The Problem of True Macroscopic Charge Quantization in Coulomb Blockade

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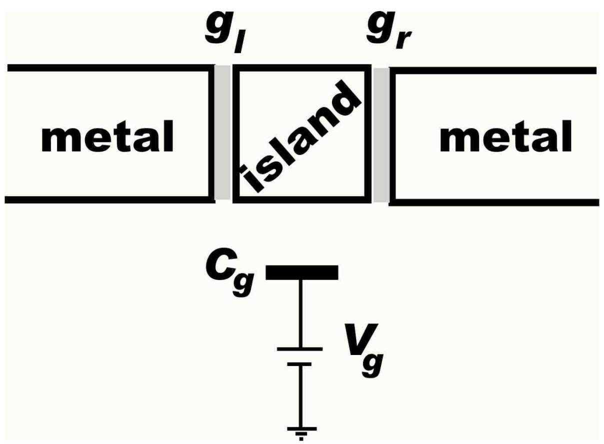
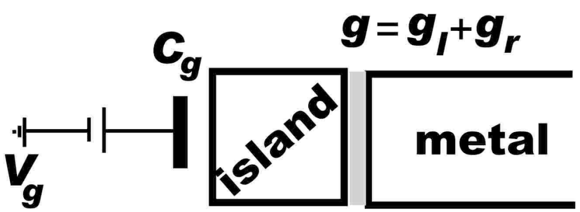
Introduction

- ***Single electron transistor (SET)***

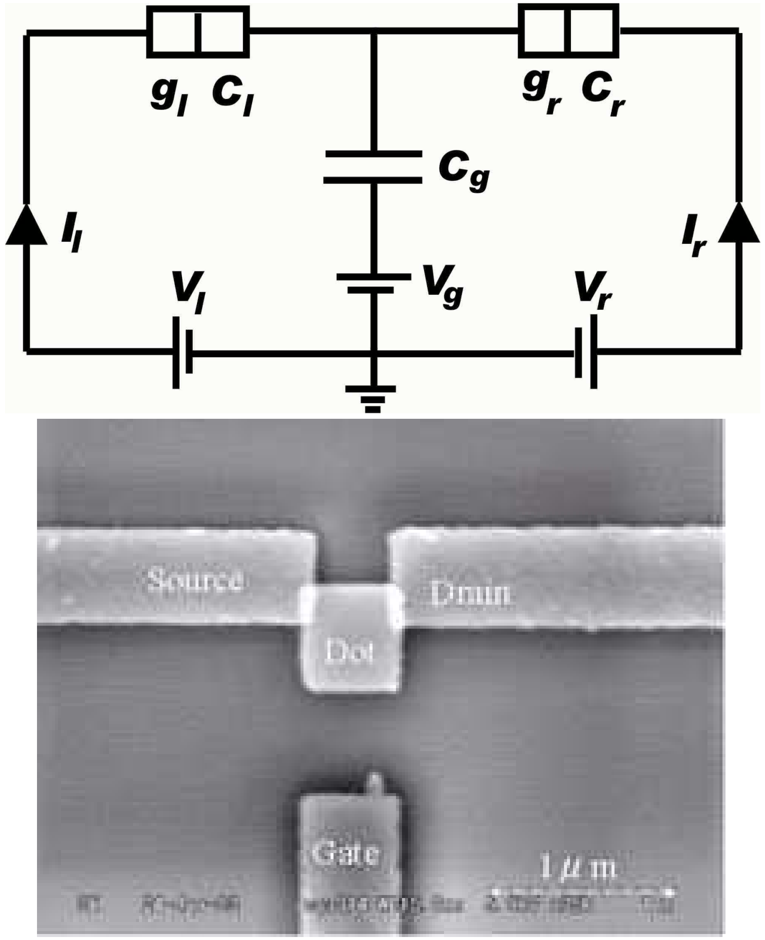


$$V_l = V_r = 0$$

- ***Single electron box (SEB)***



Equivalent circuit for SET



Single charge tunneling,
ed. by Grabert and Devoret, 1992

Introduction

The SET parameters

Number of transport channels	$N \gg 1$
Tunneling matrix elements	$\nu_{l,r}\nu_d \lvert t_{l,r}\rvert^2 \ll 1$
Tunneling conductances **[in units e^2/h]**	$g_{l,r} = 8\pi^2 N \lvert t_{l,r}\rvert^2 \nu_{l,r}\nu_d$
Charging energy	$E_c = \frac{e^2}{2C}$
Level spacing on the island	$\delta \ll E_c$
Temperature	$\max\{1, g_{l,r}\}\delta/N \ll T \lesssim E_c$
External charge	$q = C_g V_g/e$

Beloborodov, Efetov, Altland, Hekking, 2001
Efetov, Tschersich, 2003

Introduction

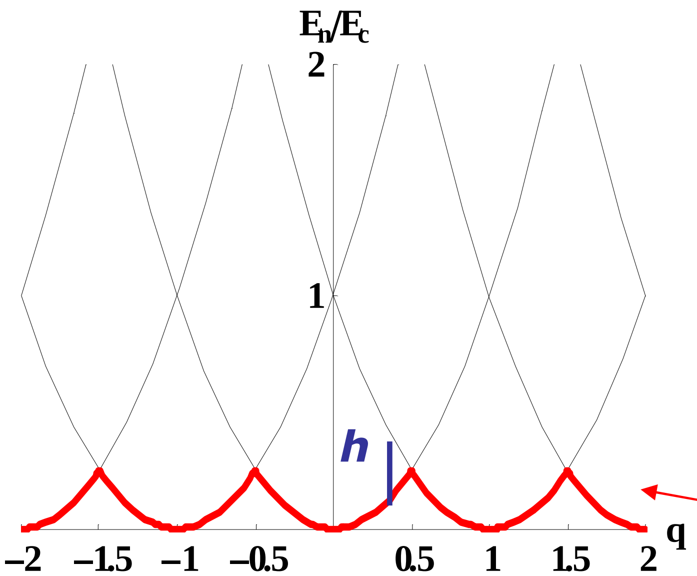
- ***How does discreteness of the charge manifest itself at T=0 ?***

Isolated island (*$g_{l,r}$=0*)

