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Dynamics of spin waves in a one-dimensional Bose ferromagnet

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Outline

Introduction

Formal statement of the problem

Trapping of spin and the logarithmic diffusion

Crossover to the open regime and the Gaussian damping

Unresolved questions

Experiments on cold atomic gases

1D, no spin

[T. Stoferle *et. al.*, PRL **92**, 130403 (2004)]

[B. Paredes *et. al.*, Nature **429**, 277 (2004)]

[T. Kinoshita *et. al.*, Science **305**, 1125 (2004)]

[T. Kinoshita *et. al.*, PRL **95**, 190406 (2005)]

[B.L. Tolra *et. al.*, PRL **92**, 190401 (2004)]

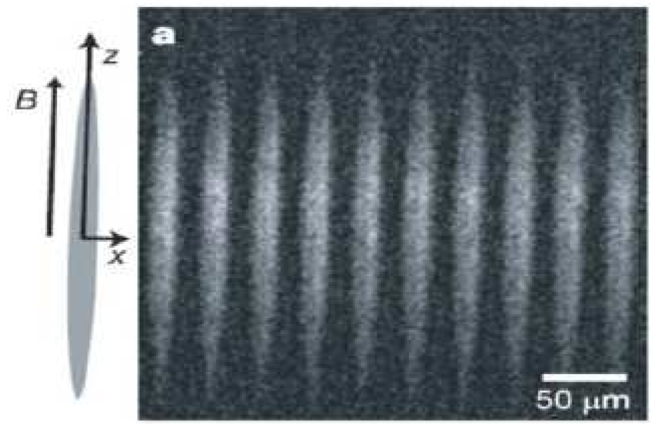
[… and others …]

populated bands: transverse 1, longitudinal > 30

quasi 1D, with spin

[L.E. Sadler *et. al.*, Nature **443**, 164 (2006)]

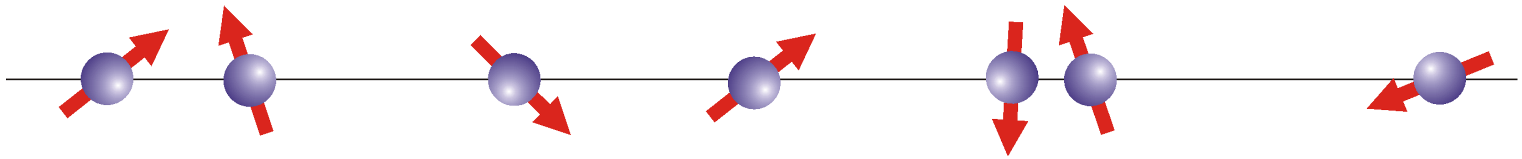
[J.M. Higbie *et. al.*, PRL **95**, 050401 (2005)]



populated bands: transverse ~ 50, longitudinal > 1000

The system

Bose particles with spin in one dimension



Assume spin-independent interactions

Q1: What is the magnetic order in the ground state?

Q2: How do excitations propagate and interact?

The model

A system of 2S+1 – component Bose particles with the Hamiltonian

$$H = \sum_{j=1}^{N} \frac{p_j^2}{2m} + \sum_{i<j} U(x_i - x_j) + hS_z$$

The spin density operator:

$$\vec{s}(x) = \sum_{j=1}^{N} \delta(x - x_j)\vec{s}_j$$

$\vec{s}_j$ is a spin operator in the 2S+1 –dimensional representation of SU(2)

Ground state

For spin-independent interactions

Truly 1D fermionic system cannot be ferromagnetic
[E.Lieb & D. Mattis, Phys. Rev. **125**, 164 (1962)]

The ground state of a 1D itinerant Bose system is completely polarized
[see e.g. E. Eisenberg & E. H. Lieb, PRL **89**, 220403 (2002)]

We investigate the propagation of spin waves in a 1D Bose <u>ferromagnet</u>

Hydrodinamics and spin-charge separation

Paramagnetic/anti-ferromagnetic ground state:

a) Linear dispersion relation (for small wave vectors) $\omega = v_S k$

b) Equations of motion – linear wave equation (sound waves)

$$\frac{\partial^2}{\partial t^2}\phi(x,t) + v_S^2 \frac{\partial^2}{\partial x^2}\phi(x,t) = 0$$

c) Effective low-energy theory – Luttinger model for all coupling strenghts

d) Spin-charge separation

$$H = H_C + H_S, \qquad [H_C, H_S] = 0$$

Hydrodinamics of a fully polarized fluid

Ferromagnetic ground state:

