Solitonic quasi-particles in electronic systems with a long range order

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Recent observations of solitons in quasi one-dimensional conductors.

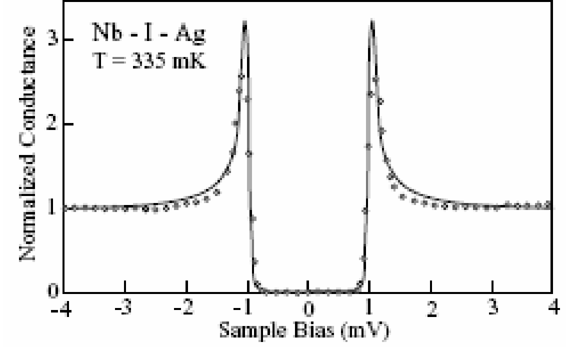
Confinements - global or local - and spin-charge reparation.

Routes to a role of microscopic topological defects in general strongly correlated electronic systems.

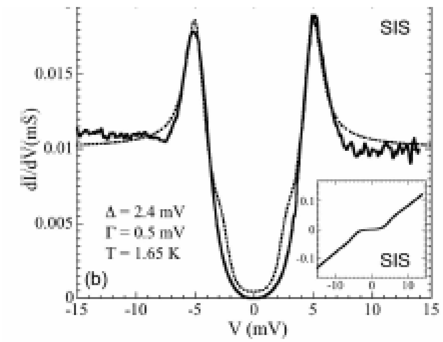
Motivation: What the CDWs can tell to
Doped Mott Insulators and to Spin-Polarized Superconductors.
From solitons in 1D models to
solitonic lattices, stripes and FFLO state and
to combined topological excitations.

Sources for this talk :
Joint work with experimental groups of
Grenoble (Monceau) and Moscow (IREE - F.Nad, Yu.Latyshev, et al),
Earlier theory collaboration
(I.Dzyaloshinskii, N.Kirova, I. Krichever, S.Matveenko, et al.)

Singlet ground state gapful systems: SCs and CDWs.
Deparing gaps as seen by tunneling experiments.
Standard BCS-Bogolubov view:
Spectra : **$E(k)= \pm(\Delta^2+(v_f k)^2)^{1/2}$**



States – linear combinations of :
electrons and holes at **$\pm p$** for SC
or of electrons at **$-p$** and **$p+2p_f$** for CDWs

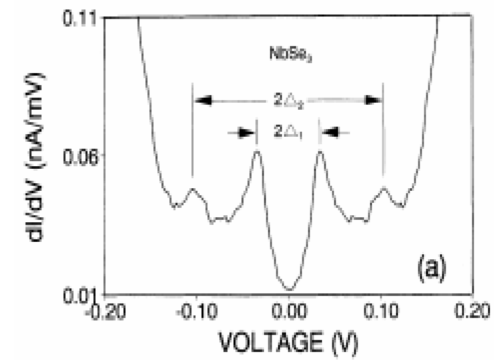


CaS$_6$

Is it always true?

Well proved “yes” for typical SCs.
Questionable for strong coupling
(real space pairing, High-T$_c$, cold atoms, bi-plolarons).
Certainly incomplete for CDWs
as proved by many experiments.
Certainly inconsistent for 1D and even quasi 1D
systems as proved theoretically.

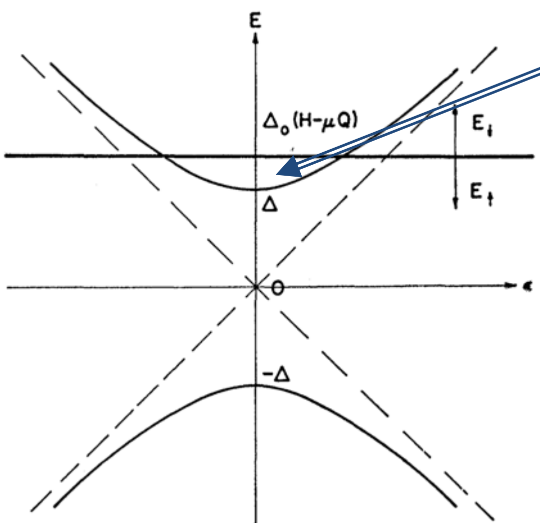
Reason – solitonic excitations competing with electrons.



NbSe$_3$ – two coexisting CDWs

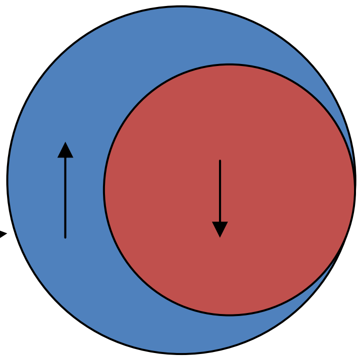
A weaker but almost general statement:
while single excitations can be of the BCS-Bogolubov type,
their finite concentration
(CDW/SDW incommensurability, Mott/AFM/SDW states dopping,
CDW or SC Zeeman breakdown, cold atom imbalance)
givez rise **not** to pockets occupying E(k) above Δ, but to
solitonic lattices in CDW = stripes in AFM = FFLO in SC

Figure by Fulde-Ferrell 1964



Fill excess spins to states
above the gap

Or make a modulation Q≠0 to erase
a mismatching at some
(all in quasi-1D !)
parts of the FS – both suggestions
FF: Δ~**exp(iQx)** & LO: Δ~ **cos(Qx)**



Or build a structure of local walls so strong as to create intra-gap states
which able to accommodate access spins.
Proved in quasi-1D, able to evolve into LO, not FF, similar to CDWs.

Solitons' workshop in organic conductors like $(TMTTF)_2X$

Discovery of charge ordering and related ferroelectricity in 2000-01
Nad, Monceau and S.B.; S. Brown et al
- Acess to switching on/off of the Mott state
and to the Zoo of solitons.

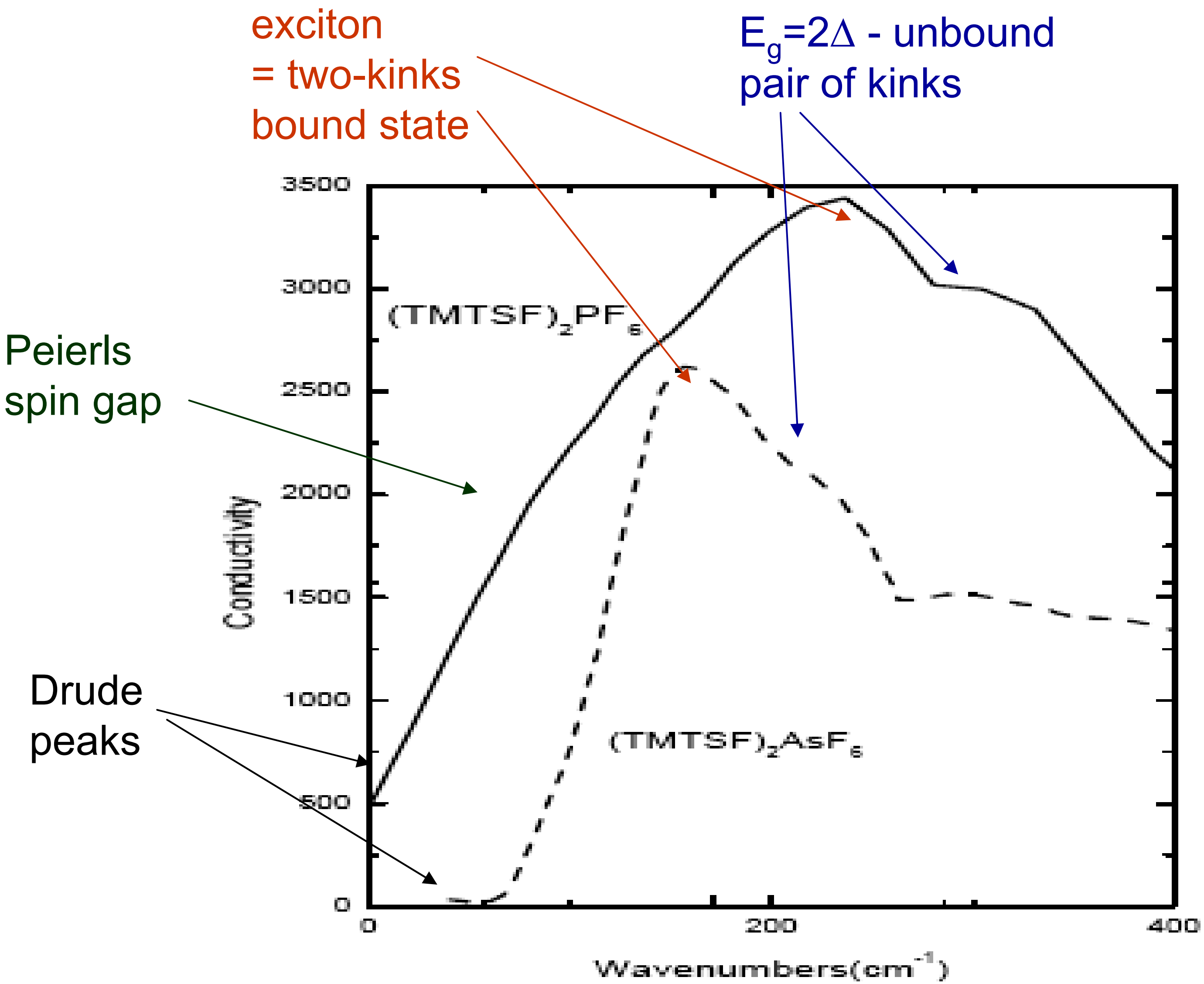
Quasi-1D Mott state = $4K_F$ CDW= commensurate Wigner crystal