Periodic in q

$$\hat{H}_c = E_c(\hat{n} - q)^2 \quad \Rightarrow \quad Z = \sum_{n=-\infty}^{\infty} \exp\left[-\beta E_c (n-q)^2\right]$$

where $\hat{n}$ is the operator of the excess particle number on the island



Ground state energy

At $T \ll E_c$

$$Q = \langle \hat{n} \rangle = \left[1 + \exp(\beta h)\right]^{-1}, \quad 0 \leqslant q \leqslant 1$$

where *h = E_c(1-2q)*

Introduction

- ***How does discreteness of the charge manifest itself at T=0 ?***

Non-isolated island $(g_{l,r} \neq 0, \quad g_{l,r} \ll 1)$

At *T=0*

Matveev, 1991
Schoeller, Schön, 1994

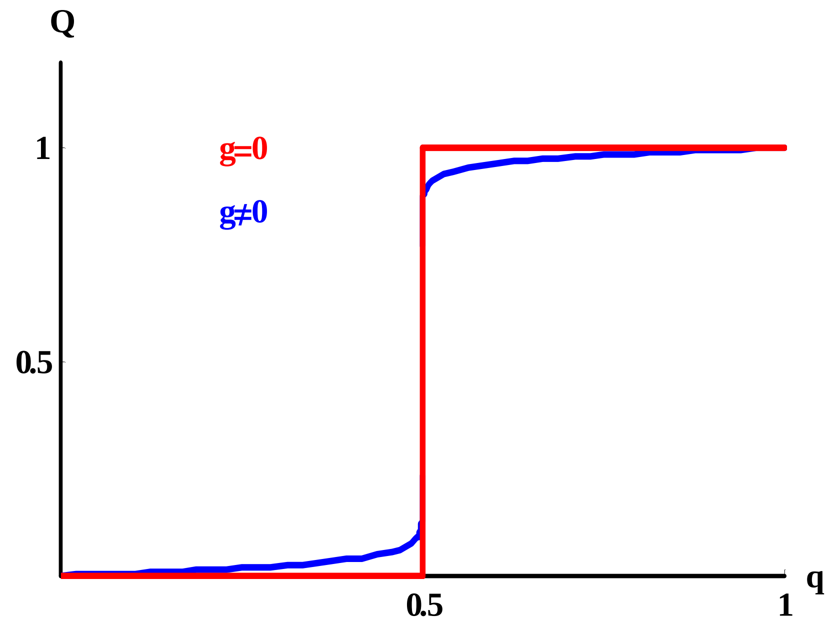
$$Q = \frac{1}{2}\begin{cases} 1 - \left[1 + \frac{g}{2\pi^2}\ln\frac{1}{|1-2q|}\right]^{-1}, & 0 < \frac{1}{2} - q \ll 1 \\ 1 + \left[1 + \frac{g}{2\pi^2}\ln\frac{1}{|1-2q|}\right]^{-1}, & 0 < q - \frac{1}{2} \ll 1 \end{cases}$$

where $g = g_l + g_r$

***Q* is not quantized at *T=0* for any *g*!**

↓

What quantity is quantized at *T=0*?



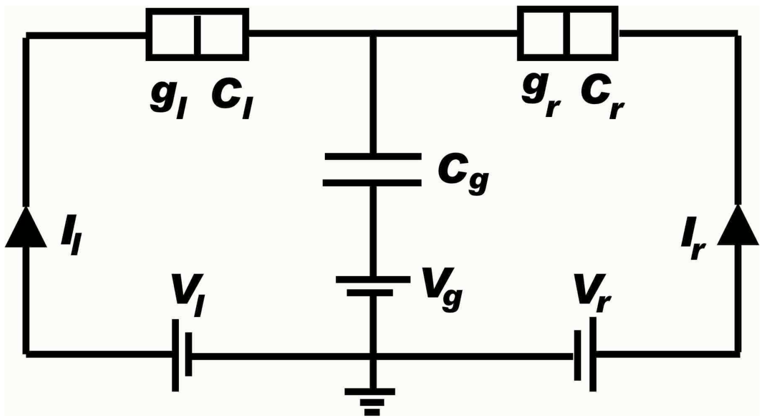
Results

N.B. commutator

- ***Novel quantity***

$$q' = Q - i\frac{(g_l + g_r)^2}{2g_l g_r}\frac{\partial}{\partial V}\int_{-\infty}^{0} dt \langle [\hat{I}(0), \hat{I}(t)] \rangle \Big|_{V=0}$$

where $\hat{I}(t)$ is current operator for the SET, *$V=V_r-V_l$* is the voltage drop

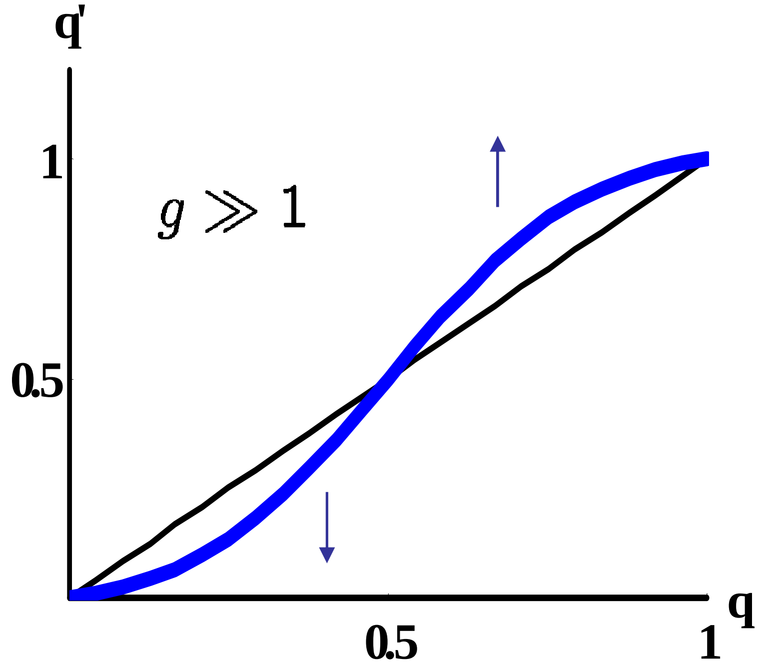


Results

- ***Temperature dependence (explicit calculations)***

Weak coupling $g \gg 1$

$$q'(T) = q - \frac{g^3 E_c}{24\pi T} e^{-g/2} \sin 2\pi q, \qquad T \gg E_c g^3 e^{-g/2}$$



Results

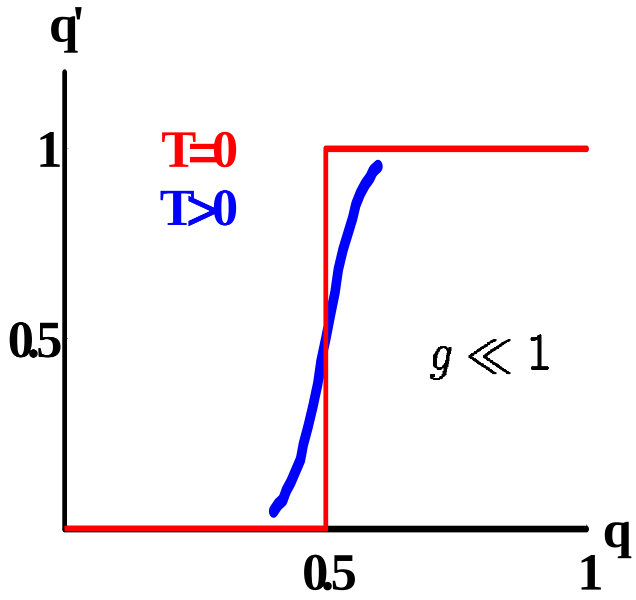
- ***Temperature dependence (explicit calculations)***

