PHASE TRANSITIONS IN HIGHLY CONSTRAINED SYSTEMS

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Work with

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Tom Pickles and Tim Saunders (Oxford)

Ludovic Jaubert and Peter Holdsworth (ENS Lyon)

Roderich Moessner (Dresden)

Pickles, Saunders + JTC, arXiv:0708.3791

Jaubert, JTC, Holdsworth, and Moessner, Phys. Rev. Lett. 100, 067207 (2008)

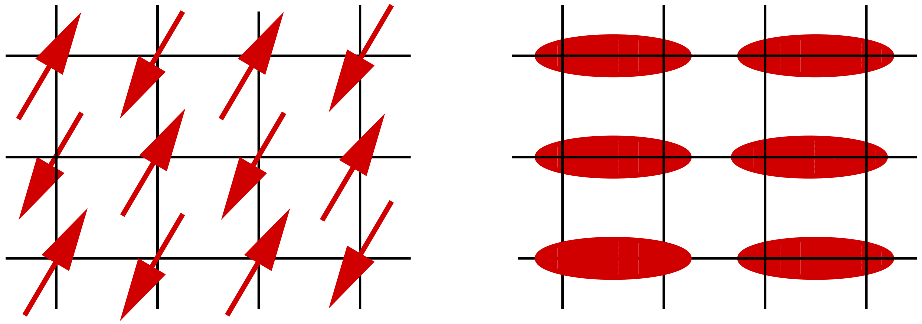
Powell + JTC, arxiv:0803.4204 + arXiv:0805.3698

Evading the Landau description of phase transitions

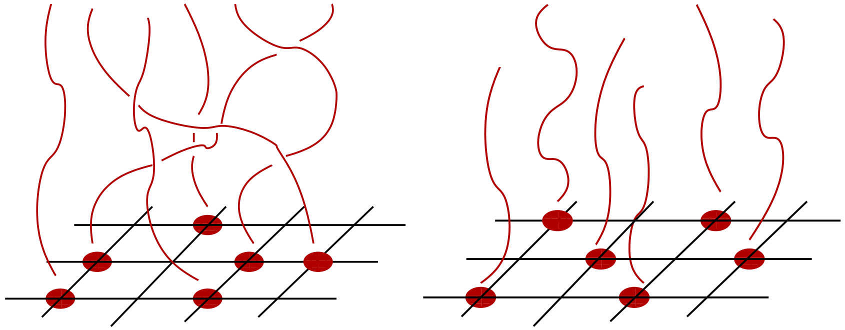
Quantum transitions

Between states with unrelated order parameters

Quantum magnet: Neel - valence bond solid



Lattice bosons: superfluid - density-wave order



Conventionally 1st order: if critical, then unconventional

Evading the Landau description of phase transitions

Quantum transitions

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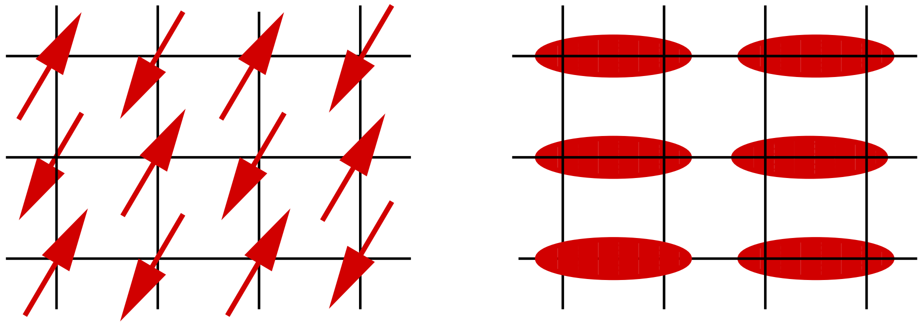

