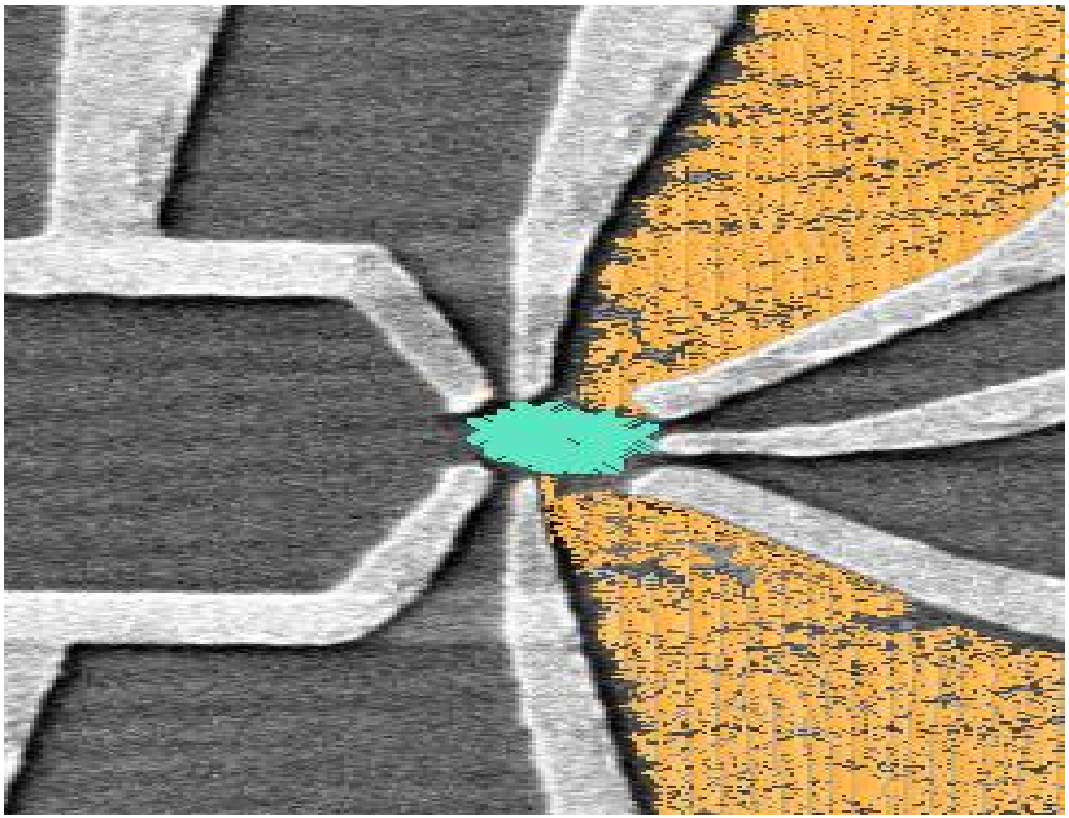
Quantum Impurities Out of Equilibrium



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Landau100 – June 2008

Quantum Impurities: strong correlations out-of-equilibrium

Experimentally well studied : *Goldhaber-Gordon et al, Cronenwett et al, Schmid et al*

Theoretically - example of interplay of strong correlation and nonequilibrium

- **Nonequilibrium - poorly understood**

- Non-equilibrium systems are all different- it is unclear what if anything they all have in common.
  - **No unifying theory such as Boltzmann's statistical mechanics**
- Many of our standard physical ideas and concepts are not applicable (Scaling? RG? Universality?)
- New inherently non-equilibrium phenomena:
  - e.g. **entropy production, dissipation**

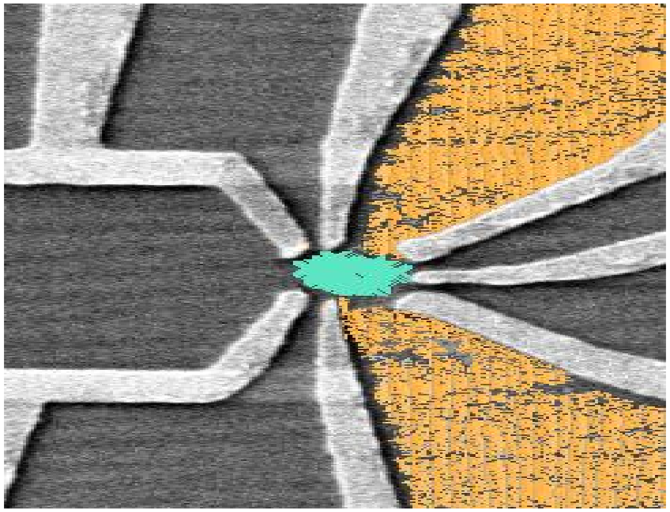
- **Strong correlations - poorly understood**

  Perturbative approaches fail
  - New degrees of freedom emerge in the infra-red
  - New collective Behavior

Can fully discuss issues – in quantum impurity context

Quantum Impurities out-of-Equilibrium

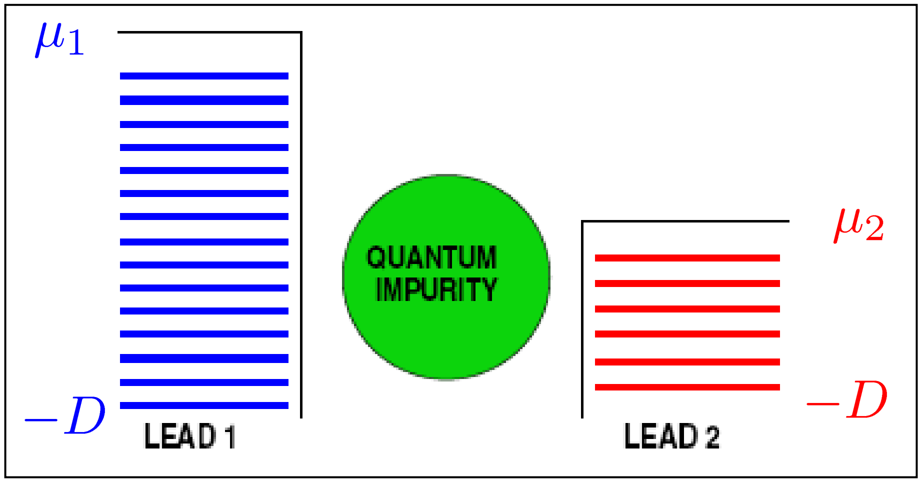
- **The quantum impurity - *experimentally*:**



- **Couple impurity to leads with** $\mu_1 \neq \mu_2$
- **<u>Non-equil steady state</u> (NESS) is established:**
  - **- currents flow time independent (after transients)**
- **Measure non-equil current in steady state**

$$I = I(V), \quad V = \mu_1 - \mu_2$$

- **The quantum impurity - *theoretically*:**



$$H = H_{leads} + H_{imp} + H_{leads-imp}$$

- How to compute $I = I(V)$?

Leads = Fermi seas, i=1,2

$$H_{leads} = \sum_{i,k} \epsilon_k c^\dagger_{i\vec{k}} c_{i\vec{k}}$$

Non-equilibrium

$$V = \mu_1 - \mu_2 \neq 0$$

Non-equilibrium: Time-dependent Description

Given H - how to set up the non-equilibrium problem?

Keldysh
- $t \leq t_o$, leads decoupled, system described by: ρ_o
- $t = t_o$, couple leads to impurity
- $t \geq t_o$, evolve with $H(t) = H_o + e^{\eta t} H_1$

Description of Nonequilibrium involves two elements: H, ρ_o or H, H_o

For T > 0 :

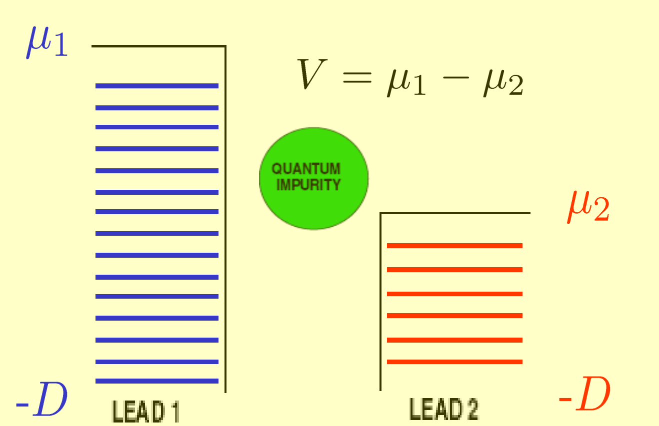
1. initial condition: ρ_o
2. evolution: $U(t,t_o) = T\{e^{-i\int_{t_o}^{t} dt' H(t')}\}$
3. density matrix: $\rho(t) = U(t,t_o)\rho_o U^\dagger(t,to)$
4. non equil value: $\langle \hat{O}(t) \rangle = Tr\{\rho(t)\hat{O}\}$

For T = 0 :