Strong coupling $g \ll 1$ **and** $\left|\frac{1}{2} - q\right| \ll 1$

$$q'(T) = \left[1 + \exp(\beta h')\right]^{-1}, \qquad T \ll E_c$$

where

$$h' = h\left[1 + \frac{g}{2\pi^2} \ln \frac{E_c}{\max\{T, h'\}}\right]^{-1}, \quad h = E_c(1-2q)$$



q^0 is quantized at $T=0$

Conjecture

q^0 is quantized at $T=0$ independent of g

AES model

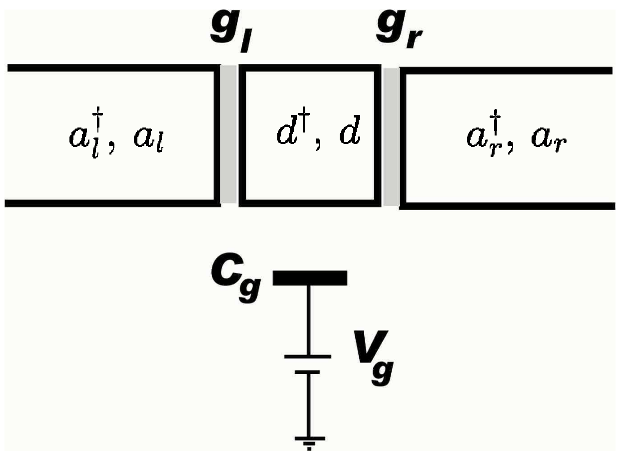
Microscopic Hamiltonian

$$\hat{H} - \mu \hat{N} = \hat{H}_0 - \mu \hat{N} + \hat{H}_c + \sum_{\alpha=l,r} \hat{H}_T^{(\alpha)}$$

$$\hat{H}_0 - \mu \hat{N} = \sum_{k,\alpha=l,r} (\epsilon_{k\alpha} - \mu_\alpha) a_{k\alpha}^\dagger a_{k\alpha} + \sum_p (\epsilon_p - \mu) d_p^\dagger d_p$$

$$\hat{H}_c = E_c \left(\sum_p d_p^\dagger d_p - q \right)^2$$

$$\hat{H}_T^{(\alpha)} = \sum_{k,p} t_{kp}^{(\alpha)} a_{k\alpha}^\dagger d_p + \text{h.c.}$$



i) Decoupling H_c by the Hubbard-Stratonovich field $\phi(\tau)$

ii) Integration over fermions to the lowest order in H_T

AES model

- ***Effective action in imaginary time***

integer

$$\mathcal{S}_{AES}[\phi] = \frac{g}{4}\int_0^\beta d\tau_1 d\tau_2 \alpha(\tau_{12}) e^{-i[\phi(\tau_1)-\phi(\tau_2)]} - iq\int_0^\beta d\tau \dot{\phi} + \frac{1}{4E_c}\int_0^\beta d\tau\, \dot{\phi}^2$$

$$\phi(\beta) = \phi(0) + 2\pi W$$

Ambegaokar, Eckern, Schön, 1982

where $\tau_{12} = \tau_1 - \tau_2, \quad \alpha(\tau) = -\frac{T^2}{\sin^2 \pi T\tau} = \frac{T}{\pi}\sum_n |\omega_n| e^{i\omega_n \tau}$

and $g = g_l + g_r, \qquad g_{l,r} = 8\pi^2 N |t_{l,r}|^2 \nu_{l,r}\nu_d$

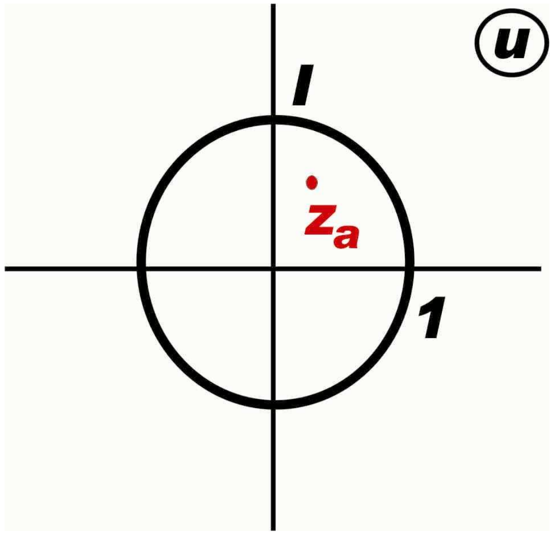
N.B. AES model is known also as circular brane model

Lukyanov, Zamolodchikov, 2003
Lukyanov, Werner, 2006

AES model

Nontrivial topology and instantons

Convenient variables $u = e^{i2\pi T\tau}, \quad f(u) = e^{i\phi(\tau)}$



$$S_{AES} = \frac{g}{2}\sum_n |n||f_n|^2 + \frac{\pi^2 T}{E_c}\sum_n n^2|\phi_n|^2 - i2\pi q\mathcal{C}[\phi]$$

Topological charge (winding number)

$$\mathcal{C}[\phi] = \frac{1}{2\pi}\int_0^\beta d\tau\dot{\phi} = W$$

Instanton ***W>0*** ***$|z_a|<1$***

Anti-instanton ***W<0*** ***$|z_a|>1$***

Instanton solution (*2|W|* zero modes)

$$f_{inst} = \prod_{a=1}^{|W|} \frac{u - z_a}{1 - u\bar{z}_a}$$

Korshunov, 1987
Bulgadaev, 1987
Nazarov, 1999

Classical action

$$S_{AES}^{inst} = \frac{g}{2}|W| - i2\pi qW + \frac{\pi^2 T}{E_c}\prod_{a,b=1}^{|W|}\frac{1 + z_a\bar{z}_b}{1 - z_a\bar{z}_b}$$

Response to change in the boundary condition

- ***Expansion in topological sectors***

$$Z = \sum_W Z[W], \quad Z[W] = \int_{\phi(\beta)=\phi(0)+2\pi W} \mathcal{D}[\phi] e^{-S_{AES}[\phi]}$$

