MOMENTUM DEFICIT IN QUANTUM GLASSES

KAPITZA INSTITUTE, MOSCOW

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CONTENTS

- Introduction: Three different states of quantum solids:
  - superfluid solids
  - normal solids
  - quantum glasses (QG) at low temperature are essentially different from both superfluid and normal solids
- Two level systems in moving quantum solids: peculiar phenomenon of momentum deficit
- Quantitative theory of QG

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Superfluid solid

A.A. and I.Lifshitz (1969); G.Chester (1970); A.Leggett (1970)

Owing to the large probability of quantum tunneling of atoms, solid helium may be superfluid

General motion of a superfluid solid is characterized by two independent velocities:

- solid bulk velocity $\mathbf{v}$
- superfluid velocity $\mathbf{v}_s$

Superfluid solid

- $\mathbf{v}_s = 0$ in a rotating superfluid solid. Rotational inertia is determined by the normal fraction
- $\mathbf{v} = 0$ in a capillary. Mass transfer is determined by the superfluid fraction

Experimental situation

Reduction of solid ^{4}He rotational inertia below 0.2K in torsional oscillator experiments (Kim and Chan, 2004) interpreted as superfluidity.

- superfluid fraction $\sim 1\%$
- critical velocity $\sim 10\mu m/s$

Further experiments (A.Rittner and J.Reppy, 2007): Superfluid fraction in highly disordered (glassy) samples $\sim 20\%$. It is absent in perfect crystals.

Experiment of J.Day *et al.* (2006) on superfluid flow under pressure difference gives the upper limit of critical velocity which is **seven** orders of magnitude smaller than the value obtained by Kim and Chan.

Experimental situation

- large reduction of rotational inertia is observed in disordered (glassy) samples of ^{4}He
- no reduction in perfect crystals
- no superflow

Glasses

P.Anderson *et al.* (1972); W.Philips (1972)

Low temperature properties of classical glasses are explained by the presence of two level systems (TLS)

TLS is an atom, or a group of atoms, which can tunnel between two localized states characterized by a small energy difference.

We show: owing to TLSs, quantum glasses (He, H_2) manifest properties which are exactly what we need to explain experimental results:

- General macroscopic motion is described by single velocity $\mathbf{v}$ of solid bulk (no supercurrent).
- Reduced inertia in oscillator experiments: momentum density is $\mathbf{j} = (\rho - \rho_d)\mathbf{v}$, where ρ_d is a mass density deficit. We calculate ρ_d in terms of TLS parameters.
- Normal behavior in uniform rotation.

TLS in a moving solid

Hamiltonian for a TLS in a stationary solid can be written as

$$H_0 = -\varepsilon\sigma_3 + J\sigma_1,$$

where $\mp\varepsilon$ ($\varepsilon > 0$) are energies of two localized states, J is the tunneling amplitude, and σ_i ($i = 1, 2, 3$) are the Pauli matrices. Let the tunneling be accompanied by displacement of a mass m by a vector $\mathbf{a}$. The coordinates of the center of gravity of the TLS can be written as $\mathbf{r}_{1,2} = \mp\mathbf{a}/2$.

TLS in a moving solid

The corresponding operator of the coordinate is

$$\mathbf{r} = -\sigma_3 \mathbf{a}/2.$$

The operator of velocity is determined by the commutator:

$$\dot{\mathbf{r}} = \frac{i}{\hbar}[H_0, \mathbf{r}] = -\frac{J\mathbf{a}}{\hbar}\sigma_2.$$

The TLS momentum in a stationary solid is

$$\mathbf{p} = m\dot{\mathbf{r}} = -\frac{mJ\mathbf{a}}{\hbar}\sigma_2.$$

TLS in a moving solid

The TLS Hamiltonian and momentum in the frame in which $\mathbf{v}$ is finite are determined by Galilean transformations

$$H_0 \to H_0 + \mathbf{pv} + mv^2/2,$$
$$\mathbf{p} \to \mathbf{p} + m\mathbf{v}.$$

Both expressions must be renormalized by inclusion the last terms to the total kinetic energy and momentum of the solid bulk.
So, the TLS Hamiltonian and momentum in the solid moving with velocity $\mathbf{v}$ are

$$H = H_0 + \mathbf{pv} \quad \text{and} \quad \mathbf{p}.$$

TLS in a moving solid

TLS Hamiltonian is the same as that for spin 1/2 in an external field. It can be written as

$$H = -h_\alpha \sigma_\alpha .$$

Here $\alpha = 1, 2, 3$, h_α is the "field" with components $h_\alpha = (-J, Ju, \varepsilon)$, where $u = \mathbf{a}\mathbf{v}m/\hbar$ is dimensionless velocity. Generally the TLS density matrix w is determined by a real vector s_α of polarization

$$w = (1 + s_\alpha \sigma_\alpha)/2.$$

We have

$$\langle \sigma_\alpha \rangle = \mathrm{Tr}\, w\sigma_\alpha = s_\alpha .$$

Dynamic equation for free TLS is

$$\hbar \dot{s}_\alpha = e_{\alpha\beta\gamma} h_\beta s_\gamma ,$$

where $e_{\alpha\beta\gamma}$ is Levi-Civita symbol.