1. initial condition: $|\phi_o, V\rangle$
2. evolution: $U(t,t_o) = T\{e^{-i\int_{t_o}^{t} dt' H(t')}\}$
3. evolved state: $|\psi(t)\rangle_V = U(t,t_o)|\phi_o, V\rangle$
4. non-equil value: $\langle \hat{O}(t) \rangle_V = \langle \psi(t)|\hat{O}|\psi(t)\rangle_V$

The initial condition at T=0:

$$|\phi_o\rangle = |\phi_o, V\rangle$$
$$= |bath1\rangle \otimes |bath2\rangle \otimes |\alpha\rangle$$



The Steady State (open system)

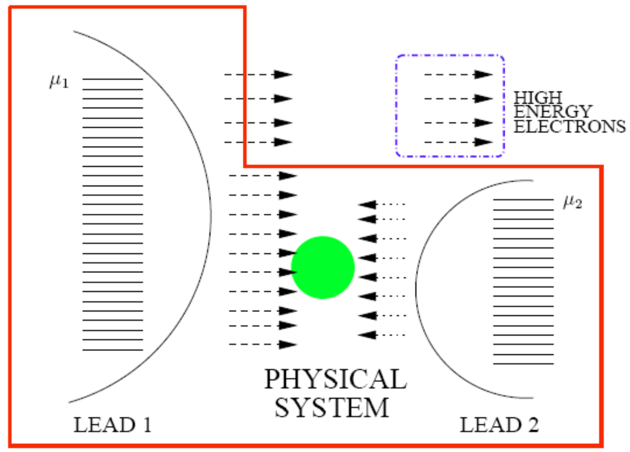
Non-equilibrium steady states (NESS): when do they occur?

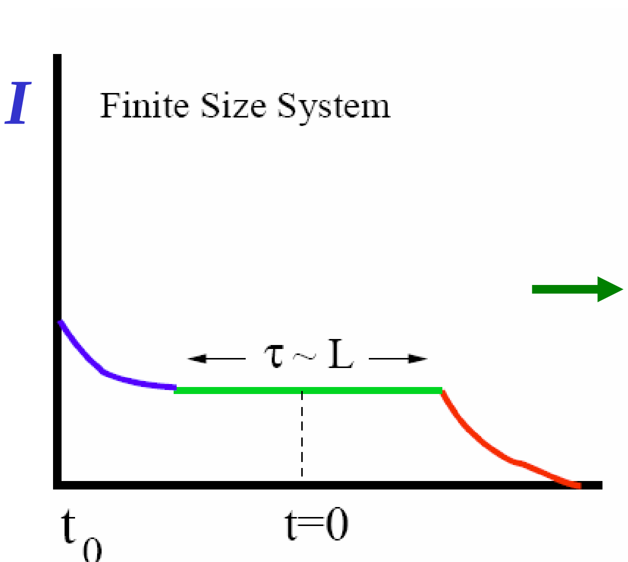
- **Leads good thermal baths, infinite volume limit - open system**

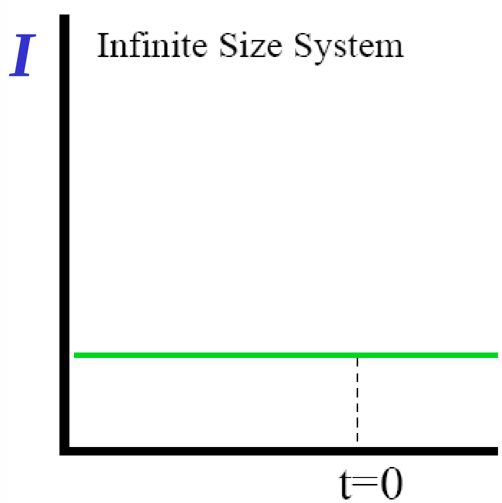
  $\Rightarrow \exists \lim_{t_o \to -\infty}$, **no *IR* divergences,** $\frac{1}{L} \ll \frac{1}{|t_0|} \ll \eta \to 0$ (**Doyon, NA 2005**)

Open system limit :

- **Dissipation mechanism**
- **Time-reversal sym. breaking**
- **Steady-state non- eq. currents**







A steady state ensues

$$\langle \hat{O}(t) \rangle = \langle \hat{O} \rangle$$

The Steady State – time independent description

The **open system** limit $\frac{1}{L} \ll \frac{1}{|t_o|} \ll \eta \to 0$ implies:

$$|\psi, V\rangle_s = U(0, -\infty)|\phi_o, V\rangle$$

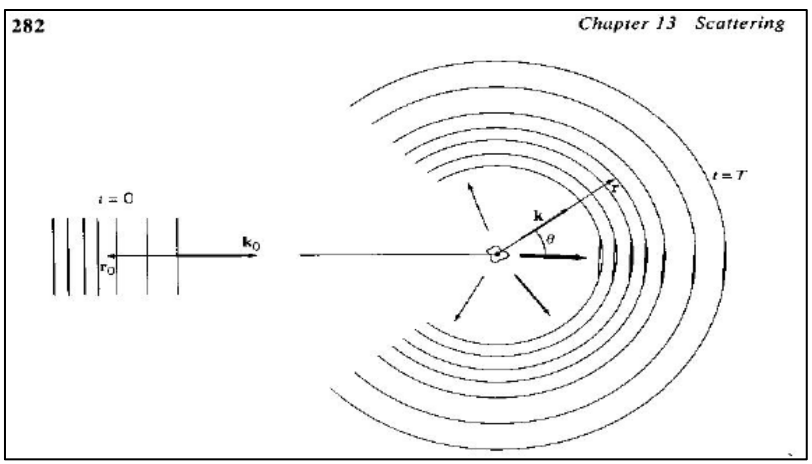
well defined state

Properties:

- $|\psi, V\rangle_s$ **eigenstate** of $H = H_0 + H_1$ (Gellman-Low thm)
- **Lippmann-Schwinger** equation, $|\phi_o, V\rangle$ -boundary condition

$$|\psi, V\rangle_s = |\phi_o, V\rangle + \frac{1}{E - H_0 + i\eta} H_1 |\psi, V\rangle_s$$

- $|\phi_o, V\rangle$: Initial condition $\to$ boundary condition
- $|\psi, V\rangle_s$ **scattering state** - eigenstate on the **infinite line**



from Merzbacher: $\psi(x)$ eigenstate of

$$H = \frac{1}{2m} p^2 + V(x)$$

with incoming boundary condition

$$\psi(x) \to \phi_o(x) = e^{i\vec{p}\cdot\vec{x}}$$

Note - both H, H_o enter description

Steady State at T>0

- *At T=0 -- single scattering state*
- *At T >0 -- many scattering states*

L-S with arbitrary boundary conditions:

- ***For T=0,*** $|\phi_o\rangle \xrightarrow{\text{L-S}} |\psi\rangle_s$ $\quad |\phi_o\rangle$ ***g.s. of*** $H_0 - \sum_i \mu_i N_i$
- ***Generally,*** $|\phi_n\rangle \xrightarrow{\text{L-S}} |\psi_n\rangle_s$ ***where*** $|\phi_n\rangle \in \mathcal{H}_o^{\perp}$
- ***For T>0,*** $''$free leads$''$ boundary conditions: $p_n^o = e^{-\beta E_n^o}/Z_o$

$$\rho_o = \sum_n p_n^o |\phi_n\rangle\langle\phi_n| \quad \longrightarrow \quad \rho_s = \sum_n p_n^o |\psi_n\rangle\langle\psi_n|_s$$

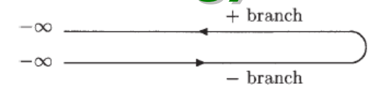
and: $\langle \hat{O} \rangle = Tr \rho_s \hat{O}$

$\rightarrow$ **In steady state - $\exists$ "non-thermal" density operator!**

Non-equilibrium Steady-states & Scattering States

- Time dependent **(Keldysh) vs.** time independent approach **(Scattering)**

$$I(V) = \langle\, \phi_o, V \,|\, U^\dagger(0,-\infty)\; \hat{I}\; U(0,-\infty) \,|\, \phi_o, V \,\rangle \equiv \langle\, T_c\, e^{-i\int_{-\infty}^{0} H(t')dt'}\; \hat{I}(0) \,\rangle$$



-Keldysh approach

$$= \langle \psi, V \,|\hat{I}\,| \psi, V \rangle_s$$

-Scattering approac

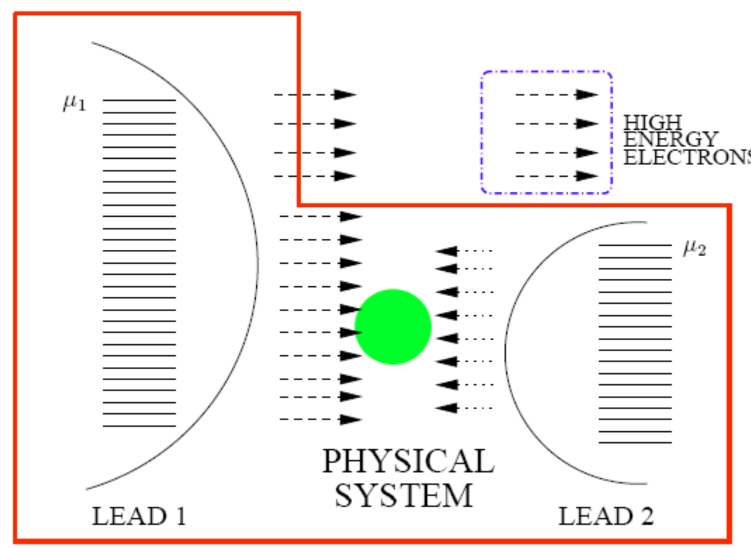
- Scattering approach: $|\psi, V\rangle_s \longleftrightarrow$ *non-perturbative Keldysh*

- **The scattering eigenstate $|\psi, V\rangle_s$ describes all aspects of non-equilibrium steady-state physics (NESS):**

- non-equilibrium currents,
- energy dissipation,
- entropy production

Q: How can an eigenstate describe dissipation, entropy production?