Let us formally consider *Z[x]* with *continuous x* and define

$$\frac{g'}{4\pi} = \mathrm{Im}\,\frac{1}{2\pi i Z}\sum_W \frac{\partial Z[x]}{\partial x}\Big|_{x=W} \qquad q' = \mathrm{Re}\,\frac{1}{2\pi i Z}\sum_W \frac{\partial Z[x]}{\partial x}\Big|_{x=W}$$

Response to change in the boundary condition

Analytic continuation

N.B. it is solution of classical equation of motion

From *W* to *W-n*

$$\phi(\tau) \to \phi(\tau) - \omega_n \tau \quad \Longrightarrow \quad Z[W-n] = Z[W]\left\langle e^{-\beta \hat{K}(i\omega_n) - i\omega_n \beta \hat{Q} - \omega_n{}^2 \beta/(4E_c)} \right\rangle_W$$

where

$$\hat{Q} = q + \frac{iT}{2E_c}\int_0^\beta d\tau \dot{\phi} \qquad \hat{K}(i\omega_n) = \frac{g}{4\beta}\int_0^\beta d\tau_1 d\tau_2 \left[e^{i\omega_n \tau_{12}} - 1\right] \alpha(\tau_{12}) e^{i[\phi(\tau_2)-\phi(\tau_1)]}$$

Analytic continuation

$$i\omega_n \to \omega + i0^+, \qquad \omega \to 0$$

Hence

$$\frac{1}{2\pi i Z}\sum_W \frac{\partial Z[x]}{\partial x}\Big|_{x=W} = Q + \frac{\partial K^R(\omega)}{\partial \omega}, \quad \omega \to 0$$

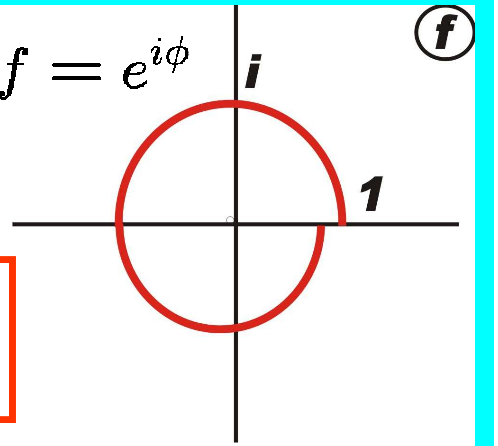
where

$$Q = q + \frac{iT}{2E_c}\int_0^\beta d\tau \langle \dot{\phi} \rangle \qquad K(i\omega_n) = \frac{g}{4\beta}\int_0^\beta d\tau_1 d\tau_2 \left[e^{i\omega_n \tau_{12}} - 1\right] \alpha(\tau_{12}) \langle e^{i[\phi(\tau_2)-\phi(\tau_1)]} \rangle$$

Response to change in the boundary condition

Response parameters

$$f = e^{i\phi}$$



Finally,

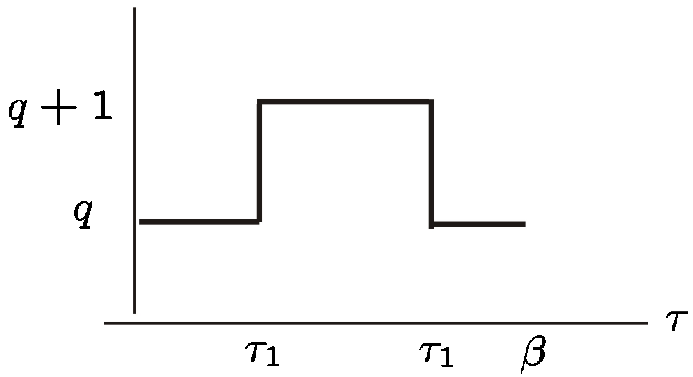
$$\frac{g'}{4\pi} = \mathrm{Im}\, \frac{\partial K^R(\omega)}{\partial \omega}\Big|_{\omega=0}, \qquad q' = Q + \mathrm{Re}\, \frac{\partial K^R(\omega)}{\partial \omega}\Big|_{\omega=0}$$

where

$$Q = q + \frac{iT}{2E_c}\int_0^\beta d\tau \langle \dot{\phi} \rangle$$

$$K^R(\omega) = \frac{g}{4\pi^2}\int dE dE' \,\mathrm{Im}\, D^R(E) \frac{E'}{E' - E + \omega + i0}[n_B(E) - n_B(E')]$$

$$D(i\omega_n) = \int_0^\beta \frac{d\tau_1 d\tau_2}{\beta} e^{-i\omega_n \tau_{12}} \langle \exp\left[i\phi(\tau_1) - i\phi(\tau_2)\right] \rangle$$



Physical observables

- ***Relation of g′ and q′ with physical quantities***

SET conductance *G* and *g′*

$$g' = \frac{(g_l + g_r)^2}{g_l g_r} G$$

It can be shown in Keldysh technique

Average charge *Q*, quantum current noise and *q′*

$$q' = Q - i\frac{(g_l + g_r)^2}{2 g_l g_r} \frac{\partial}{\partial V} \int_{-\infty}^{0} dt \langle [\hat{I}(0), \hat{I}(t)] \rangle \Big|_{V=0}$$

Weak coupling - $g \gg 1$

Perturbation theory in 1/g (W=0)

Quadratic part of the action

$$S^{(2)}_{AES} = g \sum_{n>0} \left[n + \frac{2\pi^2 T}{g E_c} n^2 \right] |\phi_n|^2 \quad \Longrightarrow \quad \langle \phi_n \phi_{-n} \rangle = \frac{1}{g} \frac{1}{|n| + 2\pi^2 T n^2/(gE_c)}$$

Correlation function

$$D(\tau_{12}) = \left\langle e^{i\phi(\tau_1) - i\phi(\tau_2)} \right\rangle \approx 1 - \frac{1}{2} \langle [\phi(\tau_1) - \phi(\tau_2)]^2 \rangle$$

$$D(i\omega_n) = \beta \delta_{n,0} \left[1 - \frac{2}{g} \sum_{m>0} \frac{1}{m + 2\pi^2 T m^2/(gE_c)} \right] + \frac{\beta}{g} \frac{1}{|n| + 2\pi^2 T n^2/(gE_c)}$$

$$\mathrm{Im}\, D^R(E) = \pi \left[1 - \frac{2}{g} \ln \frac{gE_c}{T} \right] \beta E\, \delta(E)$$

$$\Downarrow$$

Guinea, Schön, 1986 →

$$g'(T) = g - 2 \ln \frac{gE_c}{T}, \qquad q'(T) = q$$

Weak coupling - $g \gg 1$

Instanton contribution (W=± 1)