a) Linear dispersion relation for plasmons $\omega = v_S k$

b) Parabolic dispersion relation for magnons

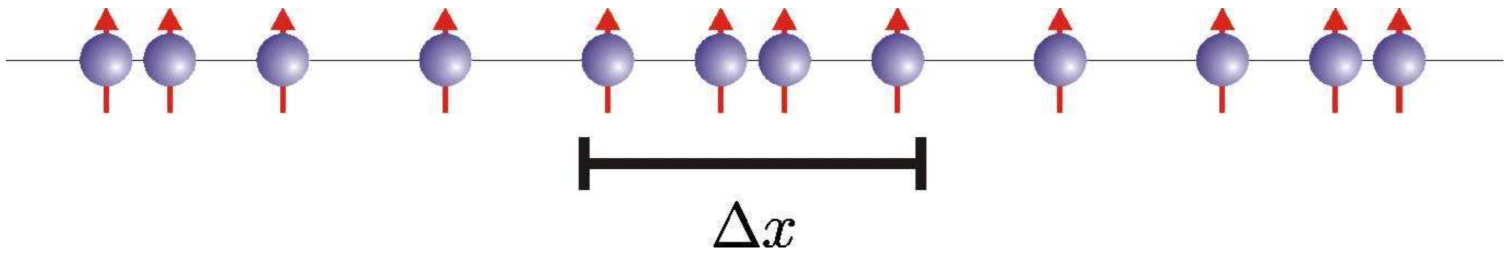
$$\omega = \frac{k^2}{2m^*}$$

c) Spin-charge separation impossible. For system polarized along z

$$s_z(x) = \varrho(x) \quad \Rightarrow \quad [\varrho(x), \vec{s}(y)] \neq 0$$

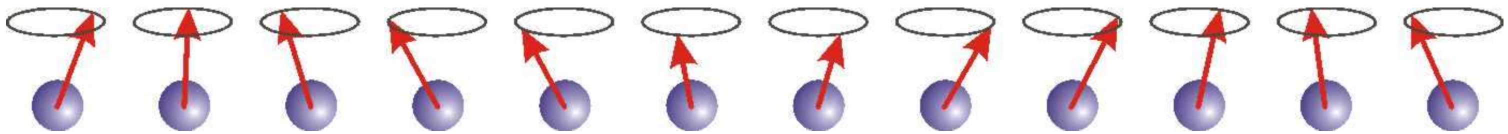
How does a spin excitation propagate?

Longitudinal and transverse spin dynamics



longitudinal spin excitations

$$\langle \Uparrow | s_z(x,t) s_z(0,0) | \Uparrow \rangle$$



transverse spin excitations

$$\langle \Uparrow | s_+(x,t) s_-(0,0) | \Uparrow \rangle$$

Longitudinal dynamics

In a fully polarized` sector $S_z = NS$

$$s_z(x) = S\rho(x)$$

$$\Downarrow$$

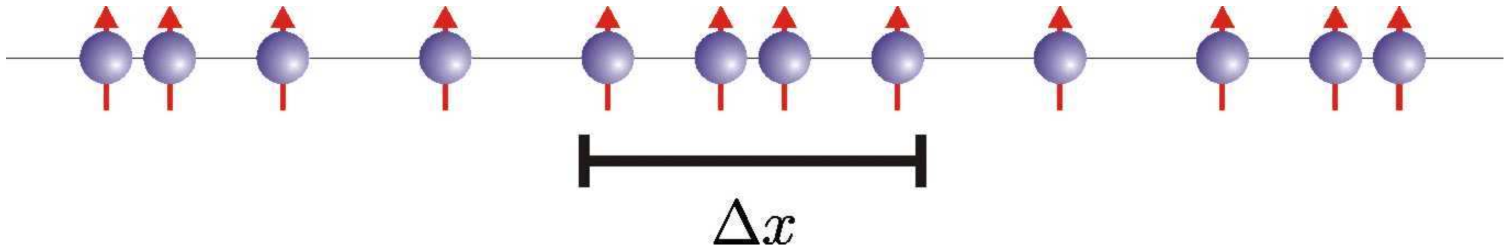
$$\langle \Uparrow | s_z(x,t) s_z(0,0) | \Uparrow \rangle = S^2 \langle \Uparrow | \rho(x,t) \rho(0,0) | \Uparrow \rangle$$

$$\Downarrow$$

In the limit of large x and t

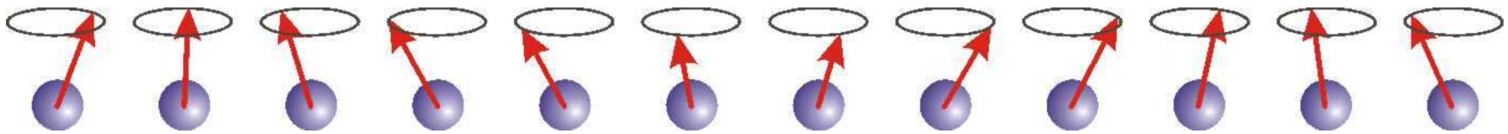
$$\langle \Uparrow | \rho(x,t) \rho(0,0) | \Uparrow \rangle = \frac{K}{2\pi^2} \frac{x^2 + v^2 t^2}{(x^2 - v^2 t^2)^2}$$

Longitudinal and transverse spin dynamics



longitudinal spin excitations = sound waves for spinless particles

$$\frac{\partial^2 s_z(x,t)}{\partial t^2} - v^2 \frac{\partial^2 s_z(x,t)}{\partial x^2} = 0$$



transverse spin excitations = ?

$$\langle \Uparrow | s_+(x,t) s_-(0,0) | \Uparrow \rangle$$

The Landau-Lifshitz equation

Hamiltonian, the gradient expansion:

$$H = \frac{1}{4m^*S}\int dx[\nabla\vec{s}(x)]^2 + \dots$$

The local spin algebra:

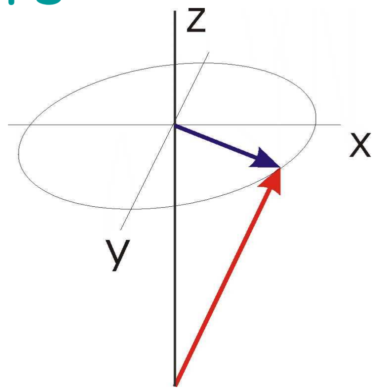
$$[s_\alpha(x), s_\lambda(y)] = i\epsilon_{\alpha\lambda\mu}s_\mu(x)\delta(x-y)$$

$$\frac{d}{dt}\vec{s}(x) = i[H, \vec{s}(x)] \quad \Rightarrow \quad \boxed{\frac{d}{dt}\vec{s} = \frac{1}{2m^*S}\,\vec{s}\times\nabla^2\vec{s}}$$

The linearized Landau-Lifshitz equation

Local ladder operators:

$$s_+ = s_x + i s_y$$
$$s_- = s_x - i s_y$$



Linearized L-L equation:

$$i\frac{\partial}{\partial t} s_- = -\frac{1}{2m^*}\nabla^2 s_-$$

Propagator of spin excitation:

$$G_\perp(x,t) \equiv \langle \Uparrow | s_+(x,t) s_-(0,0) | \Uparrow \rangle$$

$$G_\perp(x,t) = \frac{1}{\sqrt{2\pi i t / m^*}} e^{\frac{i m^* x^2}{2t}}$$

The large distance problem

The longer the distance the better the gradient expansion?