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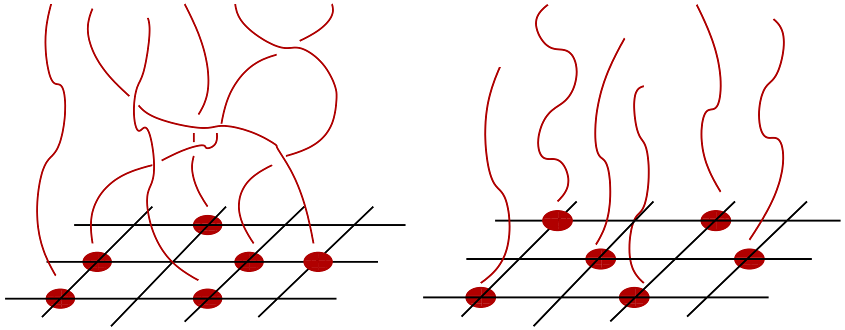


Conventionally 1st order: if critical, then unconventional

Classical transitions

In constrained systems

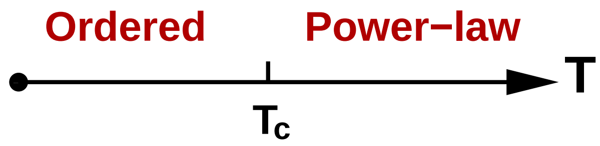
- Classical dimer models
- Classical frustrated magnets

In simplest versions:

power-law correlations

With additional interactions:

ordered low-T phase



Unconventional transition

- from ordered to power-law correlated state

Outline

Statistical mechanics in constrained systems

- Geometrically frustrated magnets, dimer models, ice
- Frustrated local interactions:
  - highly degenerate ground states + topological constraints

Correlations from constraints: the Coulomb phase

- Stable, power-law correlated phase in three dimensions

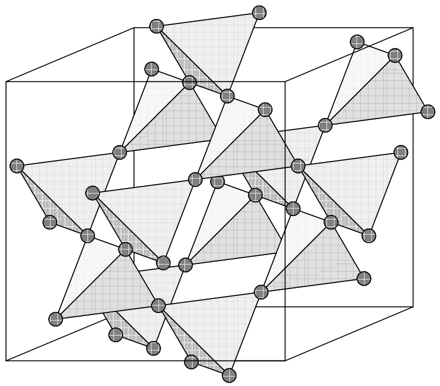
Ordering from the Coulomb phase

- Flux condensation
- A Kasteleyn transition
- Dimer ordering

Highly degenerate states in classical frustrated magnets and dimer models

Ground states of geometrically frustrated magnets

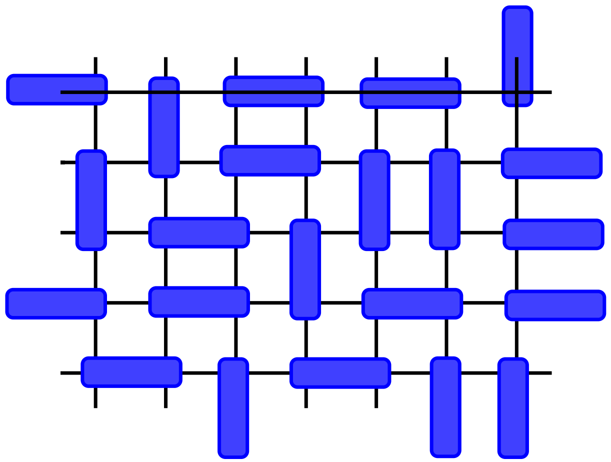
e.g. antiferromagnet on pyrochlore lattice



$$\mathcal{H} = J \sum_{\text{bonds}} \mathbf{S}_i \cdot \mathbf{S}_j \equiv \frac{J}{2} \sum_{\text{units}} |\mathbf{L}_\alpha|^2 + c$$

$$\mathbf{L}_\alpha = \sum_{i \in \text{unit}} \mathbf{S}_i$$

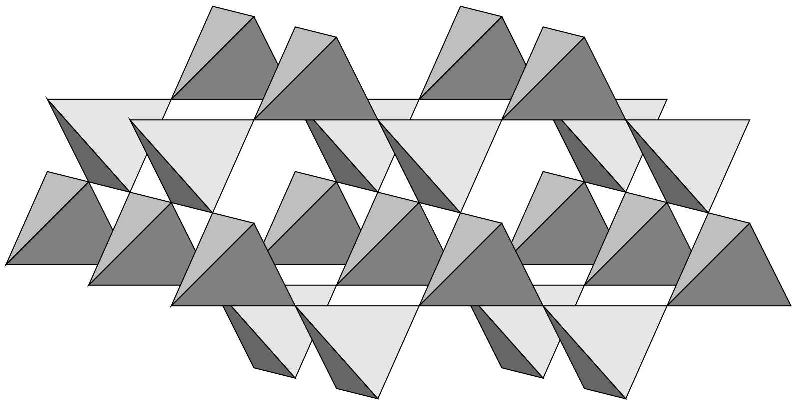
Allowed states of close-packed dimer models