Classical action

$$S_{AES}^{inst} = \frac{g}{2} \pm i2\pi q$$

Gaussian fluctuations around instanton →

$$S_{AES}^{inst} = \frac{g(\lambda)}{2} \pm i2\pi q$$

where $\lambda = \beta(1 - |z|^2)$ **and** $g(\lambda) = g - 2 \ln gE_c\lambda$

Panyukov, Zaikin, 1991
Wang, Grabert, 1996

Correlation function

$$D_{inst}(i\omega_n) = \int_0^\beta \frac{d\lambda}{\lambda^2} g\, e^{-g(\lambda)/2} \begin{cases} e^{i2\pi q}\lambda^2(1-\lambda T)^{|n|-1}, & n<0 \\ 2\beta^2(1-\lambda T)\cos 2\pi q, & n=0 \\ e^{-i2\pi q}\lambda^2(1-\lambda T)^{n-1}, & n>0 \end{cases}$$

↓ Analytic continuation

$$D_{inst}^{R}(\omega) = \int_0^\beta \frac{d\lambda}{1-\lambda T} g e^{-g(\lambda)/2} \left[e^{-i2\pi q} e^{-i\omega\beta/(2\pi)\ln(1-\lambda T)} + \left(1 - 2\frac{(1-\lambda T)^2}{\lambda^2 T^2}\right) \lim_{\eta\to 0} \frac{\eta}{\omega - \eta + i0} \cos 2\pi q \right]$$

Weak coupling - $g \gg 1$

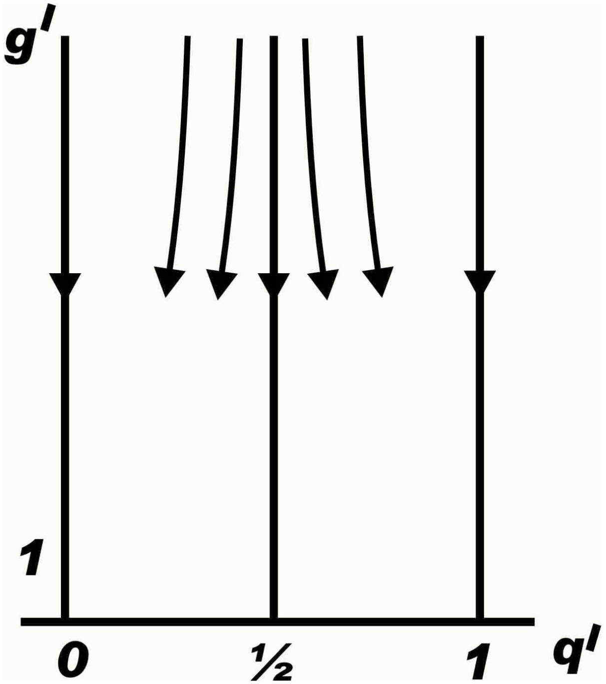
- **Instanton contribution (*W=± 1*)**

At high temperatures $T \gg E_c g^3 e^{-g/2}$

$$g'(T) = g - 2\ln\frac{gE_c}{T} - \frac{g^3 E_c}{6T} e^{-g/2}\cos 2\pi q$$

Altland, Glazman, Kamenev,Meyer, 2006

$$q'(T) = q - \frac{g^3 E_c}{24\pi T} e^{-g/2}\sin 2\pi q$$



Notice

$$Q(T) = q - \frac{g^2}{\pi} e^{-g/2}\ln\frac{E_c}{T}\sin 2\pi q$$

$$\Longrightarrow \frac{|q' - q|}{|Q(T) - q|} \sim \frac{g}{\ln \beta E_c} \gg 1$$

Oscillations in q′ is much more pronounced than in Q

Strong coupling – $g \ll 1$

- ***Effective action for |q-1/2|¿ 1 and T¿ E_c***

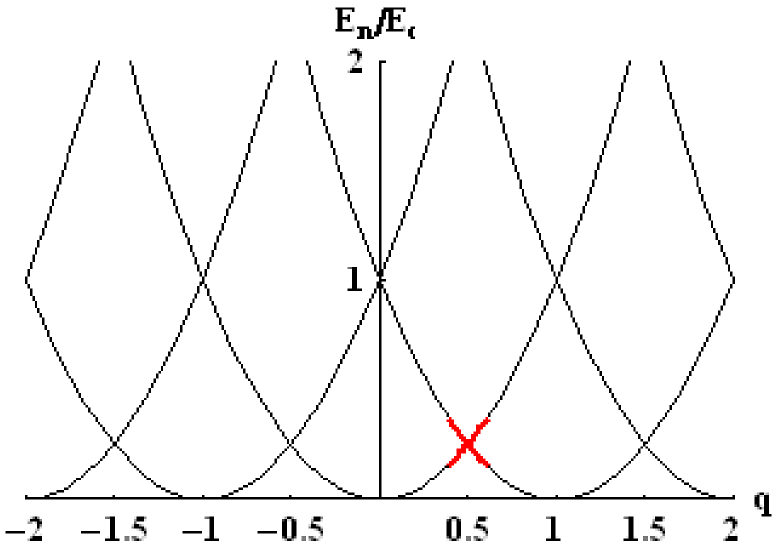
Isolated island (*g=0*)

$$Z = \sum_{n=0}^{\infty} e^{-\beta E_c (n-q)^2} \approx e^{-\beta E_c q^2}(1 + e^{-\beta h})$$

where ***$h=E_c(1-2q)$***

Two-level Hamiltonian

$$\hat{H}_0 = \frac{h}{2}\hat{\sigma}_z \longrightarrow Z = 2\cosh\frac{\beta h}{2}$$



Average charge

$$Q = \frac{1}{2} - \frac{\partial}{\partial h} T \ln Z = \frac{1}{1+e^{\beta h}}$$

Operators

$$e^{\pm i\phi} \longrightarrow \hat{\sigma}_\pm$$

Strong coupling – $g \ll 1$

- ***Effective action for*** $|q - 1/2| \ll 1$ ***and*** $T \ll E_c$ ***(XY Bose-Kondo)***

Non-isolated island $(g \neq 0)$

$$\mathcal{S}_{AES} \to \mathcal{S} = \int_0^\beta d\tau \bar{\psi} \left(\partial_\tau - \eta + \frac{h}{2}\sigma_z \right) \psi + \frac{g}{4} \int_0^\beta d\tau_1 d\tau_2 \alpha(\tau_{12}) (\bar{\psi}\sigma_- \psi)(\tau_1)(\bar{\psi}\sigma_+\psi)(\tau_2)$$

Larkin, Melnikov, 1971
Sachdev, Ye, 1993
Zhu,Si; Zarand,Demler, 2002

where $\bar{\psi}, \psi$ **–** ***pseudofermions*****, chemical potential** $\eta \to -\infty$

