact that the left-moving particles traverse the voltage in the opposite direction. In a first step, we observe that instead of considering the left-moving states, we can equivalently perform a parity transformation $x \to x^{\rm P} = -x$ and then deal with the right-moving states. The parity transformation changes the branch $E_{\rm L} \to E_{\rm L}^{\rm P} = E_{\rm R}$ as well as the momentum $k \to k^{\rm P} = -k$, such that the annihilation operators transform according to $a_{{\rm L},k} \to a_{{\rm L},k}^{\rm P} = a_{{\rm R},-k}$ and $\hat{U}_{\rm L} \to \hat{U}_{\rm L}^{\rm P} = \hat{U}_{\rm R}^\dagger$ [cf. Eqs. (13) and (28)]. Applying the parity transformation, Eq. (26) reads

$$\hat{U}_{\rm R}^\dagger|\Phi_{\rm F}\rangle_{\rm R} = B^{\rm P}|\Phi_{\rm F}\rangle_{\rm R}. \tag{29}$$

Next, we implement a particle-hole transformation $a_{{\rm R},k_{\rm F}+\varkappa}^\dagger \to \bar{a}_{{\rm R},k_{\rm F}+\varkappa}^\dagger = a_{{\rm R},k_{\rm F}-\varkappa}$ on the branch of the right-moving states which has been chosen in such a way as to keep the excitation energy $v_{\rm F}\varkappa$ (and thereby the Fermi level $k_{\rm F}$) as well as the anti-commutation relations unchanged. The particle-hole transformation can be implemented on the level of field operators

$$\begin{aligned}\Psi_{\rm R}^\dagger(x_{\rm R}) \to \bar{\Psi}_{\rm R}^\dagger(x_{\rm R}) &= \int \frac{d\varkappa}{2\pi} e^{-i(k_{\rm F}+\varkappa)x_{\rm R}}\, \bar{a}_{{\rm R},k_{\rm F}+\varkappa}^\dagger \\ &= e^{-2ik_{\rm F}x_{\rm R}} \int \frac{d\varkappa}{2\pi} e^{i(k_{\rm F}-\varkappa)x_{\rm R}}\, a_{{\rm R},k_{\rm F}-\varkappa} \\ &= e^{-2ik_{\rm F}x_{\rm R}}\, \Psi_{\rm R}(x_{\rm R}).\end{aligned}$$

Using this relation, we can transform the scattering operator

$$\begin{aligned}\overline{\hat{U}_{\rm R}^\dagger} &= \exp[-i\int dx_{\rm R}\, \phi(-x_{\rm R}/v_{\rm F}) \bar{\Psi}_{\rm R}^\dagger(x_{\rm R})\bar{\Psi}_{\rm R}(x_{\rm R})] \\ &= \exp[-i\int dx_{\rm R}\, \phi(-x_{\rm R}/v_{\rm F}) \Psi_{\rm R}(x_{\rm R})\Psi_{\rm R}^\dagger(x_{\rm R})] \\ &= c\exp[i\int dx_{\rm R}\, \phi(-x_{\rm R}/v_{\rm F}) \Psi_{\rm R}^\dagger(x_{\rm R})\Psi_{\rm R}(x_{\rm R})] \\ &= c\,\hat{U}_{\rm R},\end{aligned} \tag{30}$$

where c is a phase factor which we will set equal to one in the following.[11] After the particle-hole transformation, Eq. (29) reads

$$\hat{U}_{\rm R}|\Phi_{\rm F}\rangle_{\rm R} = \overline{B^{\rm P}}|\Phi_{\rm F}\rangle_{\rm R}. \tag{31}$$

The comparison of Eq. (31) with Eq. (16) yields $\overline{B^{\rm P}} = A^\dagger$ from where B can be obtained by inverting and subsequent usage of Eq. (17). Finally, we obtain

$$\begin{aligned}B = \overline{A^\dagger}^{\rm P} &= \left[\int \frac{dk}{2\pi} f(k-k_{\rm F})\bar{a}_{{\rm R},k}^\dagger\right]^{\rm P} \\ &= \int \frac{d\varkappa}{2\pi} f(\varkappa) a_{{\rm R},k_{\rm F}-\varkappa}^{\rm P} = \int \frac{d\varkappa}{2\pi} f(\varkappa) a_{{\rm L},\varkappa-k_{\rm F}} \\ &= \int \frac{dk}{2\pi} f(k+k_{\rm F}) a_{{\rm L},k}\end{aligned} \tag{32}$$

proving (26). The application of the voltage pulse creates a single-hole excitation with wave function $f(k+k_{\rm F})$ which moves to the left. Similar to the case of the particle excitation in the right-moving states (16), this state can be described in first-quantized language.

6. CONCLUSION

We have shown how the application of a unit-flux Lorentzian voltage pulse leads to single-particle excitations, an electron moving to the right and a hole moving to the left. These excitations can be described in a first-quantized language. For general integer-flux pulses, it was proven that the average transported charge is determined by the integral of the voltage alone, independent of the pulse shape. The proof has been outlined within a linear-spectrum approximation and using an infinitely-deep Fermi sea. The effect of the quadratic spectrum on the present results remains an interesting problem for further studies.

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[11] Note that the phase factor c appearing when commuting the operators in Eq. (30) is undefined as a factor $\delta(0)$ appears. This fact poses no further problems as an overall phase factor in a state is not measurable.

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