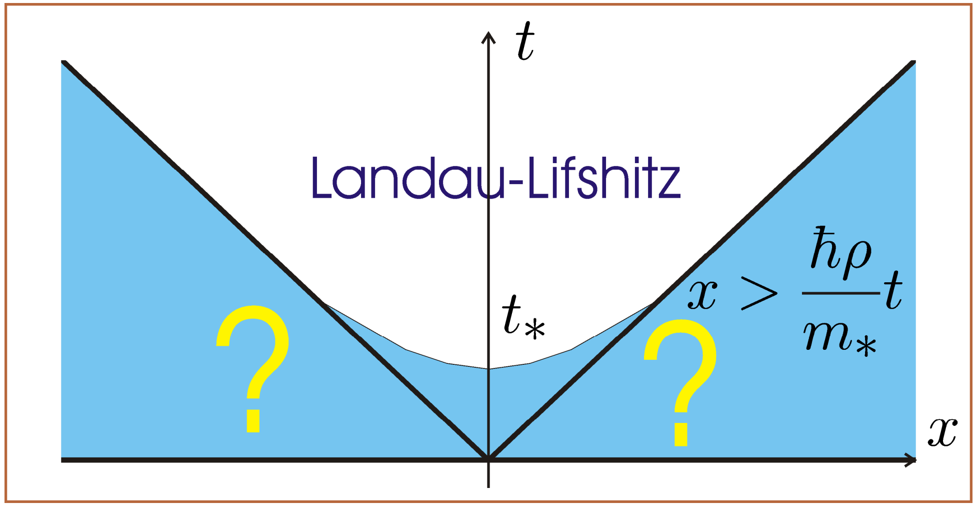
$$G_\perp(x,t) \sim e^{i\phi(x,t)} \qquad \frac{1}{\lambda} \sim \frac{d\phi(x,t)}{dx} = \frac{m^* x}{t}$$

Restriction on the wavelength:

$$\lambda\rho < 1 \quad \Rightarrow \quad x > \frac{\rho}{m^*}t$$

Emergence of a timescale

$$t^* = \frac{m^*}{\rho^2}$$



Diverging effective mass

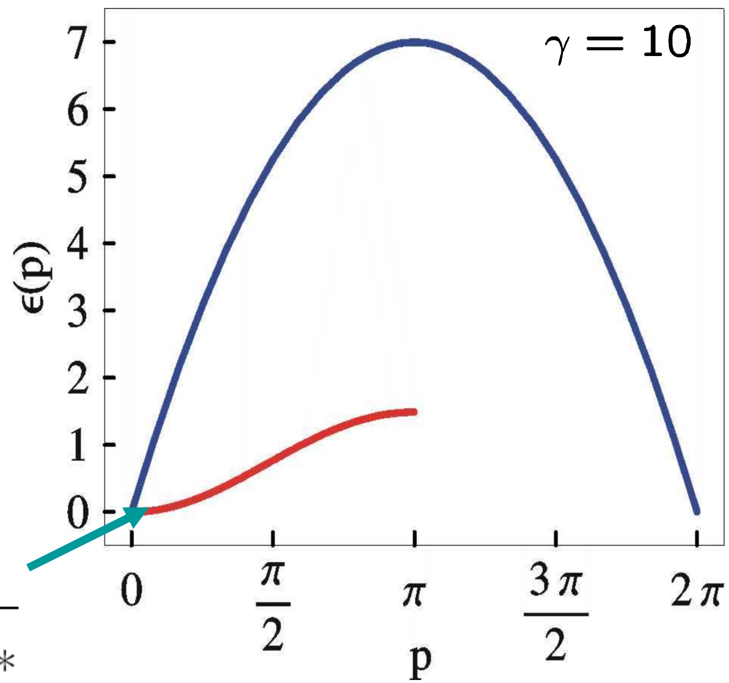
2S+1 - component bosons with spin-independent interaction

$$H = \sum_{j=1}^{N} \frac{p_j^2}{2m} + \sum_{i<j} U(x_i - x_j) \qquad U(x) = \tilde{U}(x) + \alpha\delta(x)$$

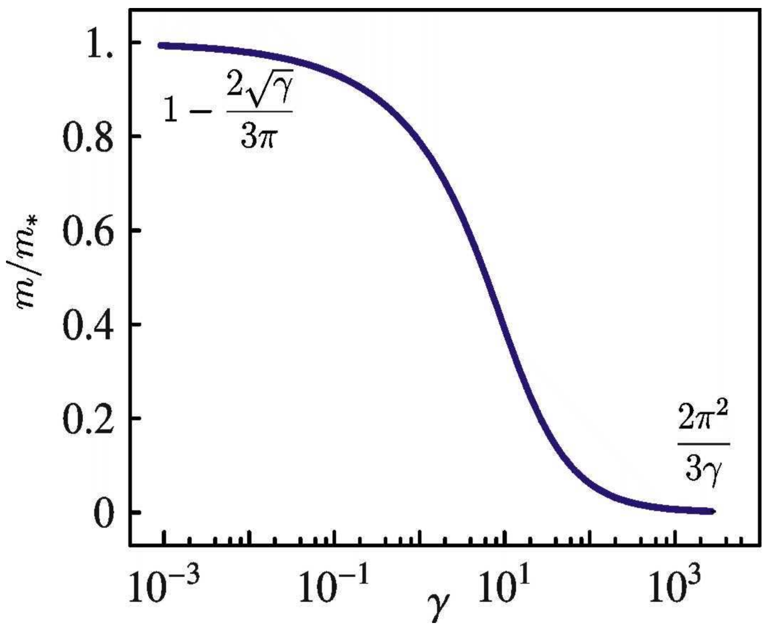
For $\tilde{U}(x) = 0$ Bethe-Ansatz solvable:

C. N. Yang, PRL **19**, 1312 (1967);
M. Gaudin Phys. Lett. A **24**, 55 (1967)

$$\gamma = \frac{m\alpha}{\hbar^2 \rho}$$

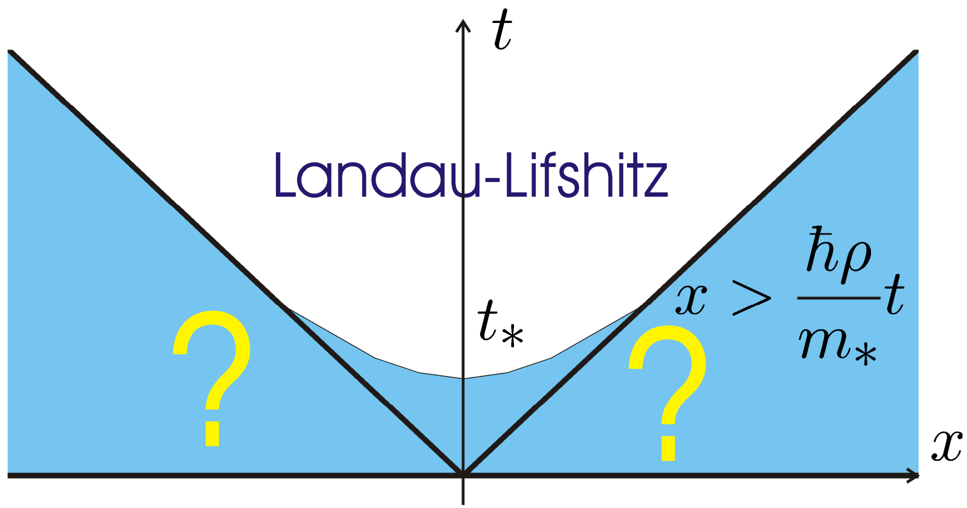


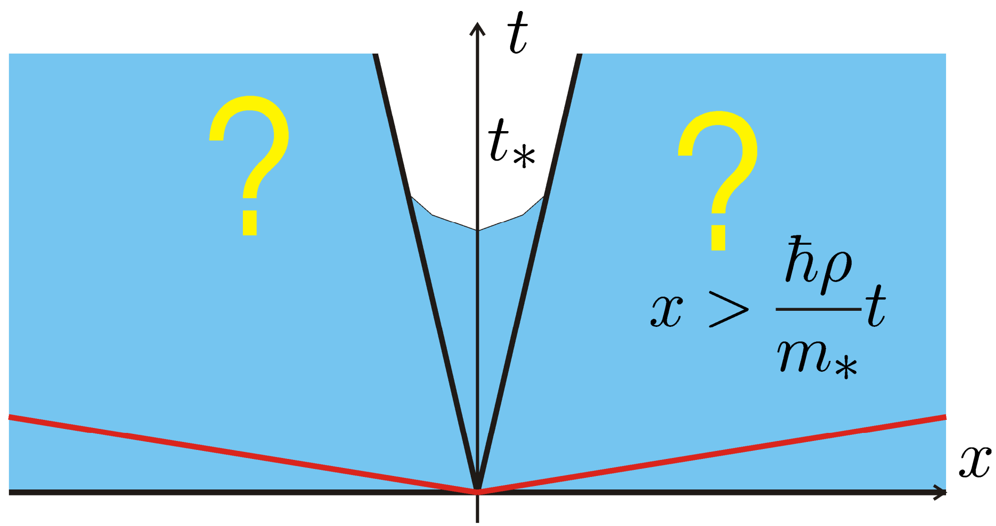
$$\frac{p^2}{2m_*}$$



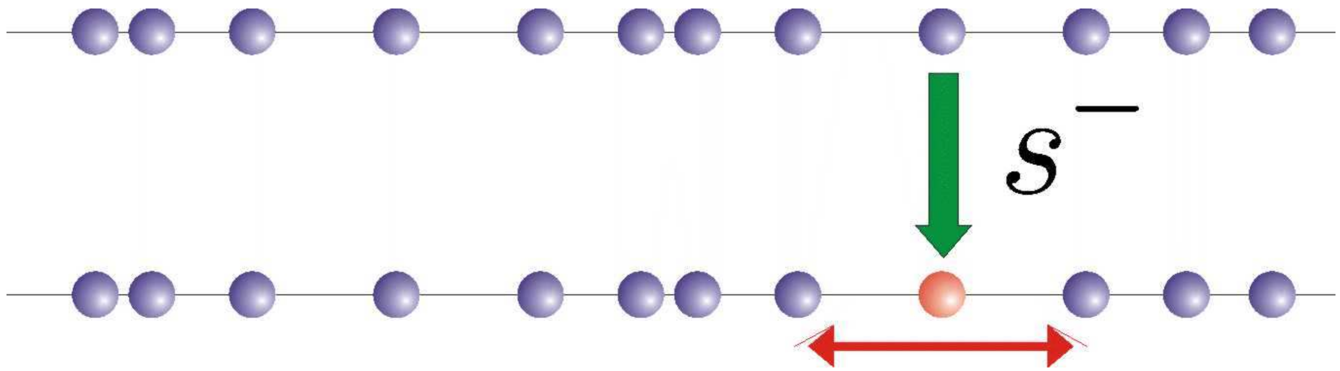
J. N. Fuchs et. al.,PRL **95**, 150402 (2005)

Strong repulsion limit

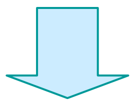




Trapping of spin:



The "red" particle is trapped between two neighbors



The spin propagator ought to be localized in space

Logarithmic diffusion:

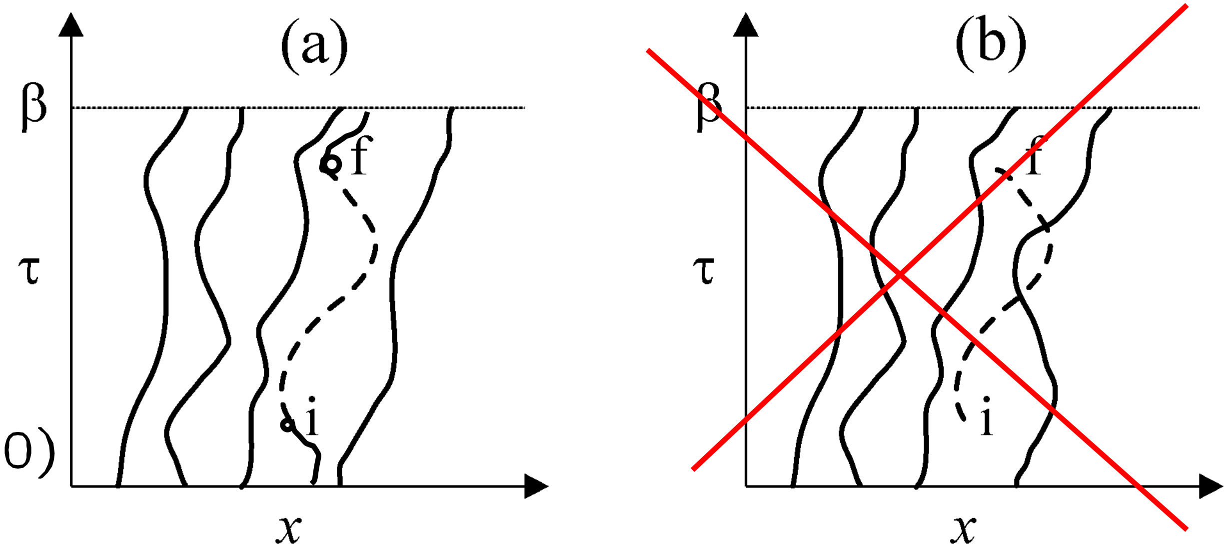
$$G_{\perp}(x,t) = \langle \Uparrow | s_{+}(x,t) s_{-}(0,0) | \Uparrow \rangle$$

1-st quantized path integral representation + bosonization
[L. Balents&G. Fiete, 2004]
Determinant representation&the asymptotic Riemann-Hilbert problem [V. Cheianov&M. Zvonarev, 2004]

$$G_{\perp}(x,t) = \frac{\pi\rho}{\sqrt{2\pi K \ln(t/t_0)}} \exp\left\{ -\frac{(\pi\rho x)^2}{2K \ln(t/t_0)} \right\}$$

Solution: from spinful problem to spinless

Word lines of the particles (path integral formalism):
Solid curves: spin up come from $\langle gs | \cdots | gs \rangle$
Dashed curve: spin down comes from $\psi_\downarrow(x,\tau)\psi_\downarrow^\dagger(0,0)$



Solid and dashed curves should not cross each other! – the only effect of spin

⇓

$$G_\perp(x,t) = \langle \delta[N(x,t)]\rho(x,t)\rho(0,0)\rangle \qquad \delta(N(x,\tau)) = \int d\lambda\, e^{i\lambda N(x,\tau)}$$

Operator $N(x,\tau)$ counts the number of crossings!

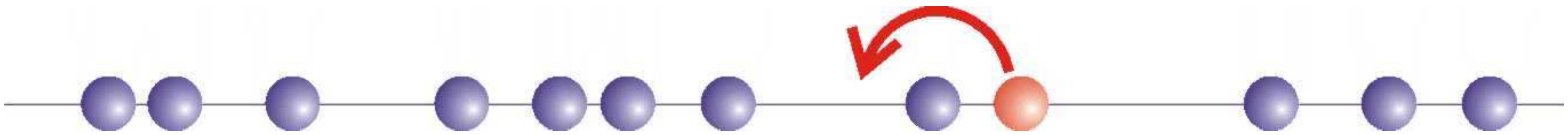
Only spinless fields!

Solid lines do not disappear ⇒ continuity equation: $\partial_x j_x + \partial_\tau j_\tau = 0$

$$\tilde{N}(x,\tau) = \int_{(0,0)}^{(x,\tau)} j_\tau dx - j_x d\tau = \rho_0 x - \frac{1}{\pi}[\phi(x,\tau) - \phi(0,0)]$$

V.V. Cheianov and M.B. Zvonarev, PRL **92**, 176401 (2004);
G.A. Fiete and L. Balents, PRL **93**, 226401 (2004)

Escaping the trap



Estimate for the escape time: $$t_*^{-1} = \frac{p}{\tau}$$

$$\frac{1}{\tau} \sim \frac{\hbar\rho^2}{m} \quad , \quad p \sim \frac{1}{\gamma^2}\cdot\gamma \quad \Rightarrow \quad \frac{1}{t^*} \sim \frac{\hbar\rho^2\gamma}{m}$$

What happens for $t > t_*$?

Perturbative treatment of tunneling processes?