Grand partition function and correlations

$$Z = \lim_{\eta \to -\infty} \frac{\partial}{\partial e^{\beta\eta}} Z_{pf}$$

$$\langle O \rangle = \lim_{\eta \to -\infty} \left[\frac{Z_{pf}}{Z} \frac{\partial}{\partial e^{\beta\eta}} \langle O \rangle_{pf} + \langle O \rangle_{pf} \right]$$

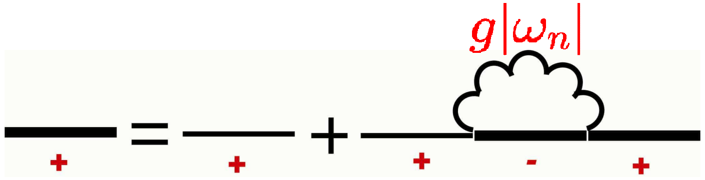
Strong coupling – $g \ll 1$

Leading logarithmic approximation – $g^n \ln^n$

Pseudofermion Green function renormalization

$$G^{-1}_{0\pm}(i\epsilon_n) = i\epsilon_n + \eta \mp h/2$$

$$G^{-1}_{\pm}(i\epsilon_n) = (i\epsilon_n + \eta)\gamma(i\epsilon_n) \mp \gamma_s(i\epsilon_n)h/2$$



Equations for γ and γ_s

$$\gamma(x) = 1 + \frac{g}{4\pi^2}\int_0^x \frac{dy}{\gamma(y)}$$

$$\gamma_s(x) = 1 - \frac{g}{4\pi^2}\int_0^x \frac{dy\,\gamma_s(y)}{\gamma^2(y)}$$

where

$$x = \ln \frac{E_c}{\max\{h\gamma)s/\gamma, |\epsilon_n|\}}$$

$$\gamma(x) = \gamma_s^{-1}(x) = \left(1 + \frac{g}{2\pi^2}\ln\frac{E_c}{\max\{h\gamma_s/\gamma, |\epsilon_n|\}}\right)^{1/2}$$

Strong coupling – $g \ll 1$

- ***Leading logarithmic approximation –*** $g^n \ln^n$

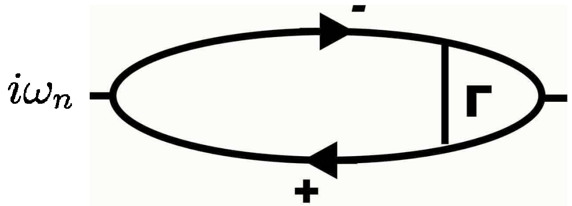
Average charge

$$Q = \frac{1}{2}\left(1 - \frac{1}{\gamma^2}\tanh\frac{\beta h'}{2}\right)$$

T=0 *- Matveev, 1991*
T>0 *- Schoeller, Schön, 1994*

where $h' = h/\gamma^2$ **and** $\gamma^2 = 1 + \frac{g}{2\pi^2}\ln\frac{E_c}{\max\{h', T\}}$

Correlation function

$D(i\omega_n) =$ 

$\Gamma = 1$

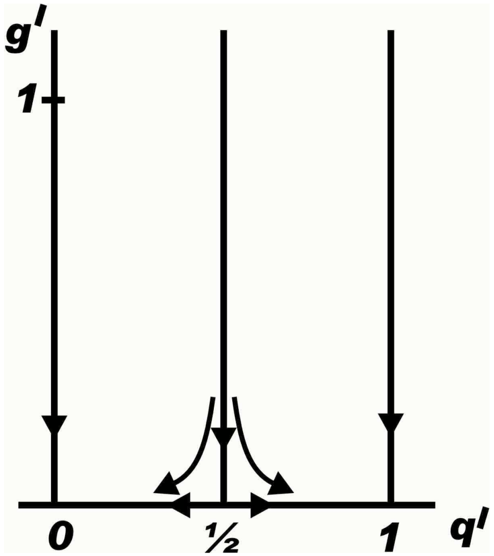
$$\Rightarrow \quad D^R(\omega) = \frac{\tanh \beta h'/2}{\gamma^2}\frac{1}{\omega + h' + i0}$$

Strong coupling – $g \ll 1$

- **Physical observables *q′* and *g′***

$$g'(T,h) = \frac{g/2}{1+\frac{g}{2\pi^2}\ln\frac{E_c}{\max\{T,h'\}}}\,\frac{\beta h'}{\sinh \beta h'}$$

(Schoeller, Schön, 1994)



$$q'(T,h) = [1+\exp(\beta h')]^{-1}$$

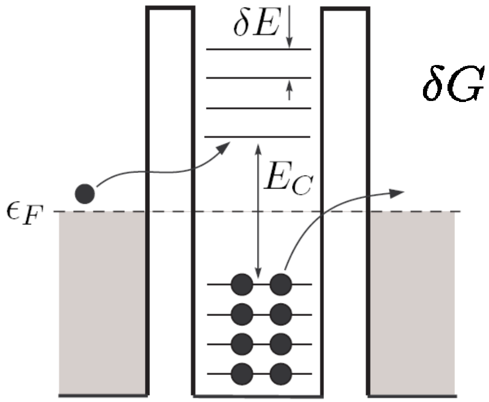
$$h' = \frac{h}{1+\frac{g}{2\pi^2}\ln\frac{E_c}{\max\{h',T\}}}$$

For $T \ll h'$

$$g'(T,h) = \frac{2\pi^2}{\ln\left(\frac{E_c e^{2\pi^2/g}}{h'}\right)}\frac{h'}{T}e^{-\beta h'}, \qquad q'(T,h) = e^{-\beta h'}$$

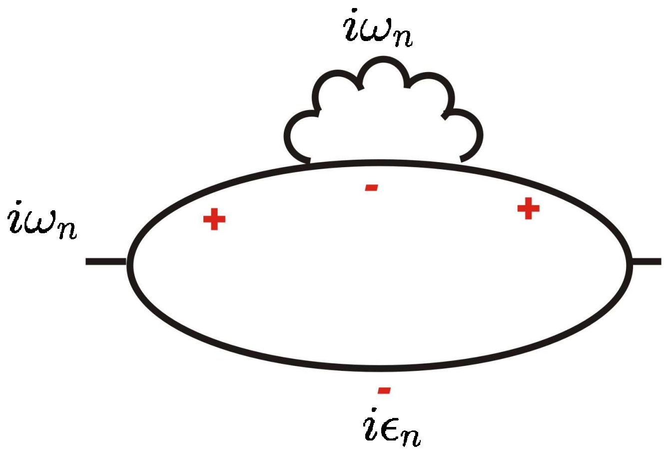
Strong coupling – $g \ll 1$

- ***Inelastic co-tunneling at*** $T \ll h'$



$$\delta G \sim g_l g_r \frac{T^2}{E_c^2}$$

(Averin, Nazarov, 1990)



$$\delta D(i\omega_n) = -\frac{g}{4\pi\gamma^4}\frac{|\omega_n|}{(i\omega_n + h')^2}$$