Correlations induced by ground state constraints

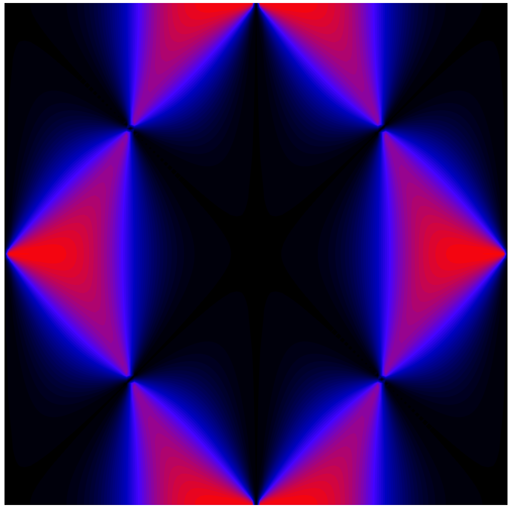
Local constraints

$$\sum_{tet} \mathbf{S}_i = 0$$



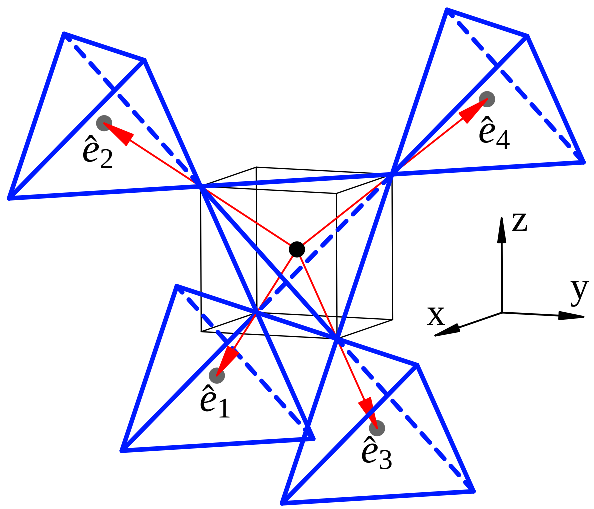
Long range correlations

Sharp structure in
$\langle \mathbf{S}_{-\mathbf{q}} \cdot \mathbf{S}_{\mathbf{q}} \rangle$



Gauge theory of ground state correlations

Youngblood *et al* (1980), Huse *et al* (2003), Henley (2004)



Construct vector fields $\mathbf{B}^a(\mathbf{r}_i)$

from each spin component S_i^a :

$$\mathbf{B}^a(\mathbf{r}_i) = \hat{e}_i S_i^a$$

$$\sum_{\text{unit}} S_i^a = 0 \rightarrow \nabla \cdot \mathbf{B}^a = 0$$

Ground state constraint

becomes flux conservation law:

$$\mathbf{B}^a = \nabla \times \mathbf{A}^a$$

Coarse-grained distribution: $P[\mathbf{B}^a(\mathbf{r})] \propto \exp(-\kappa \int |\mathbf{B}^a(\mathbf{r})|^2)$

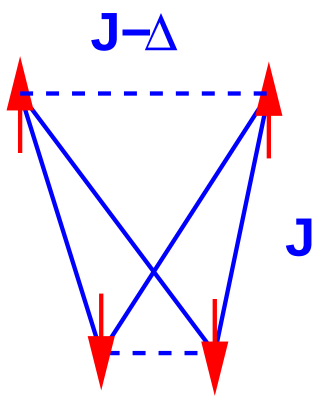
Dipolar correlators: $\langle B_i^l(\mathbf{r}) B_j^m(\mathbf{0}) \rangle \propto \delta_{lm} \frac{3r_i r_j - r^2 \delta ij}{r^5}$