A: **Scattering eigenstate describes both *system* <u>and</u> *environment (open system)***



The Scattering Bethe-Ansatz

HOW TO CONSTRUCT $|\psi\rangle_s$, $(for\ T=0)$? **OR** ρ_s, $(for\ T>0)$?

- Keldysh perturbation theory - fails in general (IR div)
- RG is inapplicable, $|\Phi_o\rangle$ highly excited

Develop a Bethe Ansatz approach to non-equilibrium:

- **Traditional Bethe-Ansatz - inapplicable**
  - Periodic boundary conditions
  - **Closed System**: Equilibrium, Thermodynamics
- **New technology ⟶ Scattering States**
  - Asymptotic Boundary conditions on the infinite line
  - **Open System**: Non-equilibrium, scattering problems

Scattering Bethe-Ansatz:

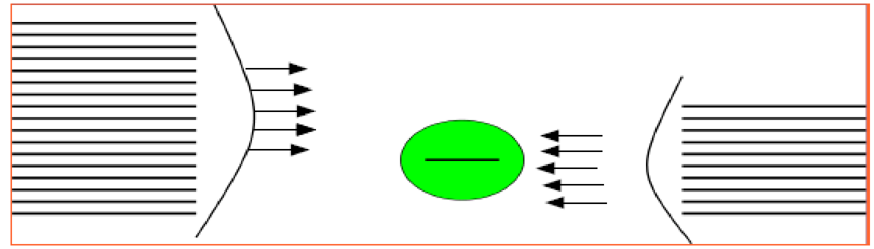
$$H|\psi\rangle_s = E|\psi\rangle_s \quad \text{with scattering BC on } \infty\text{-line}$$

Other approaches (applicable to Boundary sine-Gordon only : same bare and reno exc - non generic situation)

1. *Boltzmann + integrability (Fendley, Ludwig, Saleur 95')*
2. *Mapping to effective H (Bazhanov, Lukyanov, Zamolodchikov 98')*

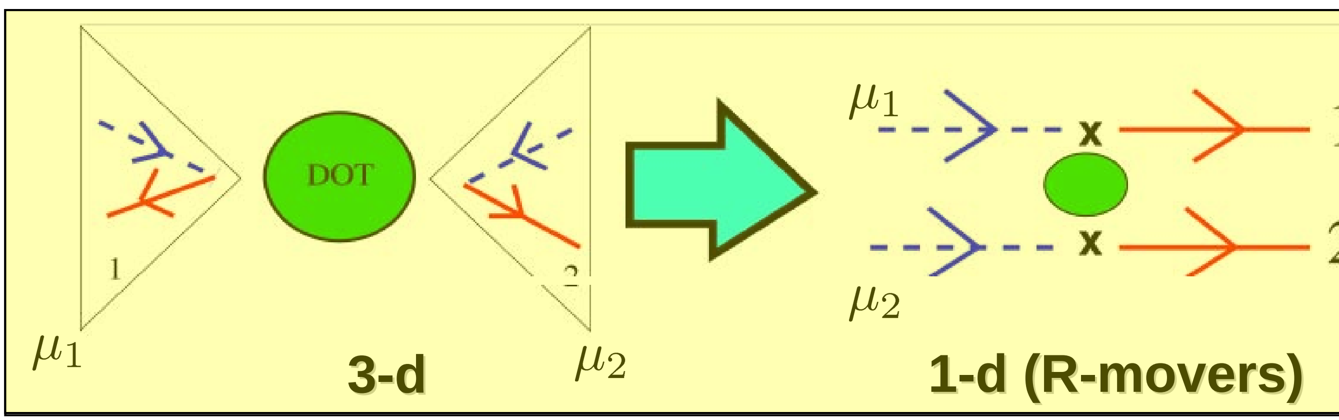
First example: The IRL model out of equilibrium

- **Interacting Resonance Level Model:**



$$H_{IRL} = \sum_{i=1,2\,\vec{k}} \epsilon_k c^\dagger_{i\vec{k}} c_{i\vec{k}} + \epsilon_d d^\dagger d + t \sum_{i=1,2\,\vec{k}} (c^\dagger_{i\vec{k}} d + h.c) + U \sum_{i=1,2\,\vec{k}} c^\dagger_{i\vec{k}} c_{i\vec{k}} d^\dagger d$$

- **Unfold quantum impurity $\longrightarrow$ 1- d theory:**



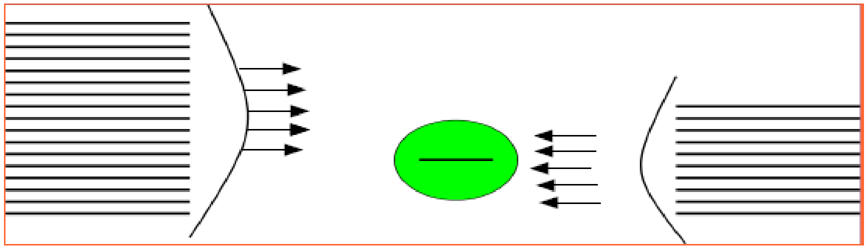
$$\psi_{i\epsilon a} \equiv \int d^3k\, \delta(\epsilon_{\vec{k}} - \epsilon)\, c_{i\vec{k}a}$$
$$\{\psi_{i\epsilon a}, \psi^\dagger_{j\epsilon' b}\} = \delta_{ab}\,\delta_{ij}\,\delta(\epsilon - \epsilon')\nu(\epsilon)$$
$$\psi_{ia}(x) = \int_{-D}^{D} \frac{d\epsilon}{\sqrt{\nu}}\, e^{i\epsilon x}\, \psi_{i\epsilon a}$$

Affleck and Ludwig 95'

- **Field Theory:**
  - low-energy physics, universality
  - cut-off to infinity, dos constant $\nu = 1/2\pi,\ v_F = 1$

IRL: The Scattering State I

- **Interacting Resonance Level Model:**



- **The 1-d field theory:**

$$H = -i\sum_i \int \psi_i^\dagger(x)\partial\psi_i(x) + \epsilon_d d^\dagger d + t\,[(\psi_1^\dagger(0)+\psi_2^\dagger(0))d + h.c.]$$

$$+U\,[\psi_1^\dagger(0)\psi_1(0) \;+\; \psi_2^\dagger(0)\psi_2(0)]\,d^\dagger d$$

***Diagonalize H via* Scattering Bethe-Ansatz:**

diagonalize directly on the infinite line (open system)

- construct 1-particle eigenstates (with boundary conditions)
- construct N-particle eigenstates out of 1-particle states

$$H|F_N\rangle = E_N|F_N\rangle \quad N=1,2...$$

IRL: The Scattering State II

Interaction vs. boundary conditions:

even/odd fields - : interactions - H

1 / 2 fields - : boundary conditions - H_o

- ***Interactions - in even/odd variables:*** $\psi_{e/o} = (\psi_1 \pm \psi_2)/\sqrt{2}$

$$H_{IRL} = H_e + H_o$$

$$\begin{cases} H_e = -i\int \psi_e^\dagger(x)\partial\psi_e(x) + \epsilon_d d^\dagger d + t\left(\psi_e^\dagger(0)d + h.c.\right) + U\psi_e^\dagger(0)\psi_e(0)d^\dagger d \\ H_o = -i\int \psi_o^\dagger(x)\partial\psi_o(x) + U\psi_o^\dagger(0)\psi_o(0)d^\dagger d \end{cases}$$

H_e integrable *(Filyov-Wiegman 1980)*

- ***Boundary conditions in 1 / 2 variables***

$$\begin{cases} \psi_1(x) = (\psi_e(x) + \psi_o(x))/\sqrt{2} \\ \psi_2(x) = (\psi_e(x) - \psi_o(x))/\sqrt{2} \end{cases}$$