Contributions to *g′* and *q′*

$$\delta g'(T,h) = \frac{2\pi^4}{3\ln^2\left(\frac{E_c}{h'}e^{2\pi^2/g}\right)}\left(\frac{T}{h'}\right)^2, \qquad \delta q'(T,h) = \frac{\pi^2}{8\ln^2\left(\frac{E_c}{h'}e^{2\pi^2/g}\right)}\left(\frac{h'}{T}\right)^2 e^{-\beta h'}$$

Conclusions

- **Novel quantity *q^0* is introduced into the Coulomb blockade problem**

- **At $g \ll 1$ it is shown that *q^0* is quantized at *T=0* (with exponential accuracy) in the leading logarithmic approximation contrary to the average charge *Q* on the SET island**

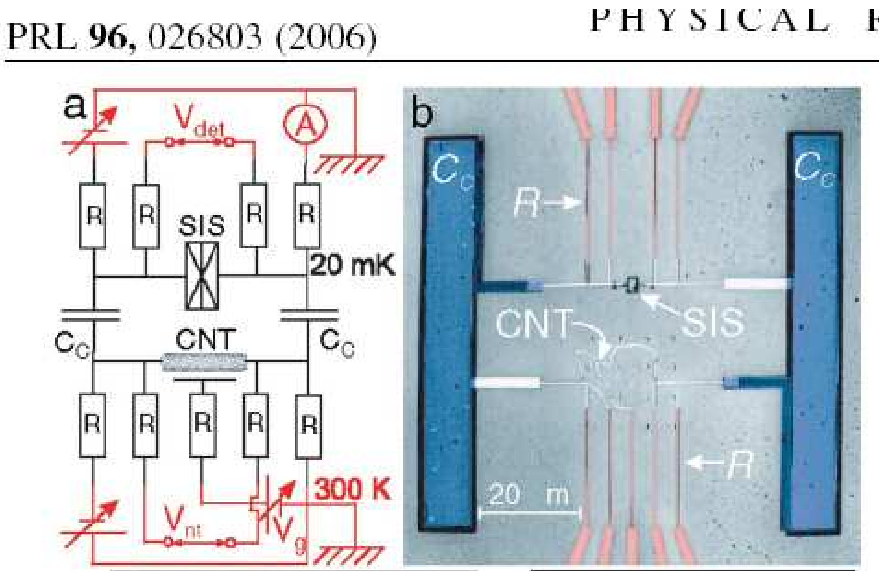
- **Inelastic co-tunneling does not destroy the quantization of q’**

Experimental realization

Main difficulty is to measure *non*-symmetrized current noise in the SET

LC curcuit inductively coupled to the SET

(Lesovik, Loosen, 1997)



(Onac, Balestro, Trauzettel, Lodewijk, Kouwenhoven, 2006)

AES model

Circular brane model

Lukyanov, Zamolodchikov, 2003
…
Lukyanov, Werner, 2006

$$\mathcal{S}_{CB}[X,Y] \;=\; \frac{g}{4\pi}\int_0^\beta d\tau \int_0^L ds[(\nabla X)^2 + (\nabla Y)^2],$$
$$X^2(\tau,0) + Y^2(\tau,0) = 1$$

where *X* and *Y* are boson fields on the surface of a half cylinder

Integrating out the 'bulk'

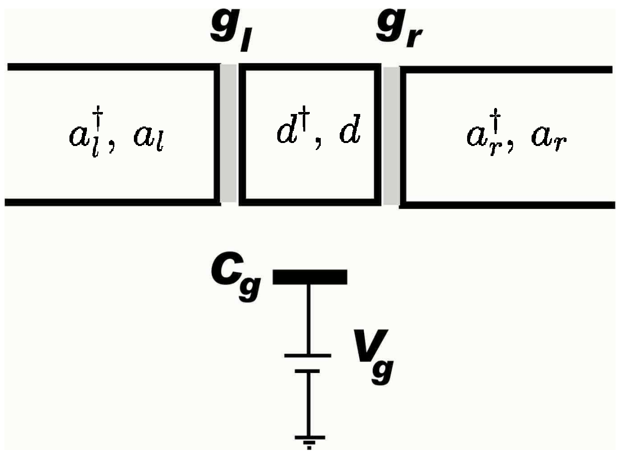
$$X(\tau,0) \;=\; \cos\phi(\tau),$$
$$Y(\tau,0) \;=\; \sin\phi(\tau)$$

$$\mathcal{S}_{AES}[\phi] = \frac{g}{4}\int_0^\beta d\tau_1 d\tau_2 \alpha(\tau_{12}) e^{-i[\phi(\tau_1)-\phi(\tau_2)]} - iq\int_0^\beta d\tau \dot{\phi} + \frac{1}{4E_c}\int_0^\beta d\tau\, \dot{\phi}^2$$
$$\phi(\beta) = \phi(0) + 2\pi W$$

with *q=0* and $E_c = \pi/gL$

Physical observables

Tunneling conductance G of the SET



Standard trick $(\alpha = l, r)$

$$\begin{aligned}\widehat{H}_T^{(\alpha)} &= X_\alpha e^{iV_\alpha t} + X_\alpha^\dagger e^{-iV_\alpha t}\\ \widehat{I}_\alpha &= iX_\alpha e^{iV_\alpha t} - iX_\alpha^\dagger e^{-iV_\alpha t},\\ \widehat{X}_\alpha &= \sum_{k,p} t_{kp}^{(\alpha)} a_{k\alpha}^\dagger d_p\end{aligned}$$

Average current

$$I_\alpha = -i\int_{-\infty}^{t} dt' \langle [\widehat{I}_\alpha(t), \widehat{H}_T^{(\alpha)}(t')]\rangle$$

Retarded correlation function

$$K_\alpha^R(\omega) = i\int_{-\infty}^{\infty} dt\, e^{i\omega t}\theta(t)\langle [X_\alpha(t), X_\alpha^\dagger(0)]\rangle$$