Engineering transitions from the Coulomb phase

Add interactions to select ordered state

Flux condensation

induced by exchange



Néel order for

$$T \ll \Delta$$

Coulomb phase for

$$\Delta \ll T \ll J$$

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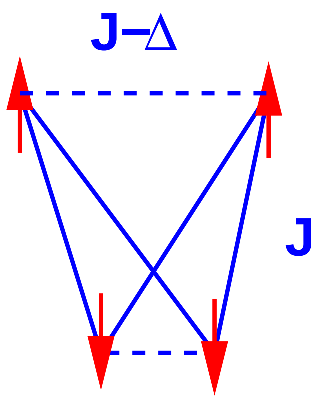

Néel order for

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Coulomb phase for

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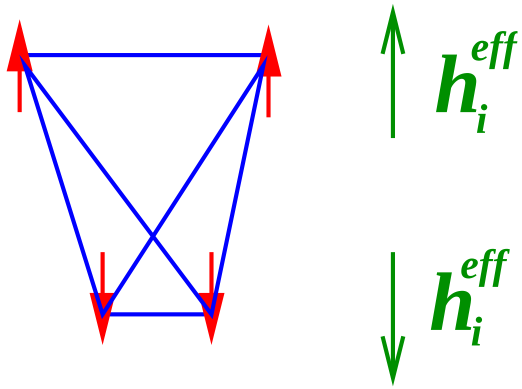
Kasteleyn transition

in staggered field



Sublattice magnetisation

vs $h^{\rm eff}/T$

for $h^{\rm eff}, T \ll J$

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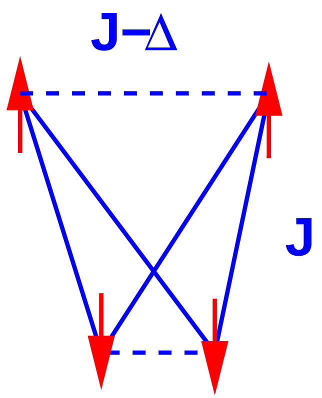

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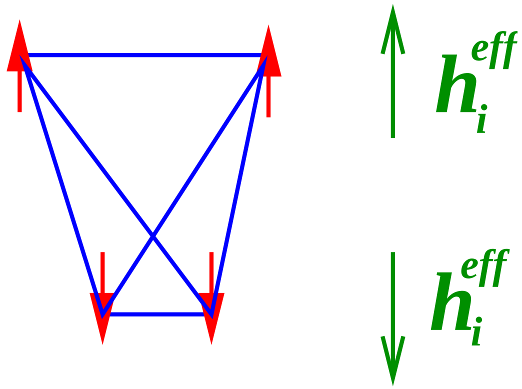