Charge degrees of freedom: phase $\varphi=\varphi(x,t)$
$2K_F$ CDW/SDW ~cos $(\varphi+x\pi/2a)$ $4K_F$ CDW~ cos $(2\varphi+x\pi/a)$

$$H\sim [v (\partial_x\varphi)^2 + (\partial_t\varphi)^2 /v] - U\cos (2\varphi -2\alpha)$$

U - $4K_F$ or CO commensurability = Umklapp scattering amplitude,
leading to the Mott state *Dzyaloshinskii & Larkin, Luther & Emery*.
U may appear at the Charge Ordering transition T_{CO}

Phase centre shift α - may appear at the ferroelectric transition T_{FE}
In our case $T_{FE}=T_{CO}$!



Interpretation of optics on metallic TMTSF in terms of firm expectations for CO (Mott insulator) state in TMTTF.
Optical conductivity is re-plotted from data of Dressel group.

Further symmetry lifting of lattice tetramerization or of spin-Peierls order mixes charge and spin: additional energy

$V\cos(\varphi-\beta)\cos\theta$ - on top of **$\sim U\cos(2\varphi-2\alpha)$**

φ and $\boldsymbol{\theta}$ -- chiral phases counting the charge and the spin

φ'/π and $\boldsymbol{\theta}'/\pi$ = smooth charge and spin densities

$\cos\theta$ sign instructs the CDW to make spin singlets over sorter bonds

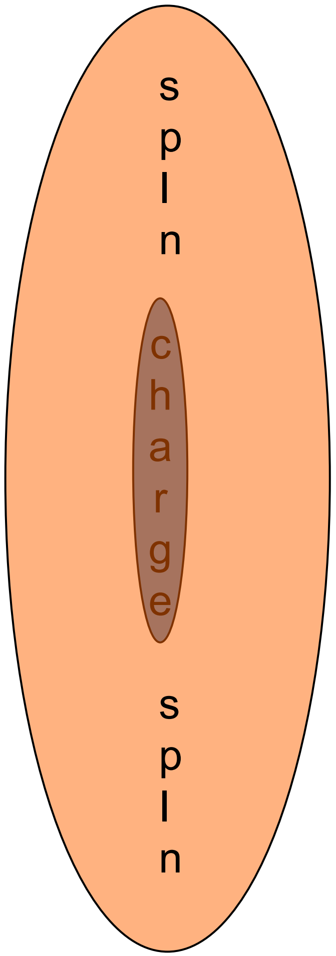
Major effects of the small V - term :

Opens spin gap $\mathbf{2\Delta_\sigma}$:

triplet pair of $\delta\boldsymbol{\theta}=\pi$ solitons at $\varphi=cnst$

- Prohibits $\delta\varphi=\pi$ solitons – now bound in pairs by spin strings
- Allows for combined spin-charge topologically bound solitons:

$\{\delta\varphi=\pi, \delta\boldsymbol{\theta}=\pi\}$ – leaves the **V** term invariant



Quantum numbers of the compound particle -- charge **e** , spin ½ but differently localized:

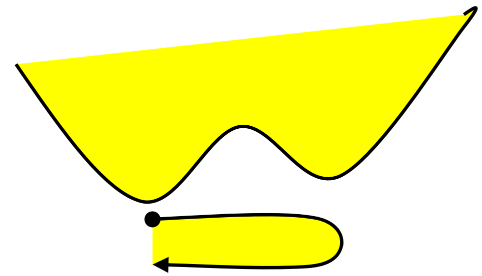
charge **e** , $\delta\varphi=\pi$ sharply within $\hbar v_F/\Delta_\rho$

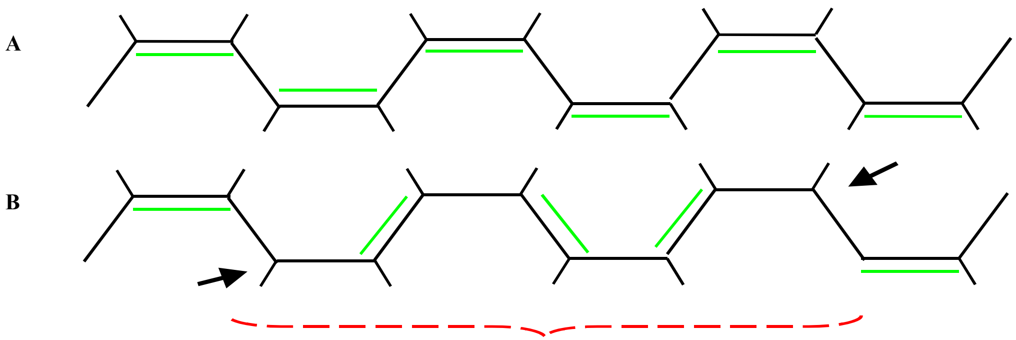
spin ½ , $\delta\boldsymbol{\theta}=\pi$ loosely within $\hbar v_F/\Delta_\sigma$

Fatal effect upon kinks: lifting of degeneracy, hence confinement.
Trivial but spectacular example: global lifting of symmetry.

Nature present -- cis-isomer of $(CH)_x$:
build-in slight inequivalence of bonds
hence lifting of ground state degeneracy,
hence confinement of solitons

Cis-$(CH)_x$:
Nonsymmetric dependence
of GS energy on dimerisation





Only a short excursion
= confined pair of kinks=
to the false GS is allowed

Confinement of kinks pair into
2e charged (bipolaron) or neutral (exciton) compex.
Symmetry determined picture of optical differences for
trans- and cis- isomers *S. B. and N. Kirova, 1981*
Photoconductivity trans-$(CH)_x$ versus photoluminescence cis-$(CH)_x$
also new optical features due to hybridization of mid-gap states

Incommensurate Charge Density Wave – **ICDW ~cos(Qx + φ)**
Minimal view : crystal of singlet electronic pairs.
Incommensurability allows for arbitrary ICDW displacements hence the complex order parameter **Acos(φ)**