IRL: The Scattering State III

- ***Single particle state:*** $|F_1\rangle = \int dx[A(g_p(x)\psi_e^\dagger(x) + e_p\delta(x)d^\dagger) + Bh_p(x)\psi_o^\dagger(x)]|0\rangle$

$$H_{IRL}|F_1\rangle = p|F_1\rangle \longrightarrow \begin{cases} -i\partial_x g_p(x) + te_p\delta(x) = pg_p(x) \\ tg_p(0) + \epsilon_d e_p = pe_p \\ -i\partial_x h_p(x) = ph_p(x) \end{cases}$$

$$g_p(x) = e^{ipx}[\theta(-x) + e^{i\delta_p}\theta(x)],$$

$$h_p^\pm(x) = \begin{cases} e^{ipx} & x \neq 0 \\ \pm\frac{e^{i\delta_p}+1}{2} & x = 0 \end{cases}$$

$$e_p = t/(p - \epsilon_d + i\Delta)$$

Discontinuous wave functions:
-"Bethe basis " for H_0
appropriate for interactions

local discontinuities consistent with linear derivatives

$\theta(\pm x)\delta(x) = (1/2)\delta(x)$

Phase shift $\delta_p = 2tan^{-1}\left[\frac{\Delta}{(p-\epsilon_d)}\right]$

Level width $\Delta = t^2/2$

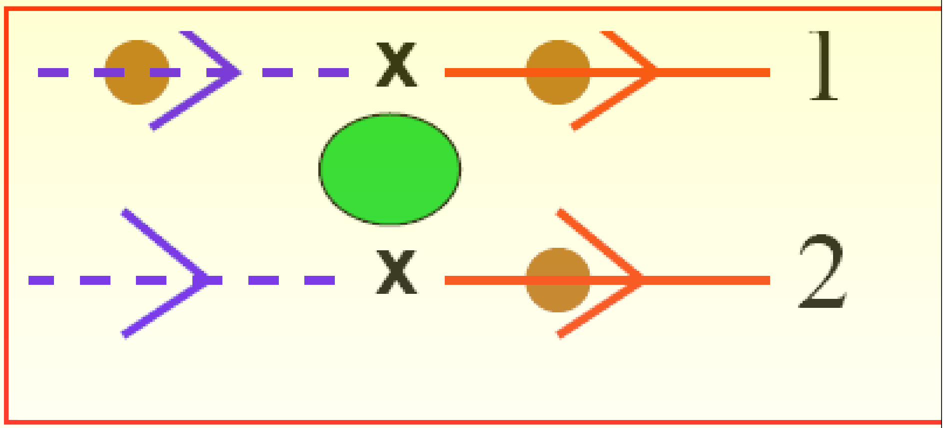
$$|F_1\rangle = \int dx\, e^{ipx}[(A+B)\theta(-x) + (Ae^{i\delta_p} + B)\theta(x)]\psi_1^\dagger(x)|0\rangle + \int dx\, e^{ipx}[(A-B)\theta(-x) + (Ae^{i\delta_p} - B)\theta(x)]\psi_2^\dagger(x)|0\rangle + e_p d^\dagger|0\rangle$$

- ***Single particle scattering state: choose A, B to impose non-eq BC***

IRL: The Scattering State IV

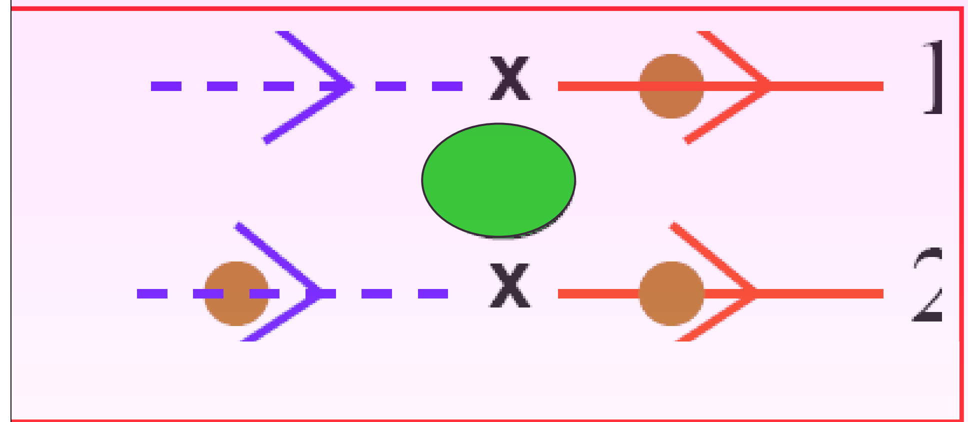
Single-particle scattering states:

$|1p\rangle$



$$|1p\rangle = \int dx\, e^{ipx}([\theta(-x) + R_p\,\theta(x)]\psi_1^\dagger(x) + T_p\,\theta(x)\psi_2^\dagger(x) + e_p\, d^\dagger\delta(x))|0\rangle = \int dx\, e^{ipx}\alpha_{1p}^\dagger(x)|0\rangle$$

$|2p\rangle$



$$|2p\rangle = \int dx e^{ipx}([\theta(-x) + R_p\,\theta(x)]\psi_2^\dagger(x) + T_p\,\theta(x)\psi_1^\dagger(x) + e_p\, d^\dagger\delta(x))|0\rangle = \int dx\, e^{ipx}\alpha_{2p}^\dagger(x)|0\rangle$$

Phase shift $\delta_p = 2tan^{-1}[\frac{t^2}{2(p-\epsilon_d)}]$

Reflection $R_p = \frac{e^{i\delta_p}+1}{2},$

Transmission $T_p = \frac{e^{i\delta_p}-1}{2}, \quad |T_p|^2 = \frac{\Delta^2}{(p-\epsilon_d)^2+\Delta^2}$

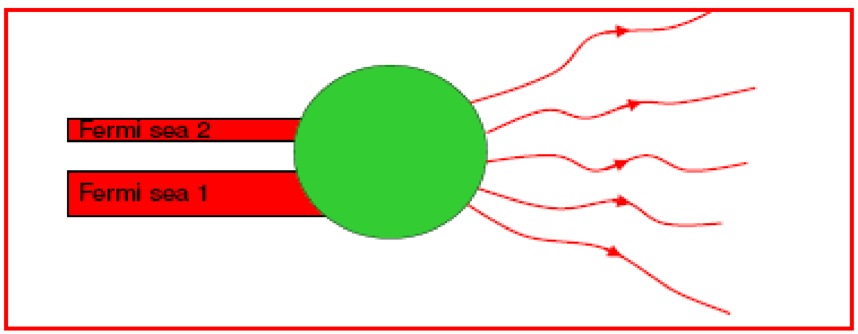
IRL: The Scattering State V

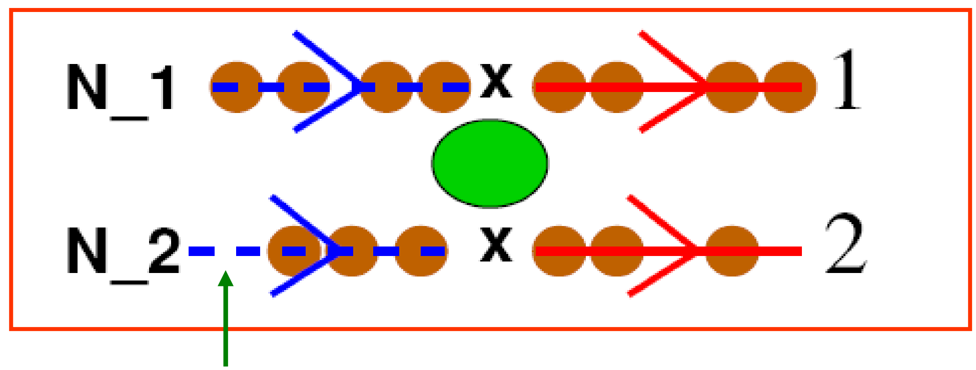
Multi-particle scattering state - *N_1 lead-1, N_2 lead-2,* $N_i \sim \mu_i$

$$|\{p\}\rangle_s = \int dx\, e^{i\sum_j^N p_j x_j} e^{i\sum_{j<l}^N \Phi(p_j,p_l) sgn(x_j - x_l)} \Pi_u^{N_1} \alpha_{1p}^\dagger(x_u) \Pi_v^{N_2} \alpha_{2p}^\dagger(x_v)|0\rangle$$

with

$$e^{2i\Phi(p_i,p_j)} \equiv S(p_i,p_j) = \frac{i - \frac{U}{2}\frac{p_i - p_j}{p_i + p_j - 2\epsilon_d}}{i + \frac{U}{2}\frac{p_i - p_j}{p_i + p_j - 2\epsilon_d}}$$





away from impurity - eigenstate of H_o Bethe basis

- $|\{p\}\rangle_s$ **eigenstate for any choice of $\{p\}$. What distribution $\rho_i(p)$ to choose?**
- ***The distributions $\rho_i(p)$ describing free leads in the Bethe basis must satisfy SBA equation***