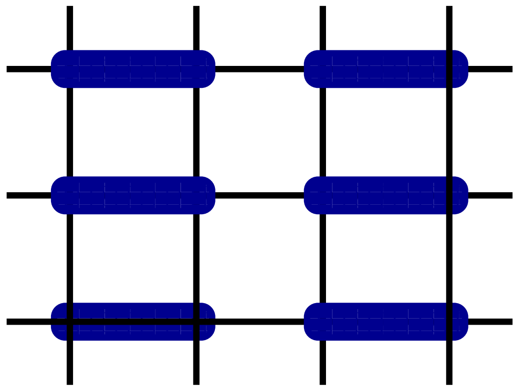
Sublattice magnetisation

vs h^{eff}/T

for $h^{\text{eff}}, T \ll J$

Dimer crystallisation

favour parallel pairs



$\mathcal{H} = -J \sum n_{\|}$

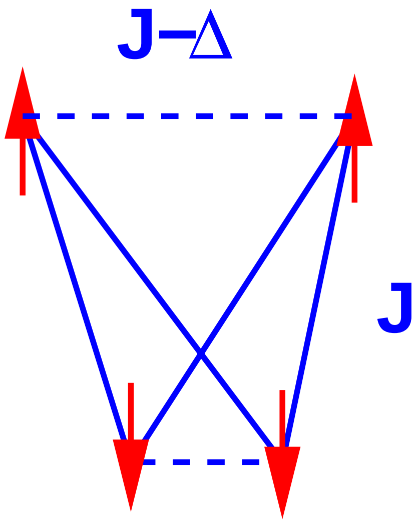
Crystal for $T \ll J$

Coulomb phase for $T \gg J$

Alet et al. PRL 2006

Néel ordering from the Coulomb phase

Pyrochlore lattice strained in $[100]$ direction



Constrain to Coulomb phase: $J \to \infty$

Study statistical mechanics vs T/Δ

Continuum description:

$$P[\mathbf{B}^a(\mathbf{r})] \propto \exp(-\mathcal{H})$$

$$\mathcal{H} = \int \mathrm{d}^3\mathbf{r} \Big\{ \sum_a \Big[\kappa |\mathbf{B}^a(\mathbf{r})|^2 - \frac{\Delta}{k_\mathrm{B}T} |\mathbf{B}^a(\mathbf{r}) \cdot \hat{\mathbf{z}}|^2 + c|\nabla \times \mathbf{B}^a(\mathbf{r})|^2 \Big] + u \Big[\sum_a |\mathbf{B}^a(\mathbf{r})|^2 \Big]^2 \Big\}$$ al

$$\kappa - \frac{\Delta}{k_\mathrm{B}T} \equiv t \propto (T - T_c)$$

Critical behaviour (for continuous transition)

Flux condensation transition

$$\mathcal{H} = \int \mathrm{d}^3\mathbf{r}\left\{\kappa \sum_a \left[|\mathbf{B}^a_\perp(\mathbf{r})|^2 + t\,|B^a_\parallel(\mathbf{r})|^2 + c|\nabla\times\mathbf{B}^a(\mathbf{r})|^2\right] + \ldots\right\}$$

Reduction to essentials: $\mathbf{B}^a = \nabla\times\mathbf{A}^a$ **with** $\nabla\cdot\mathbf{A}^a = 0$

Two modes $A^a_{\mathrm{crit}}, A^a_{\mathrm{noncrit}}$ **one critical. Rescale:** $qA^a_{\mathrm{crit}}(\mathbf{q}) = \varphi^a(\mathbf{q})$

Effective theory

$$\mathcal{H}_{\mathrm{eff}} = \int \mathrm{d}^3\mathbf{q} \sum_a \kappa\left[(1-t)\frac{q_\parallel^2}{q^2} + t + cq^2\right]|\varphi^a(\mathbf{q})|^2 + u\int\left[\sum_a|\varphi^a|^2\right]^2$$

Non-analytic dispersion $q_\parallel^2/q^2$ **equivalent to dipolar interactions**

Long-range forces → upper critical dimension is $d_u = 3$

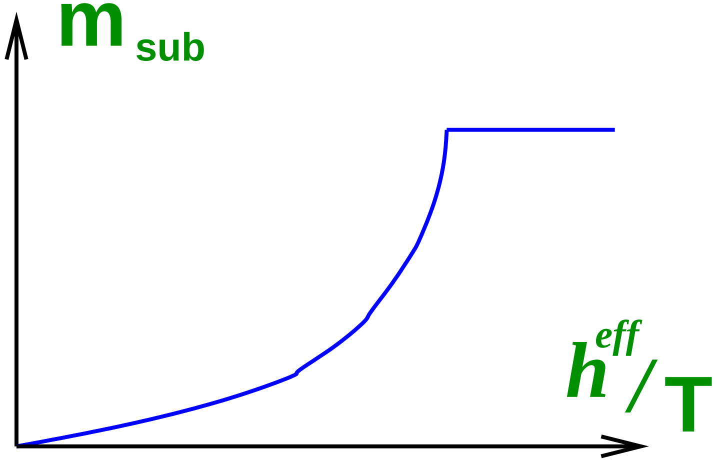
— hence critical behaviour (almost) mean field

[cf Larkin and Khemlnitskii, 1969]