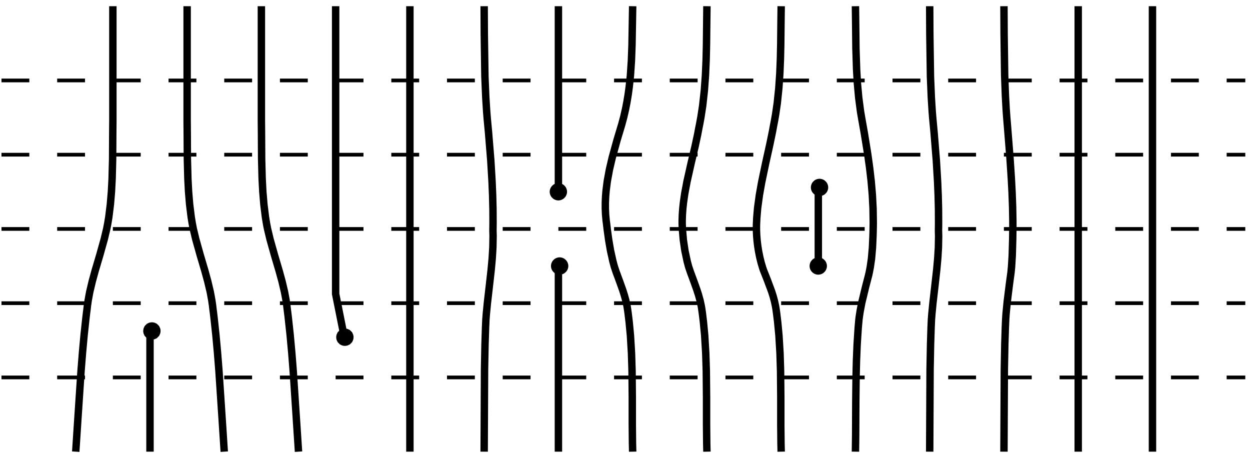
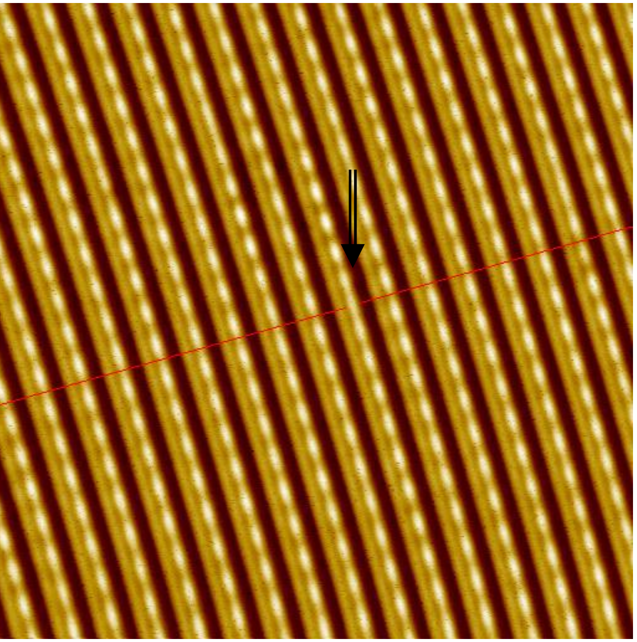


Figure from S.B. and T. Natterman Adv. In Physics 2004

Topological 2π defects in a CDW – no breaking of singlets.
Solid lines: maxima of the charge density.
Dashed lines: chains of the host crystal.
Embracing only one chain of atoms, the pairs of dislocations become a vacancy or an interstitial or $\pm 2\pi$ soliton.

Effects of nonlocal elasticity from Coulomb interactions -
S.B. & Matveenko, Hayashi & Yoshioka, Takane, Artemenko et al.

Better seeing than plotting : visualization of a 2π soliton = 2e prefabricated electrons' pair



"Shut up !
I am experiencing a divine moment of observing a single atom"
(read "single soliton" – S.B.)
R. Feinman watching the STM at Bell Labs.

C. Brun and Z.Z. Wang
STM scan of $NbSe_3$, capturing the phase soliton

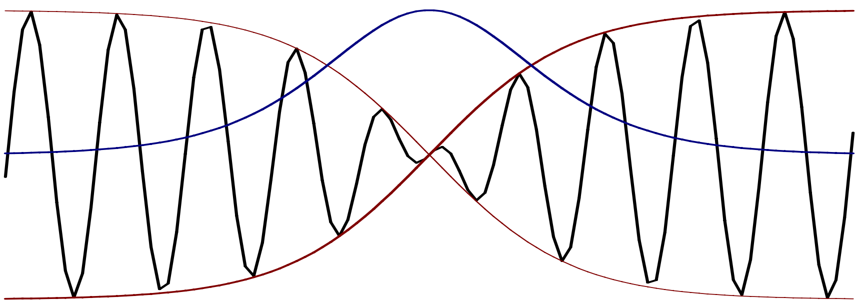
Charge Ordering was a crystal of electrons.
Conventional CDW is a crystal of electron pairs.
Its lowest energy current carrier must be a charge 2e defect of adding/missing one period at a defected chain.
It is the **±2**π soliton of the CDW ~ $A\cos(2K_F x + \varphi)$.

Recall D.Geshkenbein – flux excitations as holes in the 2D crystal of pancake vortices

But what should happen if the singlet pair is broken into spin ½ components ?

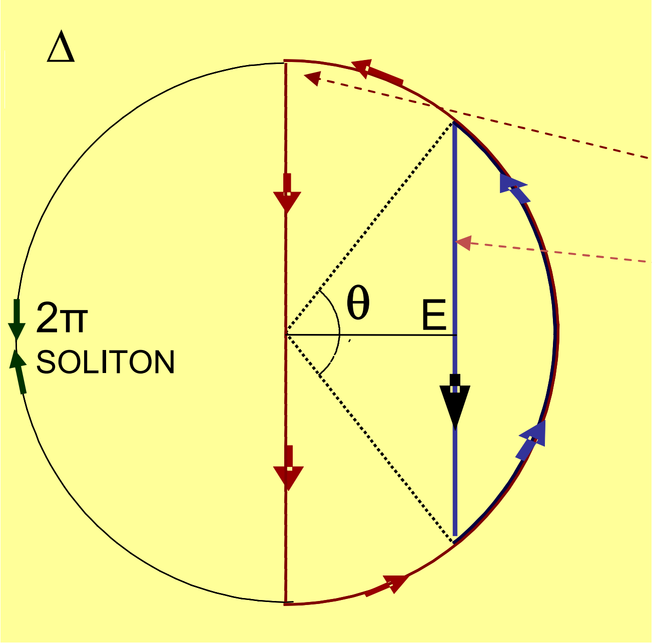
It will NOT be an expectedly liberated electron-hole pair at $\pm\Delta$,
but two spin carrying "amplitude solitons"
– zeros of the order parameter - , distributed over a number of periods.

This creature substitutes for unpaired electron *(S.B. 1978-80)* :
Amplitude soliton with energy $\approx 2\Delta/3$, total charge 0, spin ½
This is the CDW realization of the SPINON

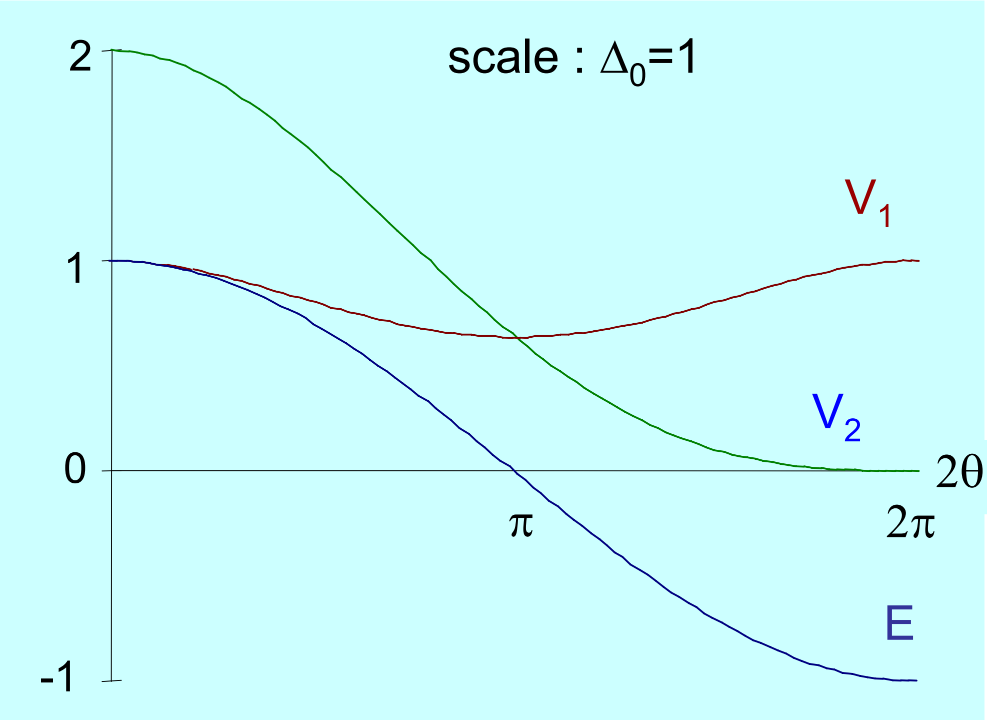


Oscillating electronic density,
Overlap soliton A(x),
Midgap state =spin distribution

Analogies and aggregated forms:
FFLO unit for spin-polarized superconductors
Unit of the CDW superstructure in HMF *(recent experiments on organics)*
Kink in the polyacetylene
Soliton lattice unit in spin-Peierls systems in HMF *(seen by the NMR)*



Soliton trajectories in the complex plane of the order parameter.
Red line: stable amplitude soliton.
Blue line: intermediate chordus soliton within chiral angle θ (black radial lines).
The value θ=100° is chosen which corresponds to the optimal configuration for the interchain tunnelling
S.Matveenko and S.B.