The Boundary Conditions VI

The boundary conditions become BA equations for: ρ_1, ρ_2

• Non-eq BC → momentum distributions $\rho_1(p),\ \rho_2(p)$:

– BA eqns with upper cut-offs $k_o^j = k_o(\mu^j)$, lower cut-off, D:

$$2\rho(p) = \frac{1}{2\pi} - \int_{-D}^{\kappa_0^2} 2\rho(k)\mathcal{K}(k,p)dk - \int_{\kappa_0^2}^{\kappa_0^1} \rho(k)\mathcal{K}(k,p)dk$$

$$\rho_2(p) = \rho(p) \quad p \leq \kappa_o^2$$

$$\rho_1(p) = \rho(p) \quad p \leq \kappa_o^1$$

$$N_j = \int_{-D}^{\kappa_o^j} \rho_j(p)$$

$$\mu_j = \partial_{N_j} \int dp\, p\, (\rho_1(p) + \rho_2(p))$$

with: $\mathcal{K}(p,k) = \frac{U}{\pi} \frac{(k-\tilde{\epsilon}_d)}{(p+k-2\tilde{\epsilon}_d)^2 + \frac{U^2}{4}(p-k)^2}$

- **BA eqns describe the free leads in the Bethe basis**
- **For U=0 distributions reduce to Fermi-Dirac distributions**

Comment:

These BA eqns valid for: $\epsilon_d \geq k_o^j$ **otherwise, eqns more complicated**

Current and Dot Occupation

The scattering state $|\{p\}\rangle_s^{\mu_1 \mu_2}$ is determined in terms of ρ_1, ρ_2

- **Current and dot-occupation:**

$$\hat{I} = \frac{i}{\sqrt{2}} t \sum_{j=1,2} (-1)^j (\psi_j^\dagger(0) d - h.c)$$

$$\hat{n}_d = d^\dagger d$$

- **Expectation values: $\hat{I}, \hat{n}_d$ in Scattering State: $|\{p\}\rangle_{L \to \infty}^{\mu_1, \mu_2}$**

$\Delta = t^2/2$

Hybridization width

$$\langle I \rangle_s^{\mu_1, \mu_2} = \int dp \, [\rho_1(p) - \rho_2(p)] \frac{\Delta^2}{(p - \epsilon_d)^2 + \Delta^2}$$

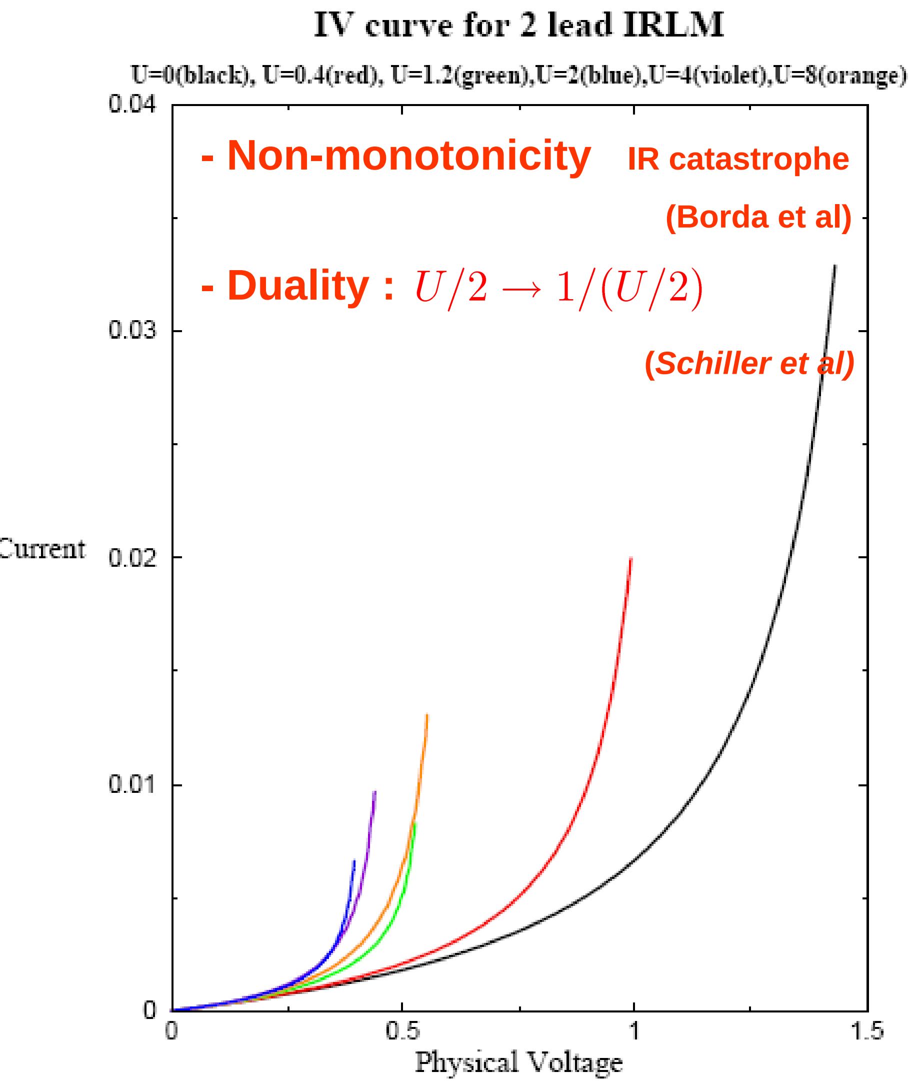
$$\langle n_d \rangle_s^{\mu_1, \mu_2} = \int dp \, [\rho_1(p) + \rho_2(p)] \frac{\Delta}{(p - \epsilon_d)^2 + \Delta^2}$$

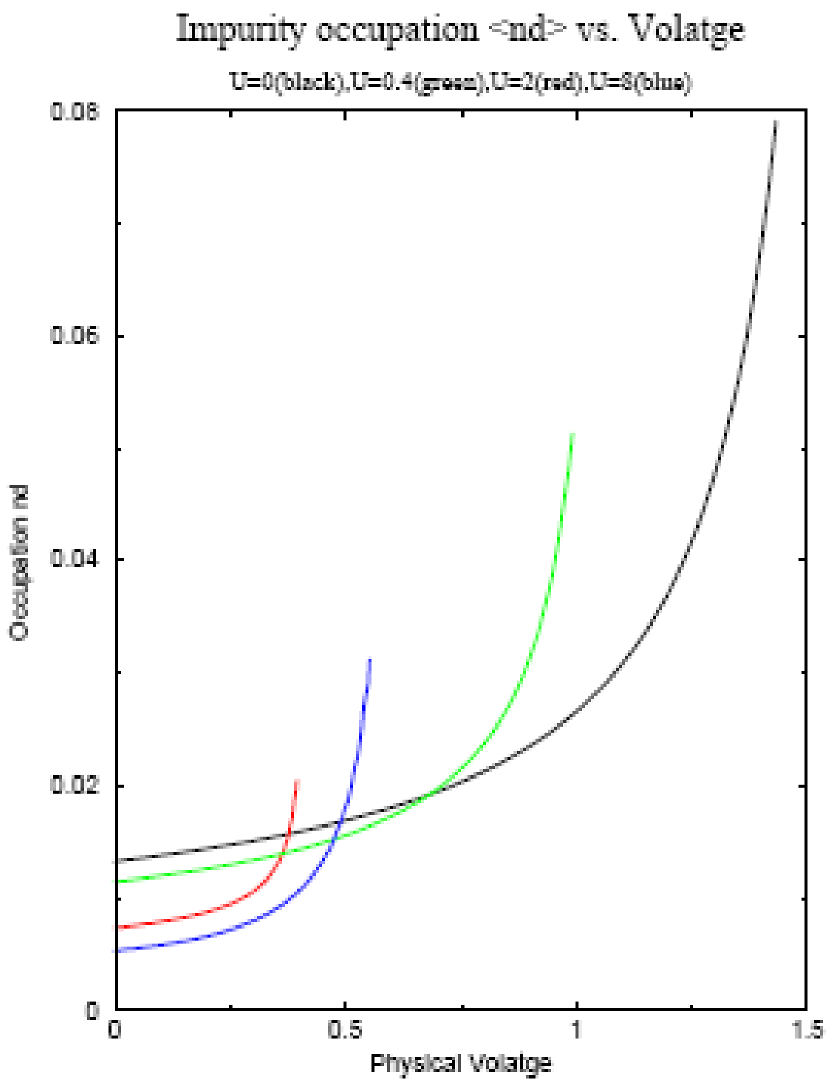
- **For U=0, Landauer-Buttiker formulas**

- **For U>0, in the Bethe-Anzatz basis, expressions look "simple":**
  - excitations undergo phase shifts only
  - $\rho_i(p)$ incorporate interactions and boundary conditions