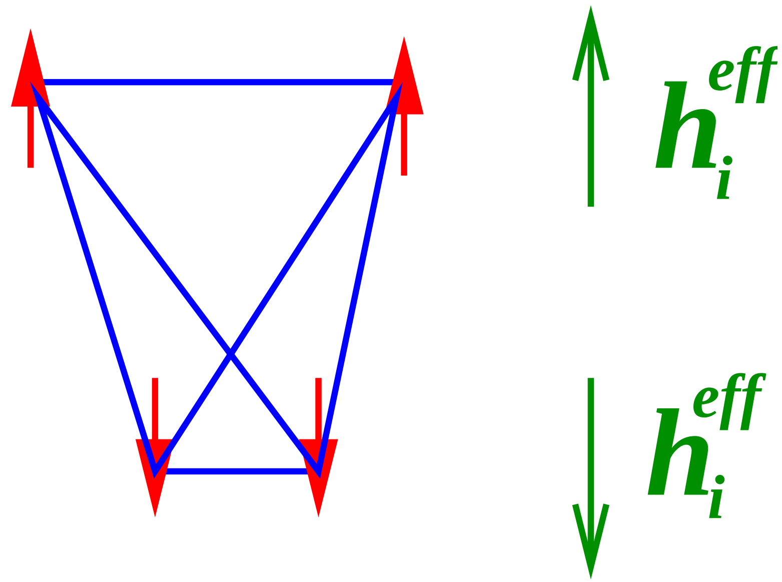
A Kasteleyn transition

Sublattice magnetisation induced by staggered field

Magnetisation vs temperature



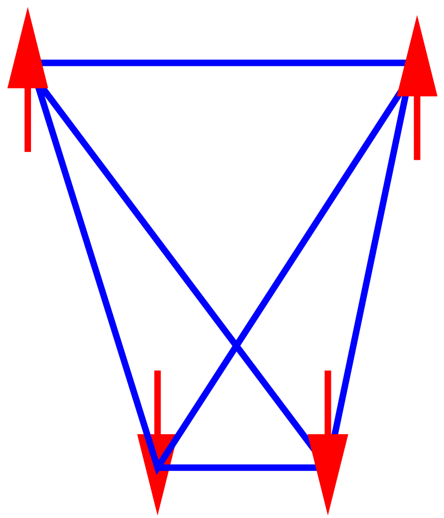
One-sided transition



- **Continuous from low-field side**
- **First-order from high-field side**

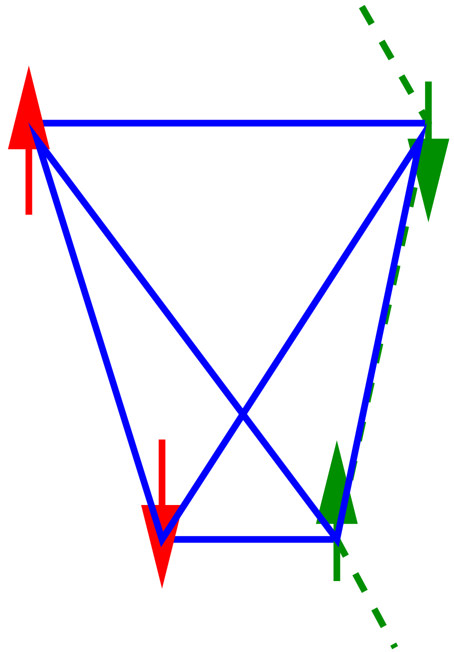
Description of the transition

Reference state: fully polarised



'Vacuum'

Excitations: spin reversals



String excitation

Thermodynamics of isolated string, length L:

Energy $L \cdot h$ **Entropy** $L \cdot k_{\mathrm{B}} \ln(2)$ **Free energy** $L \cdot [h - k_{\mathrm{B}} T \ln(2)]$