Selftrapping branches $V_n(\theta)$ for chordus solitons with fillings n =1 and n=2,
Energy E(θ) of localized split-off state - Spectral flow between gap edges $\Delta_0 \rightarrow -\Delta_0$
No barrier for selftrapping in 1D !

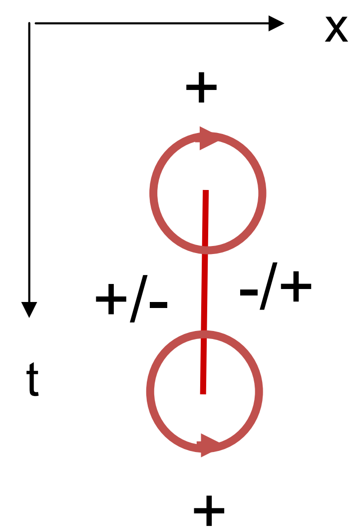
Probabilities to create combined topological defects in 1D:

AS creates the π- discontinuity $\delta\varphi=\pi$ along its world line: ($0<t<T$, $x=0$) along the interval (**0,T**) compensating for the sign change of the amplitude :

To be topologically allowed = to have a finite action S, the line must terminate with half integer space-time vortices located at (**0,0**) , (**0,T**) :

their circulation provides the jump A→-A combined with $\varphi\rightarrow\varphi+\pi$ which leaves invariant the order parameter **Aexp(iφ)**.

The phase action as a function of time **T**:



$$S_{phase} \sim \frac{v_F}{u} \ln\left(\frac{uT}{\xi_0}\right)$$

Contrary to usual 2π- vortices, connecting line is the physical singularity which tension gives

$$S_{core} = (W_s - \Omega)T$$

Total action $\mathbf{S=S_{core}+S_{phase}}$, **min{S}~ln(T)**

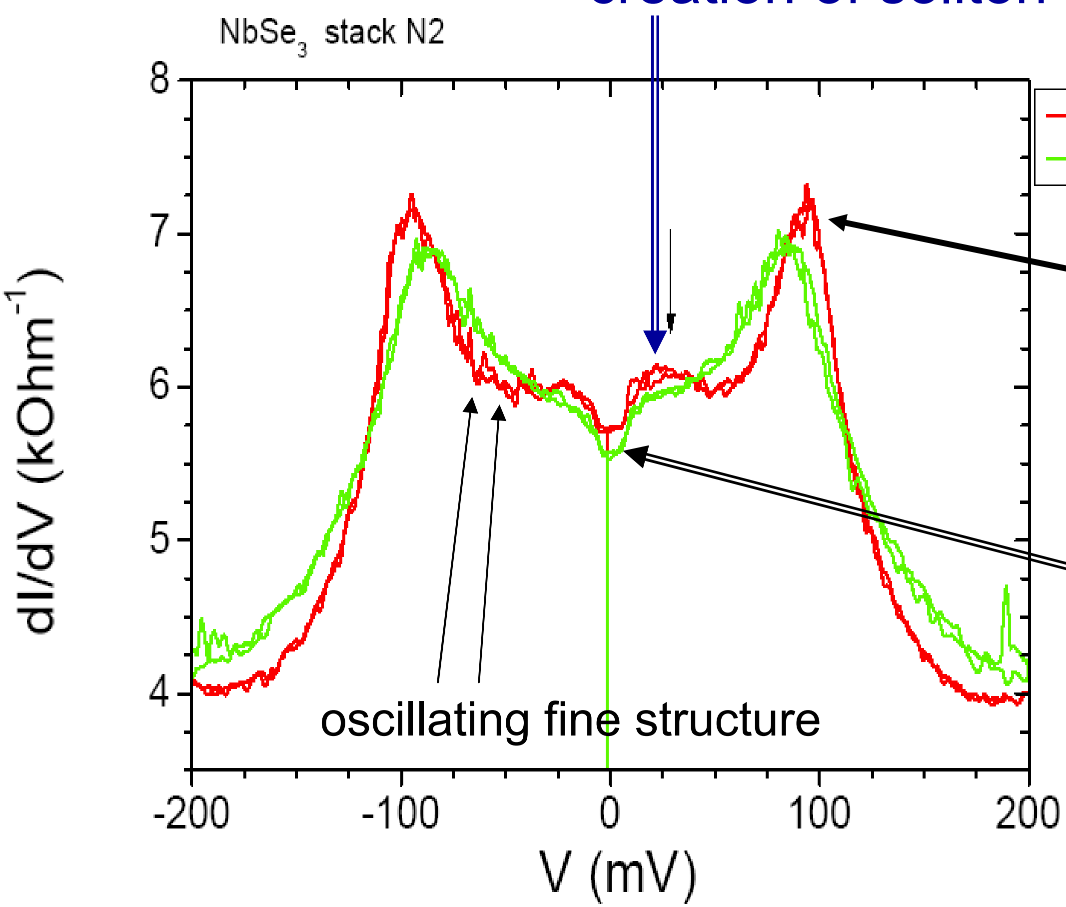
$$I(\Omega) \propto \left(\frac{\Omega - W_s}{W_s}\right)^{\beta}, \ \beta = \frac{v_F}{2u}$$

hence the power law for **I(Ω)~exp(-S)**

Direct observation of solitons and their arrays in tunneling on NbSe3

Latyshev, Monceau, SB, et al 2004-2006

creation of soliton at $E_s = 2\Delta/\pi$

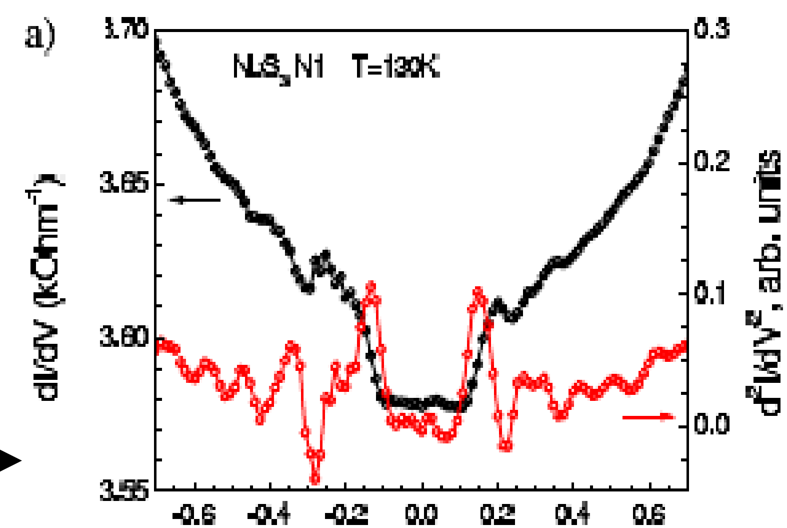


All features scale with $\Delta(T)$!

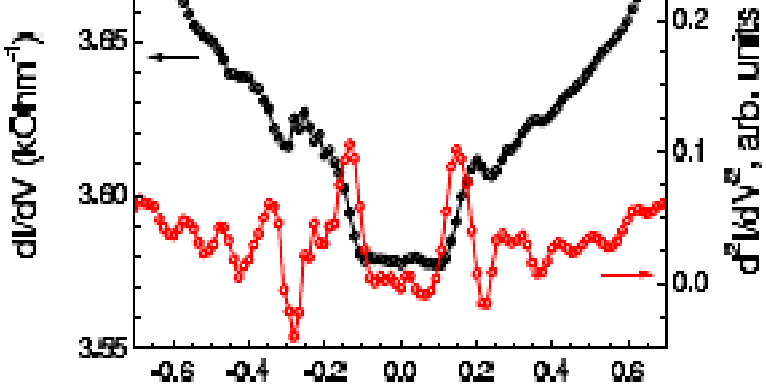
peak 2Δ for inter-gap creation of e-h pairs

absolute threshold at low $V_t \approx 0.2\Delta$

Fine structure is not a noise !
its interpretation :
sequential entering of dislocation lines into the junction area.



What does tunnel here soon above $\mathbf{V_t}$?
Only bi-elctronic pairs have that low energy.
But they do it at presence of their condensate – dislocation lines.