Current vs. Voltage IRL

- **Compute Exactly current and dot occupation as a function of Voltage:**



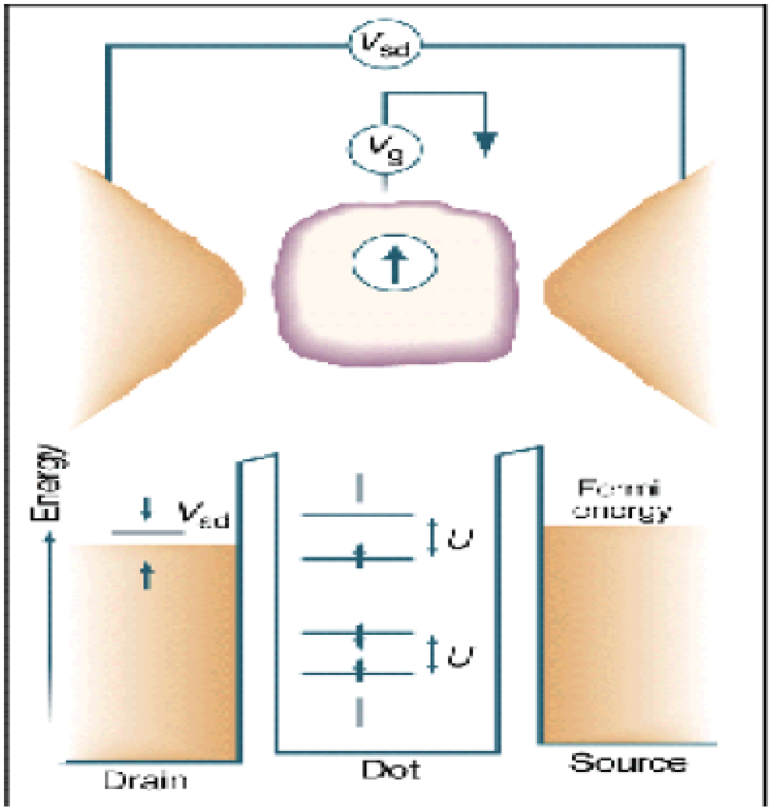


Sung Po Chao

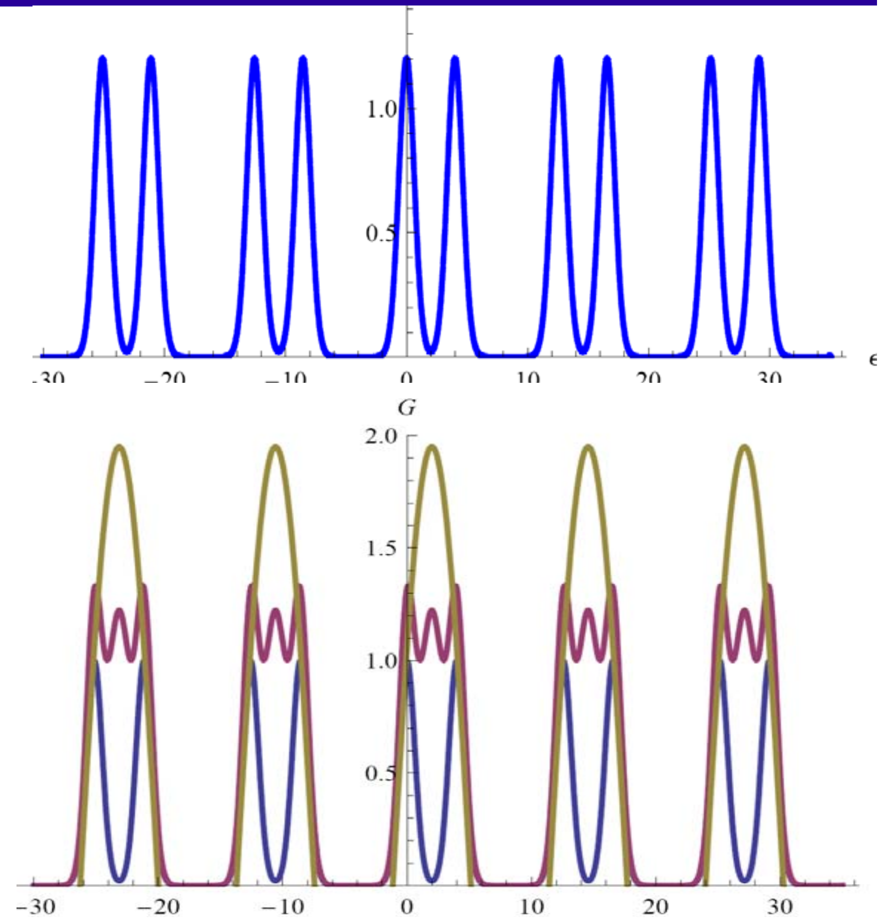
Still need express in universal form

- Can easily generalize to finite temperature case
- **Universality out of equilibrium?**: can change in **D** be compensated by change in **U** and Δ? RG invariants?

Example 2: Quantum Dot Out of Equilibrium



Inoshita:Science 24 July 1998: Vol. 281. no. 5376, pp. 526 - 527



- Can control the number of electrons on the dot using gate voltage
- For odd number of electrons- quantum dot acts like a **quantum impurity**
- New collective behaviors, e.g **Kondo effect**

$$H = -i \sum_{j=1,2} \int \psi^\dagger_{ja} \partial \psi_{ja}\, dx + \epsilon_d d^\dagger_a d_a + t \sum_{j=1,2} [\psi^\dagger_{ja}(0) d_a + h.c.] + U n_{d\uparrow} n_{d\downarrow}$$

Kondo in Quantum dot: *Glazman&Raikh, Ng&Lee,* Thermodynamic Solution: Wiegman *& Tsvelik* , *Okiji & Kawakami*