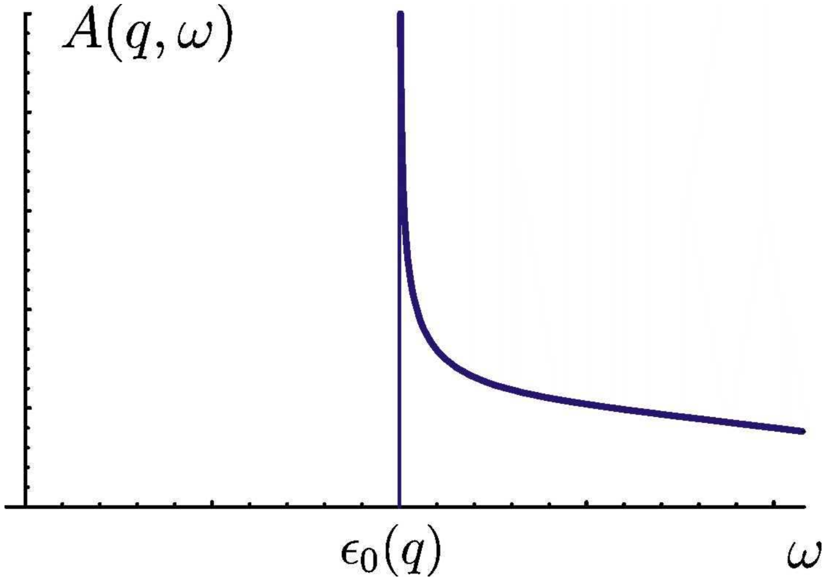
The spectral representation

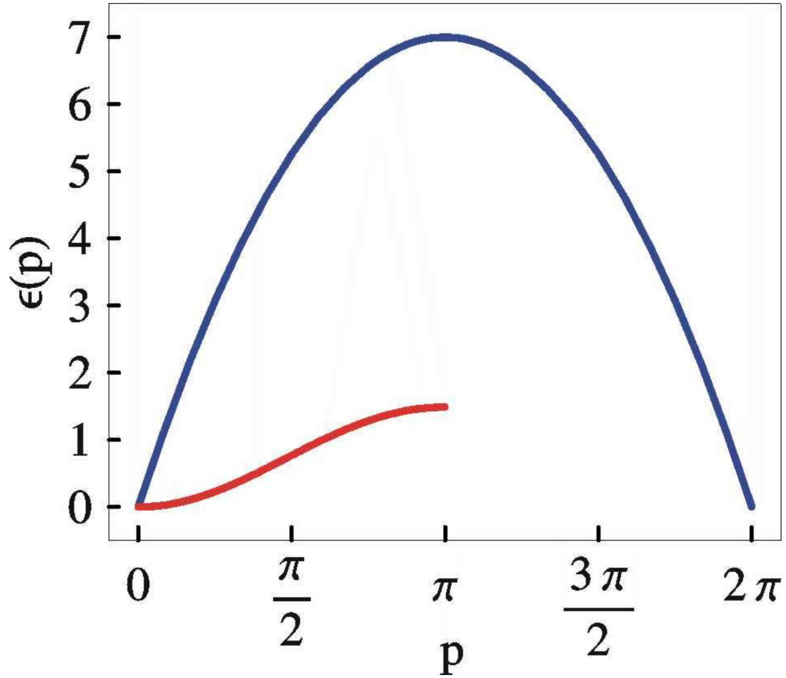
$$G(x,t) = \int \frac{dk}{2\pi} \int \frac{d\omega}{2\pi} e^{-i\omega t + ikx} A(k,\omega)$$

The large-time asymptotic behavior is controlled by the singularities of the spectral function:

$$A(q,\omega) = \sum_{\nu} \delta(\omega - E_{\nu,q}) |\langle \nu, q | \sigma_q^- | \Uparrow \rangle|^2$$

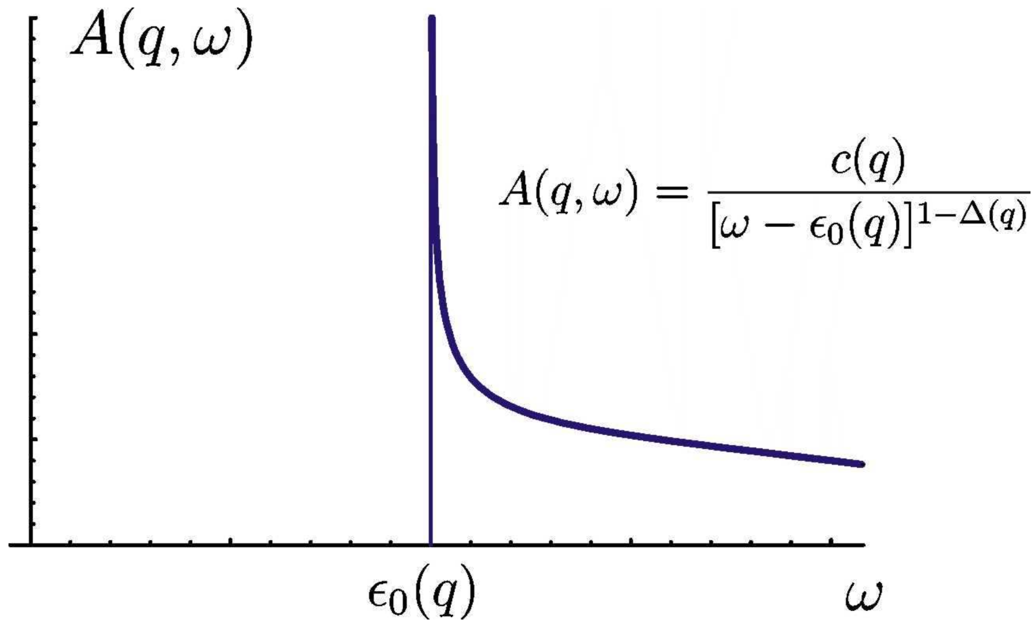
Threshold of spectral function





Above the threshold there is a continuum of states due to multiple production of longitudinal spin waves.