Origin of low threshold V_t:

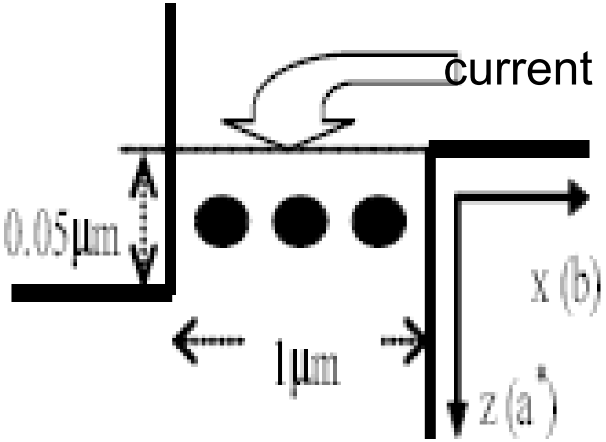
Junction internal reconstruction under applied voltage, creating an array of 2π solitons.

Result : a grid of dislocation lines - vortices of the ICDW phase.

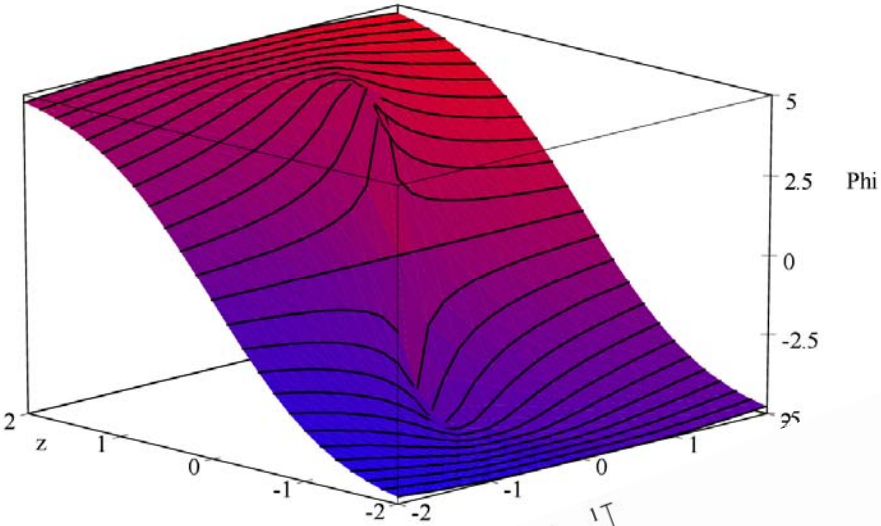
$\mathbf{V_t}$ is the vortex entry energy, like H_{c1} in superconductors.

Voltage drop, hence tunnelling, is concentrated near dislocation cores.

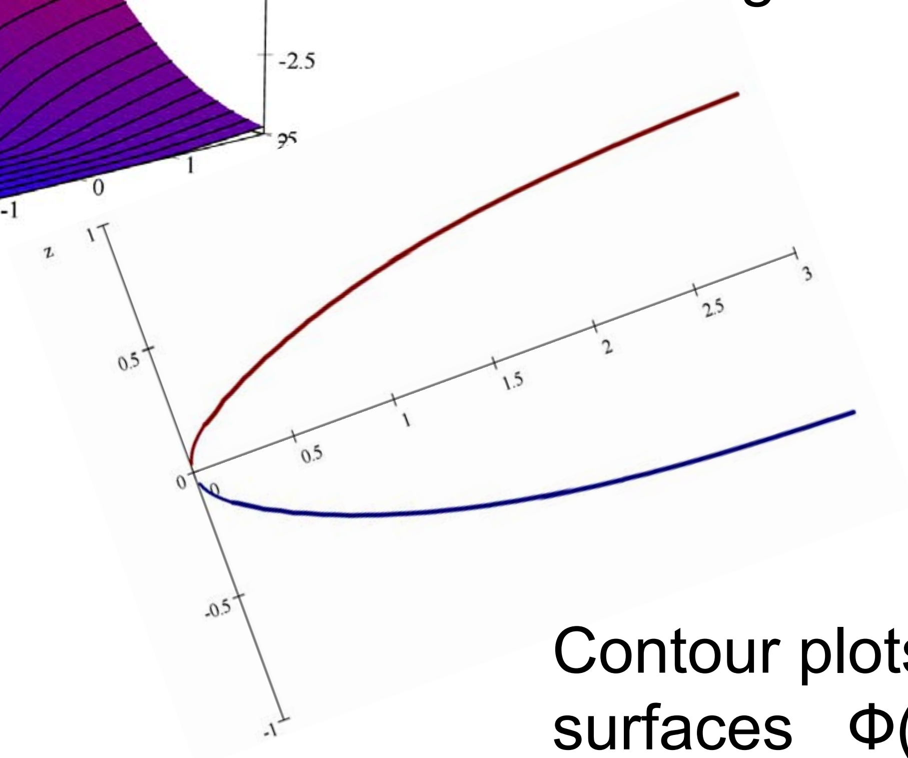
We get a self-assembled grid of filamentary nano-junctions.



Junction scheme showing crossections of dislocations



Potential distribution in a DL vicinity. Notice concentration of the potential Φ(x,z) drop, facilitating the tunneling.



Contour plots ±*z*(*x*) for surfaces Φ(x,z)±Δ where the tunneling takes place.

Pair of ±2π solitons is created by tunneling near the dislocation core, Interpretation: excitation of the dislocation line as a quantum string.

Can the solitons cross the boarder to the higher D world ?
Are they allowed to bring their anomalies like spin-charge separation or mid-gap states?
Password : *confinement.*
As topological objects connecting degenerate vacuums, solitons acquire an infinite energy unless they reduce or compensate their topological charges.

Various scenarios :

- Compensation by the gapless mode *S.B. 1980, 2000's*
- Aggregation into domain walls versus their melting by thermal deconfinement or long rage Coulomb forces
  S.B. & T.Bohr 1983, S. Teber 2001
- Coupling to structural defects in polymers.
- Binding to kink-antikink pairs, origin of bipolarons.
  S.B. & N.Kirova, 1981- 90's
- *Today's request :*
  Topological binding to gapless degrees of freedom

FINITE TEMPERATURE, ENSEMBLES OF SOLITONS, PHASE TRANSITIONS OF CONFINEMENT AND AGGREGATION. DISCRETE SYMMETRY only.

Fatal effect upon kinks: global lifting of degeneracy, hence confinement.

Nontrivial but still spectacular:

local lifting in the state with long range order.

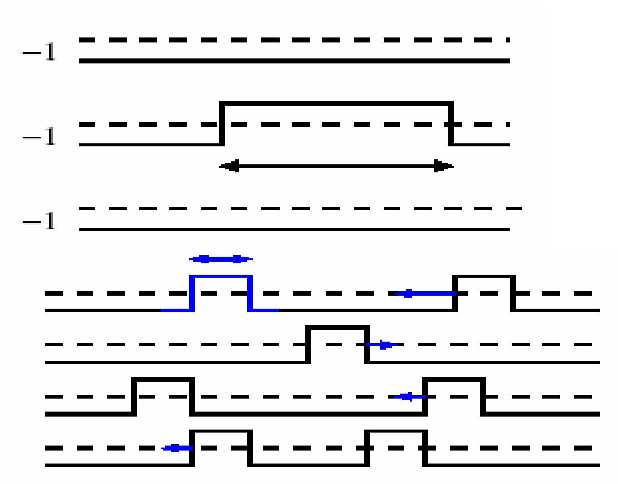
Interchain coupling of the order parameter.

$$H_I = -\sum_{<\alpha,\beta>} \int dx V_\perp \Delta_\alpha(x)\Delta_\beta(x)$$

Two competing effects:

Binding of kinks into pairs at $T<T_c$;

Aggregation into macroscopic domain walls at $T<T_0<T_c$.