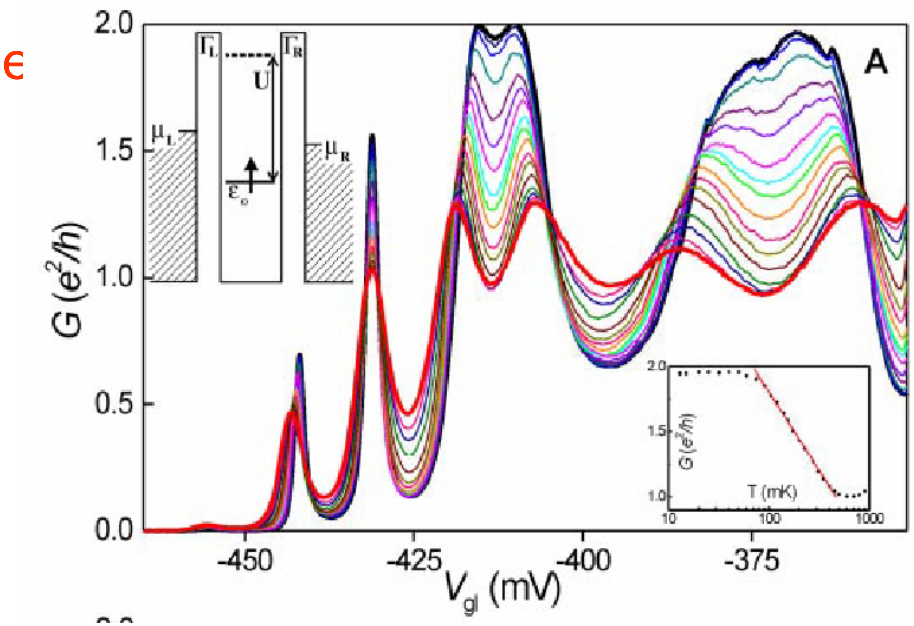
Quantum Dot – zero bias conductance

Zero bias conductance vs gate voltage
- Kondo enhancement in odd valleys

Essentially Equilibrium measurement

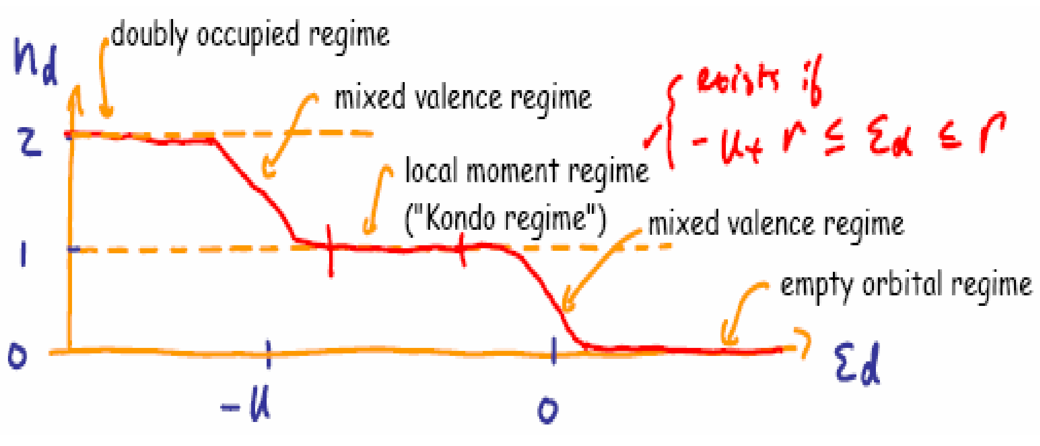
T varies in the range 15-800mK

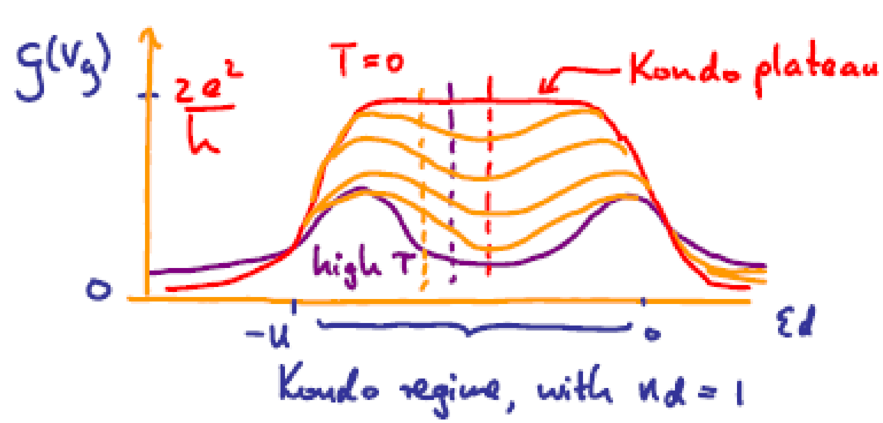
van der Wiel *et al.*, Science 2000



Interpret in terms of Friedel-Langreth sum rule:

$$G = \frac{e^2}{h} \sum_{\sigma} sin^2(\pi n_{d,\sigma})$$



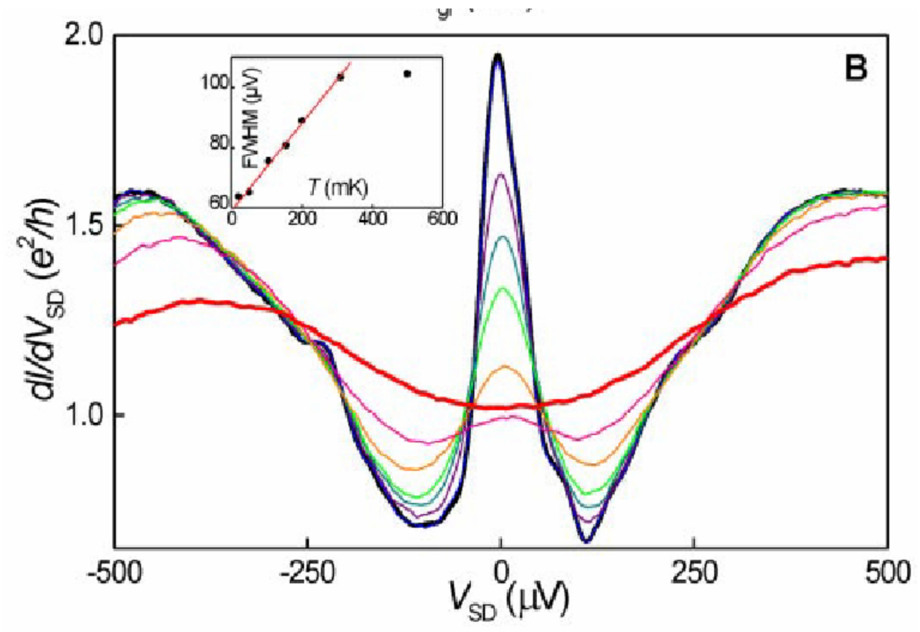


von Delft, notes

Quantum Dot out of Equilibrium

Differential conductance vs bias
- Nonequilibrium dynamics

Nonequilibrium measurement



van der Wiel *et al.*, Science 2000

What happens to the Kondo effect out of equilibrium?

Effect of a magnetic field?

Decoherence?

SBA - Anderson Model of the single-level Quantum Dot

SBA construction of Anderson model out equilibrium

- **Bethe momenta complex** $p_\pm(\lambda) = x(\lambda) \pm y(\lambda)$

$$x(\lambda) = \epsilon_d + U/2 - (Ut^2)^{1/2}(\lambda + (\lambda^2 + \frac{1}{4})^{1/2})^{1/2}$$

$$y(\lambda) = (Ut^2)^{1/2}(-\lambda + (\lambda^2 + \frac{1}{4})^{1/2})^{1/2}$$

- **Described by distributions** $\sigma_1(\lambda), \sigma_2(\lambda)$

$$\begin{aligned} 4\sigma(\lambda) &= \frac{-1}{\pi}\frac{dx(\lambda)}{d\lambda} - \int_{B_2}^{\infty} d\lambda' 4\sigma(\lambda')\frac{1}{1+(\lambda-\lambda')^2} \\ &- \int_{B_1}^{B_2} d\lambda' \sigma(\lambda')\frac{1}{1+(\lambda-\lambda')^2} \end{aligned}$$

$$\sigma_1(\lambda) = \sigma(\lambda) for \lambda > B_1$$

$$\sigma_2(\lambda) = \sigma(\lambda) for \lambda > B_2$$

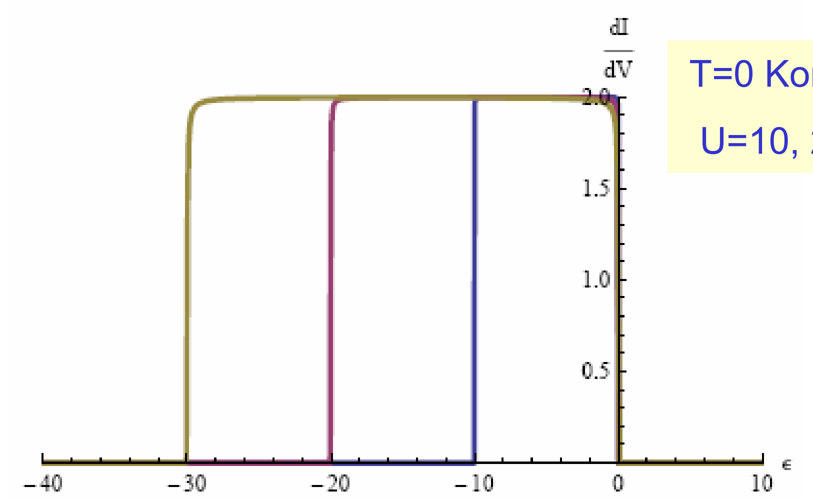
- **SBA eqns describe the free leads in the Bethe basis**

The physical chemical μ_i potential is related to Bethe chemical potential B_i by:

$$\int_{-D_0}^{\mu_i} \frac{1}{2\pi} dp = \int_{B_i}^{D} 8\sigma(\lambda) d\lambda$$

Results: conductance in and out of equilibrium

Zero Bias Conductance $G(V=0,\epsilon_d)$
vs. gate voltage ϵ_d



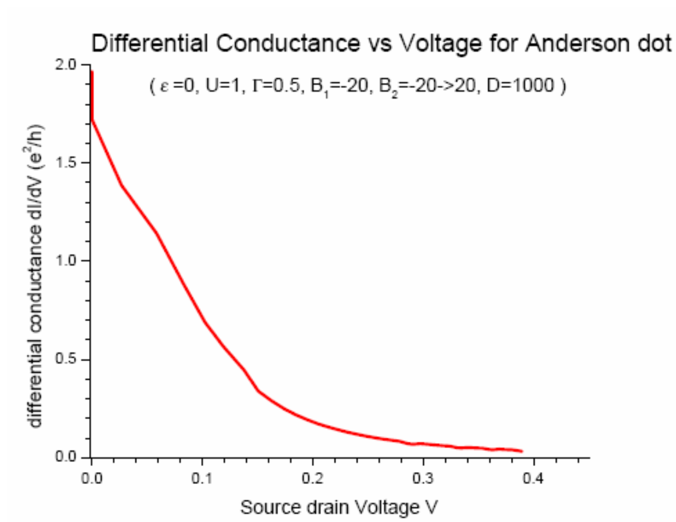
T=0 Kondo effect

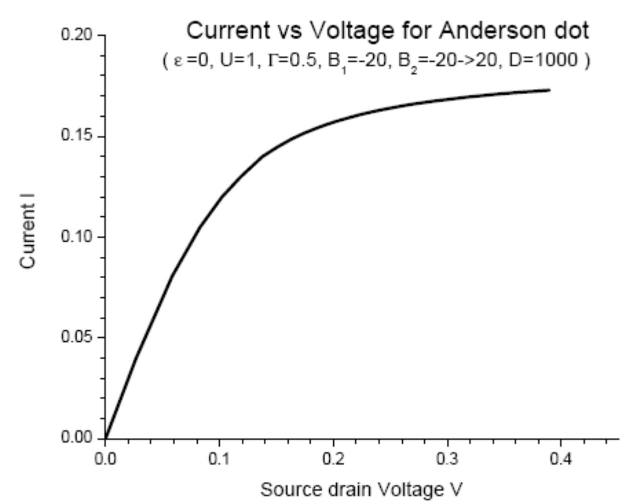
U=10, 20,30; t=1

The dependence of the Fermi level B
on ϵ_d is determined from: $\frac{dx(B)}{d\epsilon_d} = \frac{\partial x(B)}{\partial B}\frac{\partial B}{\partial \epsilon_d} + \frac{\partial x(B)}{\partial \epsilon_d} = 0$