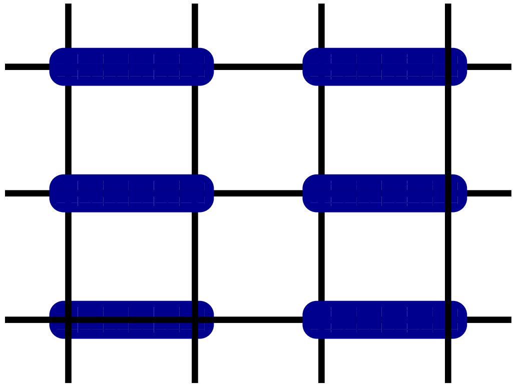
String density: finite for $h/k_{\mathrm{B}}T < \ln(2)$ **zero for** $h/k_{\mathrm{B}}T > \ln(2)$

Critical behaviour from viewing strings as boson world-lines

Classical dimer ordering in 3d and bosons in 2d

Dimer crystallisation

favour parallel pairs



$$\mathcal{H} = -J \sum n_{\parallel}$$

Crystal for $T \ll J$

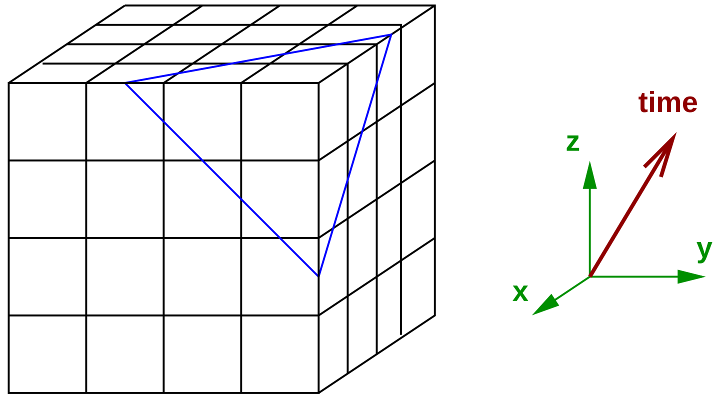
Coulomb phase for $T \gg J$

Simulations:

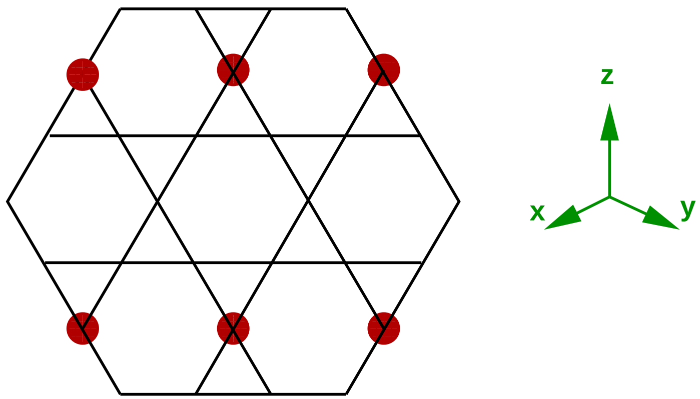
- **continuous transition**
- **unidentified critical behaviour**

Alet et al. PRL 2006

From 3d classical to (2+1)d quantum



Map to bosons on kagome lattice



1/6 filling with hard-core repulsion

Dimer liquid maps to superfluid

Dimer crystal maps to boson crystal

Critical Theory

Approach: Balents et al, Phys. Rev. B 71, 144508 (2005).

See also: Motrunich and Vishwanath, Phys. Rev. B 70, 075104 (2004)

Dual description of superfluid in terms of vortex excitations

Vortex field: φ (2-cpt, complex)

Vortex interactions mediated by gauge field: A

vortex condensation $\equiv$ boson crystallisation

crystal order parameter: $m_\mu = \varphi \cdot \sigma_\mu \cdot \varphi$

SU(2)-invariant gauge theory

$$\mathcal{L} = |(\nabla - \mathrm{i}A)\varphi|^2 + s|\varphi|^2 + \kappa|\nabla \times A|^2 + \mathcal{L}_{\mathrm{int}}$$

Summary

States of classical frustrated magnets and dimer models:

- **Macroscopic degeneracy**
- **Topological constraints**

— Constitute a Coulomb phase

Ordering from this manifold of states:

- **Symmetry-breaking:**

  non-standard critical behaviour at Néel transition

- **Symmetry-sustaining:**

  one-sided Kasteleyn transition

- **Dimer crystalisation:**

  equivalent to superfluid - density wave transition