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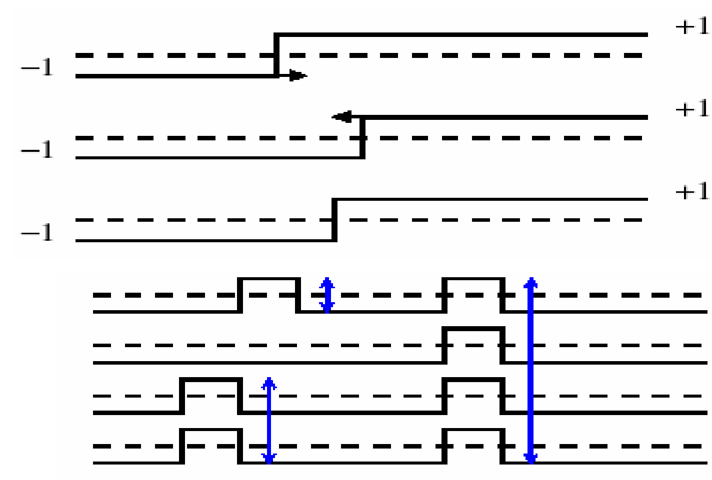


T < To

Solution for a statistical model *T.Bohr and S.B. 1983, S.Teber et al 2000's*

Recall N. Nagaosa on superionic conductors

Major and unifying observation :

combination of a discrete and continuous symmetries

Solitons are stable energetically but not topologically

Special significance: allowance for a direct transformation of

one electron into one soliton.

(Only 2→2 are allowed for kinks in discrete symmetries)

Complex Order Parameter

O= A exp[iφ] ; **A** - amplitude , φ - phase

Ground State with an odd number of particles:

In 1D - *Amplitude Soliton* AS **O(x=-∞) ⇔ - O(x=∞)**

via **A ⇔-A** at arbitrary φ=cnst

Favorable in energy in comparison with an electron, but:

Prohibited to be created dynamically even in 1D

Prohibited to exist even stationary at D>1

RESOLUTION – Combined Symmetry :

A ⇔-A combined with φ→φ+π – semi-vortex of phase rotation compensates for the amplitude sign change

SPIN-GAP cases, bosonisation language - ICDW

H_{1D}~$(\partial\theta)^2$ -Vcos(2θ)} +$(\partial\varphi)^2$

V - from the backward exchange scattering of electrons

In **1D** : Spinon as a soliton θ→ θ+π hence **s=1/2**

+ Gapless charge sound in φ.

CDW order parameter ~ $\psi^{\dagger}_{+\uparrow}\psi_{-\uparrow} + \psi^{\dagger}_{+\downarrow}\psi_{-\downarrow}$ ~ **exp [iφ] cosθ**

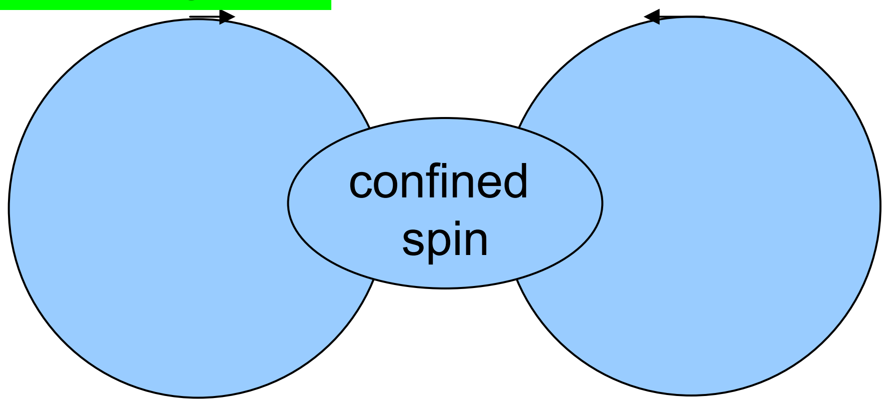
- Its amplitude **cosθ** changes the sign along the allowed π soliton

At higher D : allowed mixed configuration

θ → θ+π , s=1/2 **φ → φ+π , e=1**

↑spin soliton ↑ ↑charged wings↑

Spinon as a soliton +
semi-integer dislocation loop =
π - vortex of φ ≡ confined spin +
semi dislocation loop

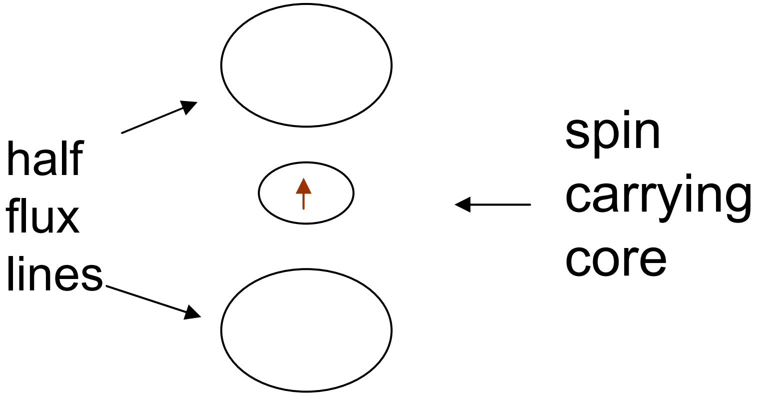


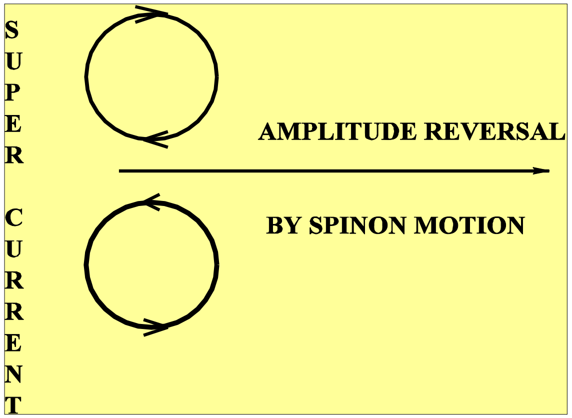
Singlet Superconductivity

$O_{SC} \sim \psi_{+\uparrow}\, \psi_{-\downarrow} + \psi_{+\downarrow}\, \psi_{-\uparrow} \sim \exp[i\chi] \cos\theta$

$\theta \to \theta+\pi$ s=1/2 $\qquad\qquad \chi \to \chi+\pi$

↑spin soliton↑ $\qquad\qquad$ ↑wings of supercurents↑





Quasi 1d view : spinon as a π- Josephson junction in the superconducting wire (*applications: Yakovenko et al*).

2D view : pair of π- vortices shares the common core bearing unpaired spin.

3D view : half-flux vortex stabilized by the confined spin.

Best view: nucleus of melted FFLO phase in spin-polarized SC

Half filled band with repulsion.
SDW rout to the doped Mott-Hubbard insulator.

$$H_{1D} \sim (\partial\varphi)^2 - U\cos(2\varphi)\} + (\partial\theta)^2$$

U - Umklapp amplitude

(*Dzyaloshinskii & Larkin ; Luther & Emery*).

φ - chiral phase of charge displacements

θ - chiral phase of spin rotations.

Degeneracy of the ground state:

$\varphi \rightarrow \varphi+\pi$ = translation by one site

Excitations in 1D :

holon as a π soliton in φ, spin sound in θ

Higher D : A hole in the AFM environment.

Staggered magnetization $\equiv$ *AFM=SDW order parameter:*

$O_{SDW} \sim \cos(\varphi)\exp\{\pm i(Qx+\theta)\}$, amplitude $A=\cos(\varphi)$ changes the sign

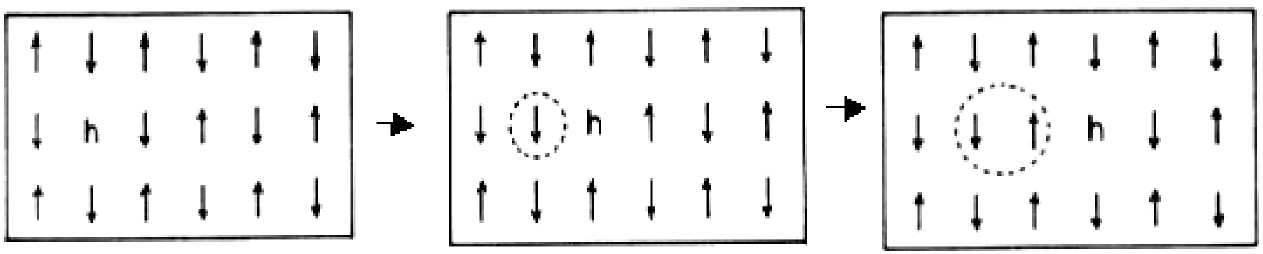
To survive in D>1 :

The π soliton in φ : $\cos\varphi \rightarrow -\cos\varphi$

enforces a π rotation in θ to preserve O_{SDW}

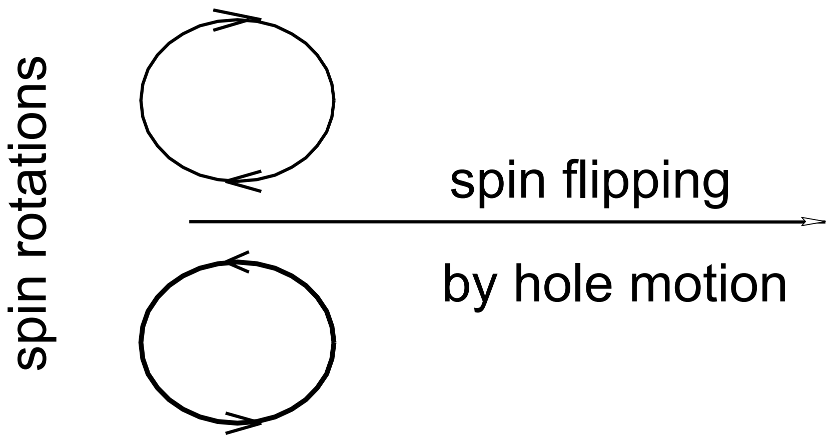
Propagating hole as an amplitude soliton.