So that $$\epsilon_d^* = \frac{-U}{2} + \frac{U}{2}\frac{(\epsilon_d + \frac{U}{2})}{\sqrt{|B|}}$$

Conductance $G(V,\epsilon_d)$
vs. bias voltage V
(very preliminary)





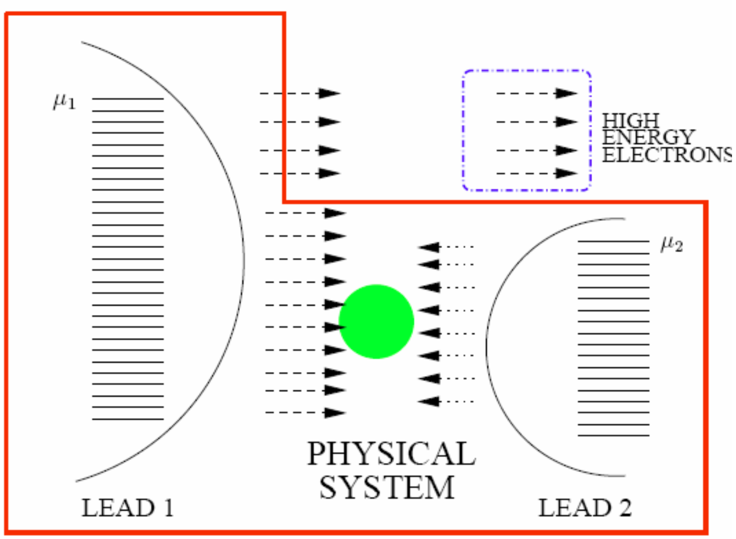
Entropy production I

Non-equilibrium currents dissipate heat into environment:

$$\delta Q_i = dE_i - \mu_i dN_i$$

- **Scattering state describes system +environment**
- **Dissipation mechanism: electrons reaching infinity**
- **Lost high energy electrons generate entropy (entanglement)**

"Discontinuous System"



Recall: currents ~ 1
leads ~ L → infty

$$\frac{dE_1}{dt} \equiv \langle \frac{d\hat{E}_1}{dt} \rangle_s = \langle i[\hat{H}, \hat{H}_{01}] \rangle_s = -\langle I_E \rangle_s$$

$$\frac{dN_1}{dt} \equiv \langle \frac{d\hat{N}_1}{dt} \rangle_s = \langle i[\hat{H}, \hat{N}_1] \rangle_s = -\langle I_N \rangle_s$$

Entropy production II

- **“Thermodynamic” approach:** (discontinuous system - defined w.r.t. quasi-equil , L ~ infty)

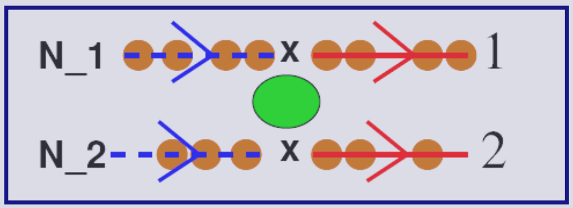
$$\sigma \equiv \frac{dS}{dt} = \frac{1}{T_1}\frac{\delta Q_1}{dt} + \frac{1}{T_2}\frac{\delta Q_2}{dt} = \left(\frac{1}{T_1} - \frac{1}{T_2}\right)\langle I_E\rangle_s + \left(\frac{\mu_1}{T_1} - \frac{\mu_2}{T_2}\right)\langle I_N\rangle_s$$

- **“Boltzmannian” approach – (distributions)**

scattering → change of distribution:

$\{f_i^- \text{ in}\}$

equilibrium distributions



$\{f_i^+ \text{ out}\}$

nonequilibrium distributions

$$\sigma = \sum_i \int dp\, v_F (f_i^+(p) - f_i^-(p)) \frac{p-\mu_i}{T_i}$$

$$f_i^+(p) = f_i^-(p)|R(p)|^2 + f_{\bar{i}}^-(p)|T(p)|^2$$

- **“Information Theory” approach – (in the infinite volume limit) :**

$$\sigma = \lim_{L\to\infty} \frac{1}{L} \sum_{i=1,2} v_F[(S_i^+ - S_i^-) + v_F D_{KL}(f_i^+||f_i^-)]$$

$$S_i^\pm = -\sum_\alpha f_i^\pm(p_\alpha)\ln f_i^\pm(p_\alpha) - \sum_\alpha [1 - f_i^\pm(p_\alpha)]\ln[1 - f_i^\pm(p_\alpha)]$$

$$D_{KL}(f^+||f^-) = \sum_\alpha f^+(p_\alpha)\ln\frac{f^+(p_\alpha)}{f^-(p_\alpha)}$$

Mixing + Relaxation

mixing = ΔS

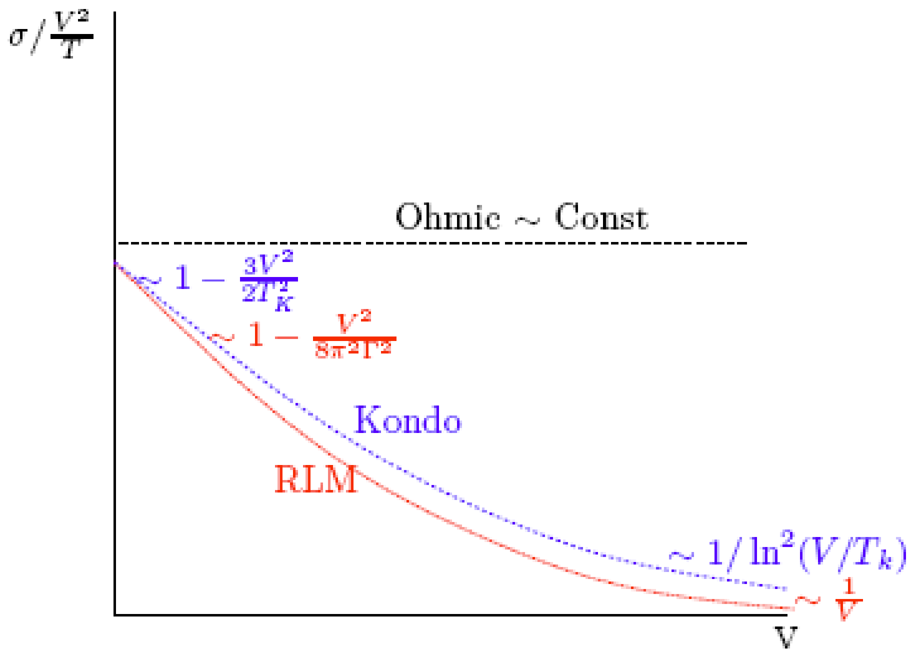
relaxation = $D_{KL}(f^+||f^-)$

Kullback-Leibler divergence: amount of work obtained when f^+ relaxes to f^-

- **Entropy production rate strictly positive,** $\sigma > 0$

Entropy Production: Effects of Correlations

How does the Kondo effect manifest itself?



- The RLM describes the Kondo model at Strong coupling
- Stronger correlations suppress entropy production
- To measure: perform spectroscopy of emerging electrons

Traditional vs Scattering BA

The construction of $|\psi\rangle_s$ is an example of the SBA approach:

	SBA	TBA
System	Infinite	Finite
Boundary condition	asymptotic (open)	periodic
Wavefunctions	used explicitly	not used
Thermodynamics	difficult	easy
Scattering Properties	possible	not possible
Nonequilibrium Generalization	Yes	No

More applications:

- **Scattering S-matrix of electrons off magnetic impurities**
  - *elastic and inelastic cross sections*
- **Calculation single particle Green's functions, spectral functions**
  - *finite temperature resistivity (resistance minimum)*

Conclusions

- **Showed:**
  Scattering eigenstates with non-eq BC – Steady States

- **Computed:**
  Steady state current, entropy production rate

- **Many Generalizations and applications:**

Non-equilibrium Impurity

- **Non-equilibrium in other impurity models**
  Multichannel versions
- **Non-equilibrium at** $T > 0, T_1 \neq T_2$**, thermal currents**
- **More leads: non-equilibrium DOS** ***(Lebanon&Schiller)***

Non-equilibrium Wire

- **The Luttinger liquid (*e.g.* nanotubes)**
- **AB Interferometers (with/without impurities)**

Scattering

- **Inclusive, exclusive scattering amplitudes**
- **Elastic, inelastic scattering amplitudes** $T > 0$

Quantum full counting statistics, Entropy fluctuations, noise, Onsager relations