Uniform rotation

Equilibrium density matrix of TLS in **uniformly** rotating frame is

$$\mathrm{e}^{(f'-H')/T},$$

where f' and H' are the free energy and Hamiltonian in this frame. We have

$$H' = H - \omega \mathbf{M} = H_0 + \mathbf{pv} - \omega \mathbf{M},$$

where ω is the angular velocity, $\mathbf{M} = \mathbf{R} \times \mathbf{p}$, $\mathbf{v} = \omega \times \mathbf{R}$, and $\mathbf{R}$ is the TLS coordinate with respect to the rotation axis. We obtain

$$H' = H_0.$$

Uniformly rotating QG behaves like normal solid — our prediction.

Adiabatic processes

However, suppose that the solid bulk velocity depends on time $\mathbf{v} = \mathbf{v}(t)$ and is "switched on" adiabatically. We consider two different cases:

1.
   - switching time t_s (or the torsional oscillator period) is much longer than the relaxation time in the solid
   - but much shorter than the time during which the solid can be regarded as thermally insulated — Kapitza thermal resistance.
2. switching time t_s is much shorter than the TLS relaxation time. TLSs can be regarded as free. Here the process is adiabatic if

$$t_s \gg \hbar/|h_\alpha|. \qquad \text{liberal condition}$$

Adiabatic process in equilibrium

The mean value of the TLS momentum is

$$\langle \mathbf{p} \rangle = \left\langle \frac{\partial H}{\partial \mathbf{v}} \right\rangle = \left(\frac{\partial f}{\partial \mathbf{v}} \right)_T ,$$

where

$$f = -T \log \left(\mathrm{Tr}\ \mathrm{e}^{-H/T} \right).$$

is the TLS free energy and $\mathbf{v} = \mathbf{v}(t)$. We get

$$p_i = -m_{ik}^{(d)} v_k ,$$

where

$$m_{ik}^{(d)} = \frac{J^2 m^2}{\hbar^2 E} a_i a_k \tanh \frac{E}{T},$$

$$E = \left(\varepsilon^2 + \Delta^2 \right)^{1/2},$$

$$\Delta = J \left(1 + u^2 \right)^{1/2},$$

Adiabatic process for free TLS

There are two integrals of motion in adiabatic process for free TLS: $|s_\alpha|$ and the "angle" between s_α and h_α. Until the velocity is switched on:

$$|s_\alpha| = \tanh \frac{(\varepsilon^2 + J^2)^{1/2}}{T}.$$

After it is switched on:

$$s_2 = \frac{Ju(t)}{E}|s_\alpha|.$$

Finally,

$$p_i = -\frac{mJa_i}{\hbar}s_2 = -m_{ik}^{(d)}v_k,$$

where

$$m_{ik}^{(d)} = \frac{J^2m^2}{\hbar^2 E}a_i a_k \tanh \frac{(\varepsilon^2 + J^2)^{1/2}}{T},$$

Momentum deficit

The total momentum density $\mathbf{j}$ is determined by

$$\mathbf{j} = \int \mathbf{p} N d\varepsilon,$$

where $Nd\varepsilon$ ($N = \text{const}$) is the number of TLSs per unit volume of the solid and per interval of the energy half-difference $d\varepsilon$ near some ε such that $\max(\Delta, T) \ll \varepsilon \ll U$. Here U is the characteristic height of the energy barriers in the solid.

With logarithmic accuracy we get the same result for both cases considered above

$$j_i = \rho v_i - \rho_{ik}^{(d)} v_k,$$

where the density deficit tensor is

$$\rho_{ik}^{(d)} = \left\langle m^2 J^2 a_i a_k \right\rangle \frac{N}{\hbar^2} \int_{\max(\Delta, T)}^{U} \frac{d\varepsilon}{\varepsilon}.$$

Momentum deficit

For an isotropic system (glass) we have $\rho_{ik}^{(d)} = \rho_d \delta_{ik}$, where

$$\rho_d = \frac{N}{3\hbar^2} \left\langle m^2 J^2 a^2 \right\rangle \log \frac{U}{\max(\Delta, T)}.$$

- The critical temperature of the phenomenon is of the order of $\Delta = J\left(1 + u^2\right)^{1/2}$
- The critical velocity v_c is determined by $u_c \sim 1$ or $v_c \sim \hbar/(ma)$. Experimentally v_c is very small. This suggests the macroscopic character of TLSs.
- ^{3}He impurities must bind to TLSs destroying them.