Its motion permutes AFM sublattices ↑,↓
creating a string of the reversed order parameter:
staggered magnetization. It blocks the direct propagation.



Nagaev et al , Brinkman and Rice

Adding the semi-vorticity to the string end heals the permutation thus allowing for propagation of the combined particle.

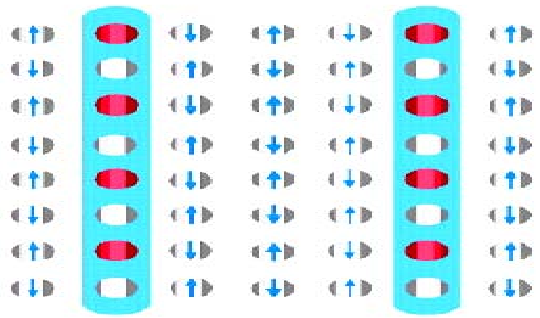


Alternative view:
Nucleus of the stripe phase or the minimal element of its melt.

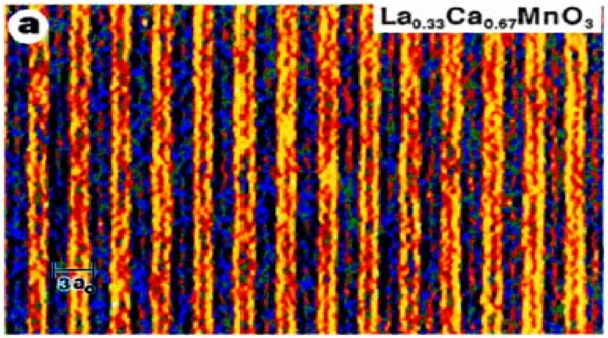
Inverse rout: from stripes to solitons

1D → quasi 1D → 2D,3D route to dopping of AFM insulator.
Aggregation of holes (extracted electrons) into stripes.

Left: scheme derived from neutron scattering experiments.
Right: *direct visualization via electron diffraction microscope.*

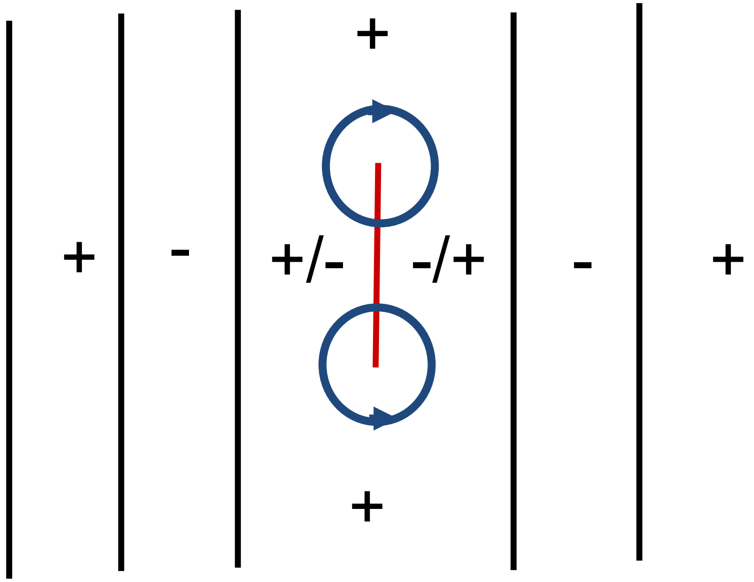


J.Orenstein et al Science 288, 468 (2000)



S.Mori et al Nature 392, 473 (1998)

Equivalence for spin-gap cases:
Fulde-Ferell-Larkin-Ovchinnikov FFLO phase in superconductors
Solitonic lattices in CDWs above the magnetic breakdown
Solitonic lattices in spin-Peierls GeCuO in HMF - Grenoble

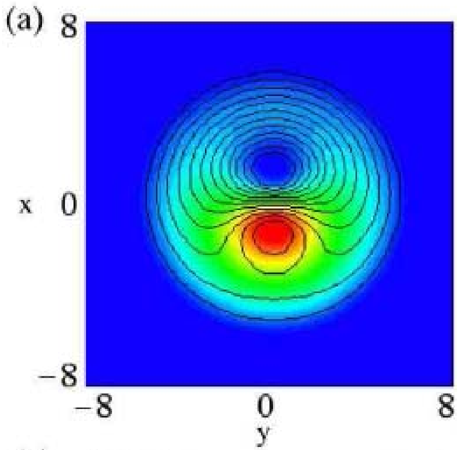


Kink-roton complexes as nucleuses of melted lattices: FFLO phase for superconductors or strips for doped AFMs.

Defect is embedded into the regular stripe structure (black lines).
+/- are the alternating signs of the order parameter amplitude.
Termination points of a finite segment L (red color) of the zero line must be encircled by semi-vortices of the π rotation (blue circles) to resolve the signs conflict.
The minimal segment corresponds to the spin carrying kink.

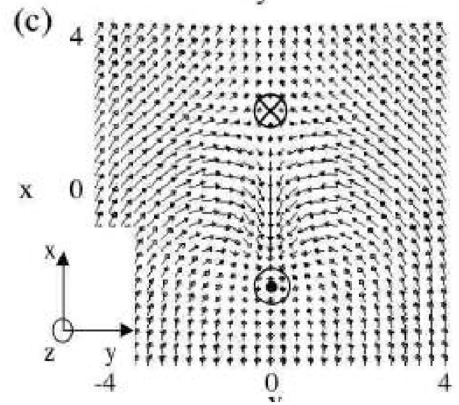
Vortices cost $\mathbf{\sim E_{phase} \log L}$ is always less than the gain $\mathbf{\sim -\Delta L}$ for the string formation at long L.
For smallest L it is still valid in quasi 1D : $\mathbf{E_{phase} \sim T_c < \Delta}$
For isotropic SCs - $\mathbf{E_{phase} \sim E_f}$ – strong coupling is necessary.

In absence of microscopic theory for a strong coupling vortex (with a single intra-gap state), we search the literature for numeric, and still phenomenological models. And it works !
At presence of unpaired spins, the vortex created by rotation (magnetic field) splits into two semi-vortices.



Spatial Line Nodes and Fractional Vortex Pairs in the FFLO Vortex State of Superconductors
D. F. Agterberg, Z. Zheng, and S. Mukherjee 2008

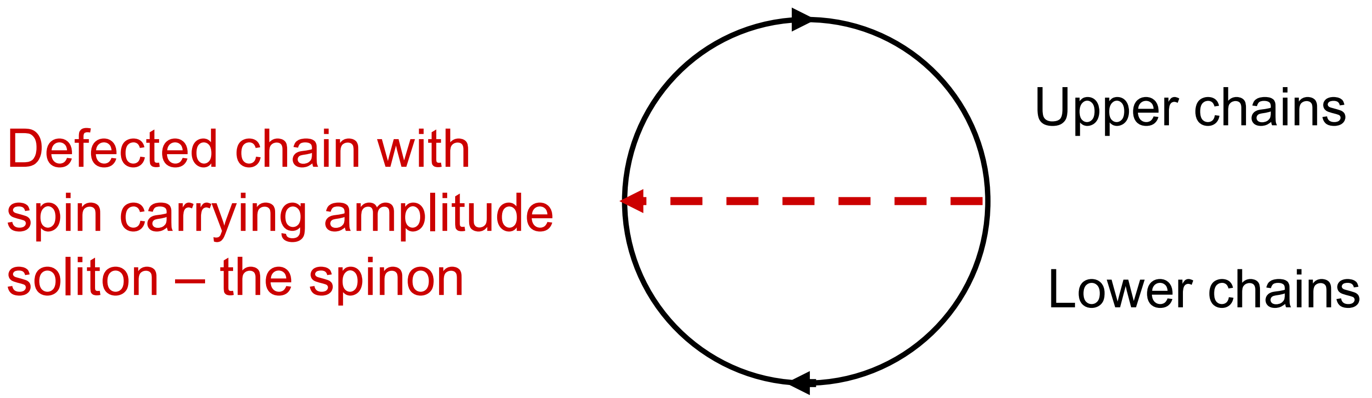
Vortex molecules in coherently coupled two-component Bose-Einstein condensates
K. Kasamatsu, M.Tsubota, and M. Ueda 2004



Last step: reformulate these results inversely – unpaired spin creates the vortex pair at NO rotation/MF.