$\Rightarrow$

$$I_\alpha = 2\,\mathrm{Im}\, K_\alpha^R(\omega = V_\alpha)$$

Matsubara correlation function

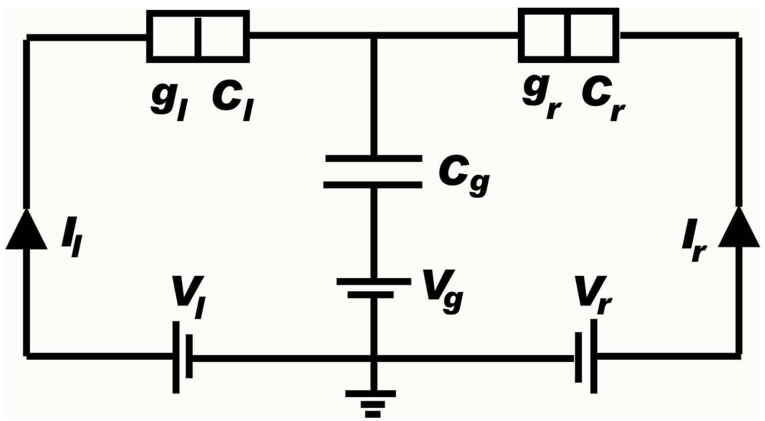
$$\begin{aligned}K_\alpha(i\omega_n) &= \frac{1}{2}\int_{-\beta}^{\beta} d\tau e^{i\omega_n\tau}\langle T_\tau X_\alpha(\tau)X_\alpha^\dagger(0)\rangle\\ &= \frac{g_\alpha}{4\beta}\int_0^\beta d\tau_1 d\tau_2 e^{i\omega_n\tau_{12}}\alpha(\tau_{12})D(\tau_{21}) \equiv \frac{g_\alpha}{g}K(i\omega_n)\end{aligned}$$

$\Rightarrow$

$$K_\alpha^R(\omega) = \frac{g_\alpha}{g}K^R(\omega)$$

Physical observables

Tunneling conductance G of the SET



Average current

$$I_\alpha = \frac{1}{2\pi}\frac{g_\alpha g'}{g}V_\alpha, \qquad g' = 4\pi\,\mathrm{Im}\frac{K^R(\omega)}{\omega}, \quad \omega \to 0$$

By definition

$$I \equiv I_l = -I_r = \frac{G}{2\pi}(V_r - V_l) \equiv \frac{G}{2\pi}V$$

$$G = \frac{g_l g_r}{(g_l + g_r)^2}g'$$

Ben-Jacob, Mottola, Schön, 1983

Strong coupling – $g \ll 1$

Pseudofermions

Spin language

$$\hat{H}_0 = \frac{h}{2}\hat{\sigma}_z \longrightarrow Z = e^{\beta h/2} + e^{-\beta h/2}$$

Fermion language

$h/2$

$-h/2$

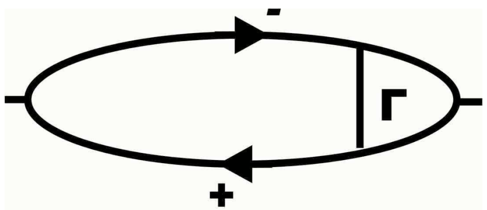
$$\begin{array}{ccc} 0\,0 & \rightarrow & \cancel{1} \\ 1\,0 & \rightarrow & e^{-\beta h/2} \\ 0\,1 & \rightarrow & e^{\beta h/2} \\ 1\,1 & \rightarrow & \cancel{1} \end{array} \quad \xrightarrow{N_{pf}=1} \quad Z = e^{\beta h/2} + e^{-\beta h/2}$$

$$Z_{pf} = \sum_{N_{pf}} e^{\beta\eta N_{pf}} \, \mathrm{Tr} e^{-\beta\hat{H}} \quad \rightarrow \quad Z = \lim_{\eta\to-\infty} \frac{\partial}{\partial e^{\beta\eta}} Z_{pf}$$

Strong coupling – $g \ll 1$

Correlation function $D(i\omega_n)$

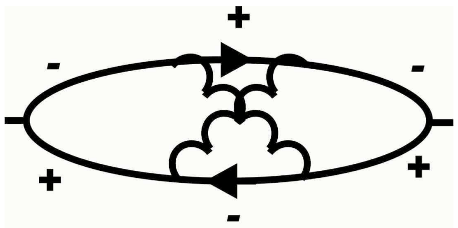
$$D(i\omega_n) = -\frac{1}{2\cosh\beta h'/2}\lim_{\eta\to-\infty}\frac{\partial}{\partial e^{\beta\eta}}T\sum_{\epsilon_m}\frac{1}{(i\epsilon_m+\eta)\gamma-\gamma_s h/2}\frac{1}{(i\epsilon_m+i\omega_n+\eta)\gamma-\gamma_s h/2}$$
$$= \frac{\tanh\beta h'/2}{\gamma^2}\frac{1}{i\omega_n+h'}$$



$\Gamma = 1$

$$D^R(\omega) = \frac{\tanh\beta h'/2}{\gamma^2}\frac{1}{\omega+h'+i0}$$

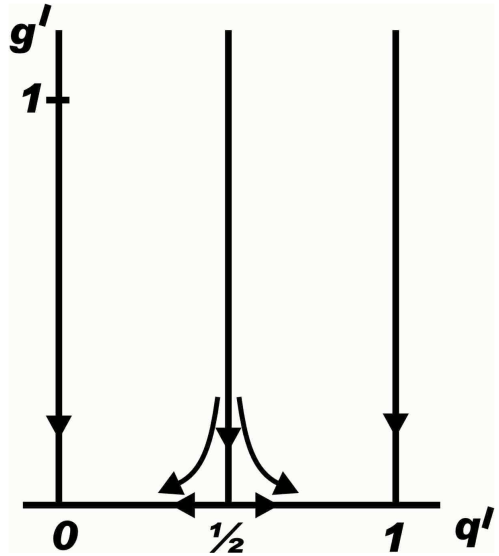
No vertex renormalization !



$$\propto g^2 \ln\frac{E_c}{\max\{h,T\}}\frac{1}{i\omega_n+h}$$

Strong coupling – $g \ll 1$

Physical observables q' and g'



Correlation function $K(\omega)$ for $\omega \to 0$

$$\mathrm{Im}\,\frac{K(\omega)}{\omega} = \frac{g}{8\pi\gamma^2}\frac{\beta h'}{\sinh\beta h'}$$

$$\mathrm{Re}\,\frac{K(\omega)}{\omega} = -\frac{g}{4\pi^2\gamma^2}\ln\frac{E_c}{\max\{T,h\}}\tanh\frac{\beta h'}{2}$$

$$g'(T,h) = \frac{g/2}{1+\frac{g}{2\pi^2}\ln\frac{E_c}{\max\{T,h\}}}\frac{\beta h'}{\sinh\beta h'}$$

(Schoeller, Schön, 1994)

$$q'(T,h) = \left(1+e^{\beta h'}\right)^{-1}$$

with $$h' = \frac{h}{1+\frac{g}{2\pi^2}\ln\frac{E_c}{\max\{h,T\}}}$$

q^0 is quantized at $T=0$ independent of g !