Competitor to the above scenario of
"split vortices with a joint spin-carrying core":

A single vortex with a half-filled mid-gap core – extrapolation of the Caroli-De Gennes-Matricorne staircase to the single zero-energy level.



Pro: compatible with the quasi-1D limit, similar local energy scales

Contra: Log growing energy – requires for rotation/MF
or for a high temperature above the BKT transition.

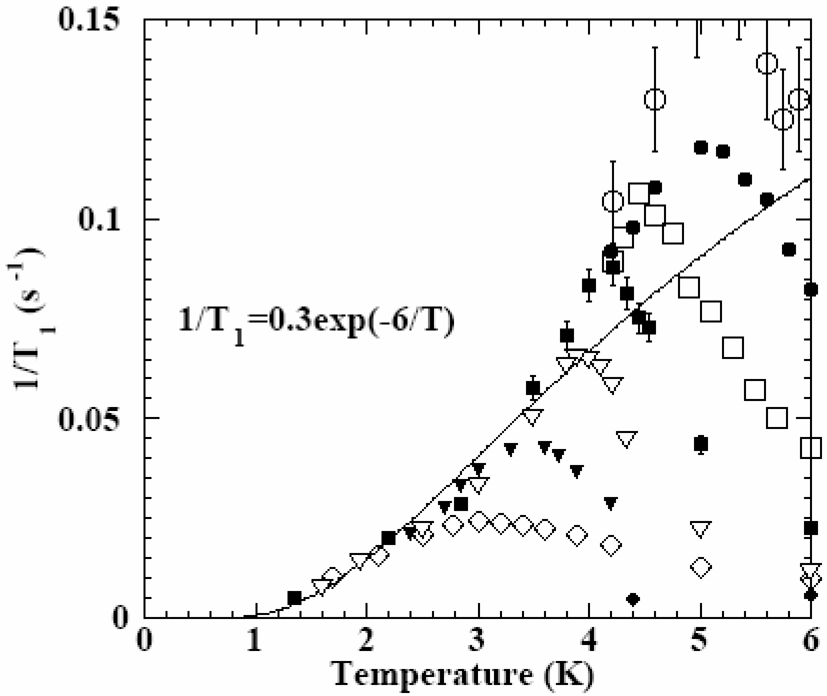
Still keep in mind a possibility that rotation/MF can erase the FFLO stripes by placing unpaired spins to the vortex core.

A distant hint to reality of unbound microscopic vortices
(“**pancakes**” – Artemenko et al; Efetov; Feigelmann, Geshkenbein et al; Klemm)

H.Mayaffre, P.Wzietek, D.Jérome, S.Brazovskii
"2D vortex melting in organic superconductors and NMR relaxation induced by vortex structure defects.",
Phys. Rev. Lett., 1996

Demonstrated coexistence of a gas of microscopic defects and their aggregates – vortex lines.

Only very mobile pancakes provide an efficient and homogeneous relaxation of nuclear spins.



At various magnetic fields.
6K = evaporation energy ?
Maxima follow the “irreversibility line” – presumably melting of the vortex lattice in favor of pancakes’ liquid.

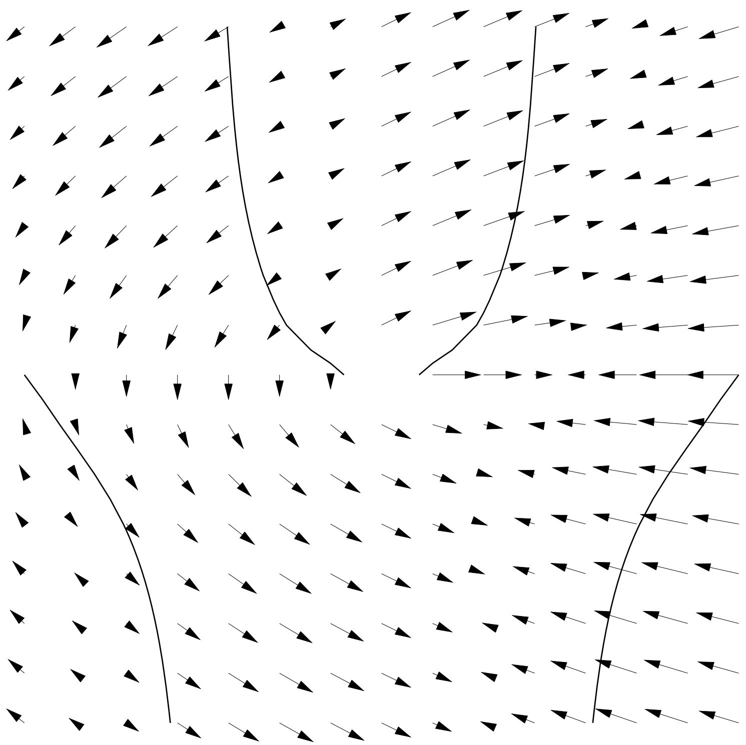
TOPOLOGICAL COUPLING OF DISLOCATIONS AND VORTICES IN INCOMMENSURATE Spin DENSITY WAVES

N. Kirova, S. Brazovskii, 2000

An attempt to rehabilitate the Density Waves against more fascinating symmetries: He^3 (Volovik et al), skyrmions in QHE (Yu.Bychkov et al, B. Doucot et al for bi-layers)

ISDW order parameter: O_{SDW} ~**m** cos(Qx+φ)

m – staggered magnetization vector



Three types of self mapping for the O_{SDW} :

1. normal dislocation, 2π translation:

 $\varphi\to\varphi+2\pi$, $\mathbf{m}\to\mathbf{m}$

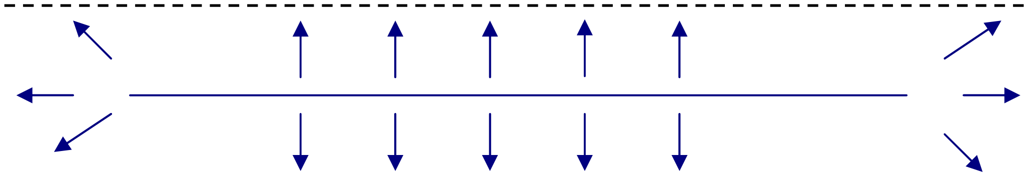
2. normal **m** - vortex, 2π rotation:

$\mathbf{m}\to R_{2\pi}\mathbf{m}$, $\varphi\to\varphi$

3. combined object :

$\varphi\to\varphi+\pi$, $m\to R_{\pi}\, m = -m$

Coulomb energy favors splitting the phase dislocation at a smaller cost of creating spin semi-vortices.



Effect of rotational anisotropy:
String tension binds semi-vortices

SUMMARY

- Existence of solitons is proved experimentally in single- or bi-electronic processes of 1D regimes in quasi 1D materials.
- They feature self-traping of electrons into mid-gap states and separation of spin and charge into spinons and holons, sometimes with their reconfinement at essentially different scales.
- Topologically unstable configurations are of particular importance allowing for direct transformation of electrons into solitons.
- Continuously broken symmetries allow for solitons to enter D>1 world of long range ordered states: SC, ICDW, SDW.
- They take forms of amplitude kinks topologically bound to semi-vortices of gapless modes – half integer rotons
- These combined particles substitute for electrons certainly in quasi-1D systems – valid for both charge- and spin- gaped cases
- The description is extrapolatable to strongly correlated isotropic cases. Here it meets the picture of fragmented stripe phases