Scaling of spectral function at threshold



Assumptions:

(a) $\Delta(q) = \alpha + \beta q^2 + \ldots$

(b) $c(q) = c_0 + c_1 q^2 + \ldots$

In the integrable case assumption (a) can be verified and the function $\Delta(q)$ can be calculated explicitly

Asymptotics of spin propagator

$$G(x,t) = \int \frac{dq}{2\pi} \frac{c(q)}{t^{\Delta(q)}} e^{i\epsilon_0(q)t}$$

Use saddle point approximation:

$$\Delta(q) = \alpha + \beta q^2 + \ldots$$

$$\epsilon_0(q) = \frac{q^2}{2m_*} + \ldots$$

The main result

$$G(x,t) \sim \frac{1}{t^{\alpha}} \frac{e^{-\frac{1}{2}\frac{x^2}{\beta \ln(t/t_0)+it/2m_*}}}{\sqrt{\beta \ln(t/t_0) + it/2m_*}}$$

M. Zvonarev, V. Cheianov, T. Giamarchi (2007)
Shimul Akhanjee, Yaroslav Tserkovnyak (2007)

$$\alpha = 0$$
$$\beta = \frac{K}{(2\pi\rho)^2}$$

by matching with the $t < t_*$ result

No $1/\gamma$ corrections in the integrable case!!!

Improved estimate for the escape time:

$$\beta \ln t_* = \frac{t_*}{2m} \quad \Rightarrow \quad t_* \approx \frac{3K}{8\pi^2}\frac{\hbar}{\epsilon_F}\gamma \ln \gamma$$

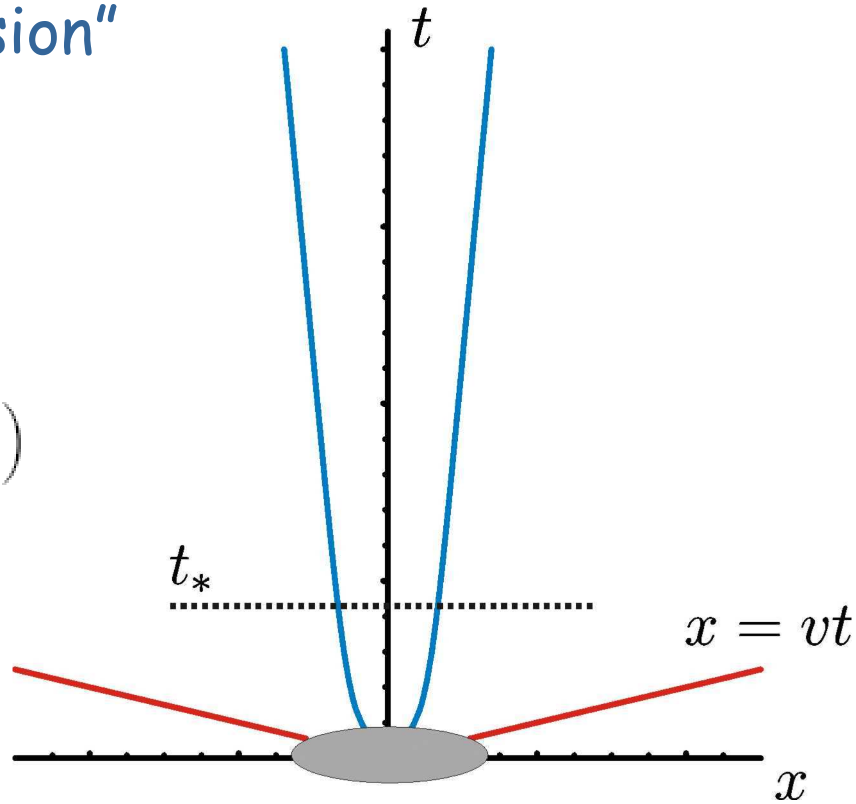
Space-time diagram:

$$|G(x,t)|^2 \sim e^{-x^2/(\Delta x)^2} \qquad \Delta x^2 = \frac{\beta^2 \ln^2 t + \left(\frac{t}{2m_*}\right)^2}{\beta \ln t}$$

For $t < t_*$ "logarithmic diffusion"

For $t > t_*$ Gaussian damping:

$$G_\perp(x,t) = e^{-\frac{x^2}{2\Delta x^2}} G_{LL}(x,t)$$



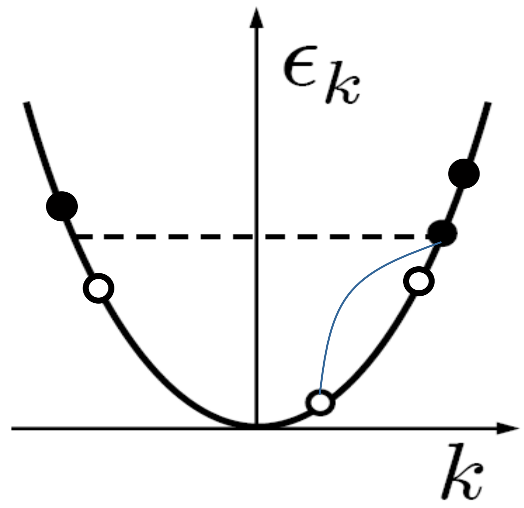
Integrable case

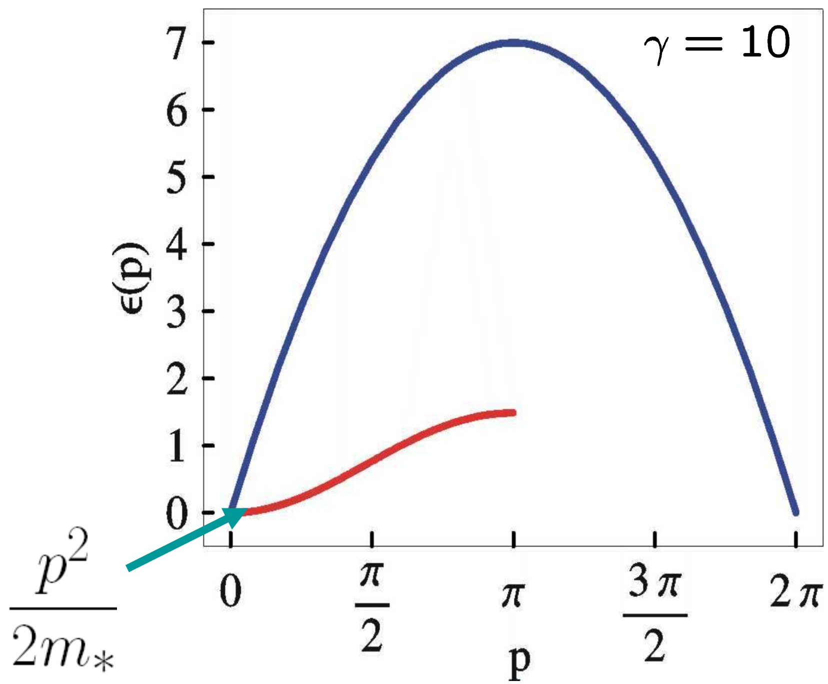
- Infinite hierarchy of local integrals of motion
- Wave functions are known explicitly
- Simple classification of excitations in the large N limit
- In some cases form factors are also known explicitly

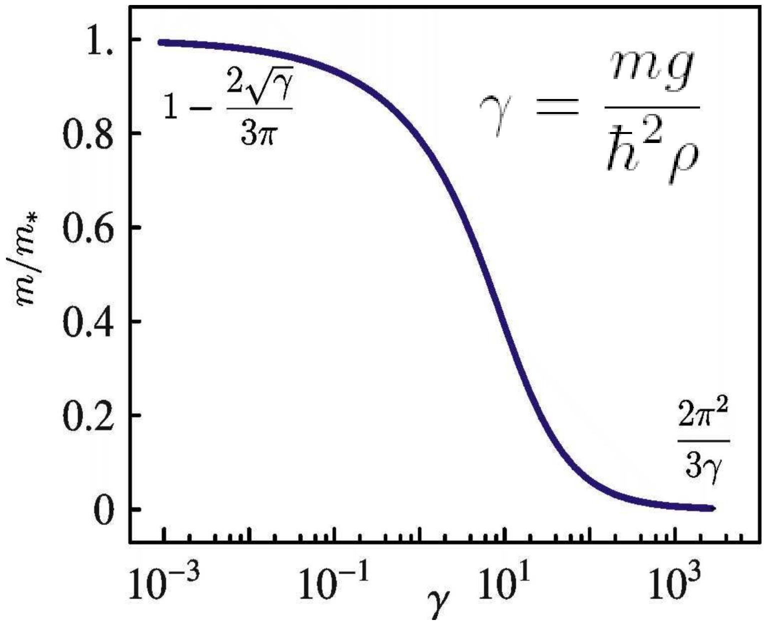
In the integrable case it is possible to reformulate the high energy problem at the threshold as a low energy problem for a different local Hamiltonian

Excitations

Excitations are electron-hole pairs + one massive particle (magnon)





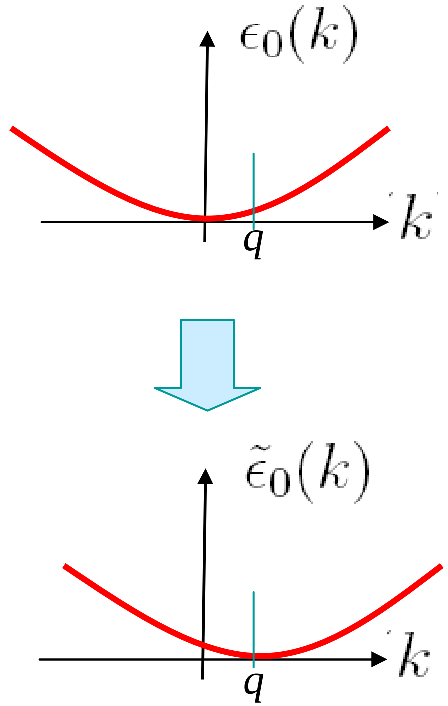


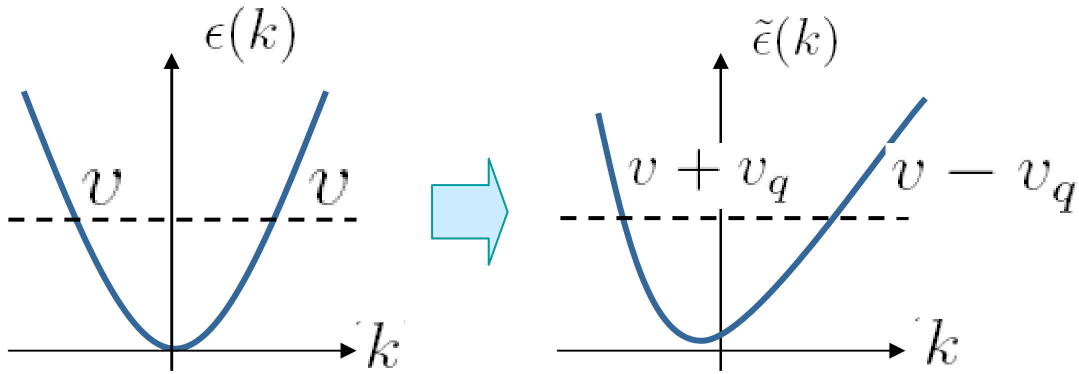
Pseudo-Hamiltonian

A linear combination of local integrals of motion

$$\mathcal{H}_q = \sum_{j=1}^{Q} c_n(q) I_n$$

- The same eigenstates
- The same classification of excitations
- Different dispersion relations





Pseudo-Spectral Function

For the spectral function generated by pseudohamiltonian

$$\tilde{A}(q,\omega) = \sum_{\nu} \delta(\omega - \tilde{E}_{\nu}) \langle \nu | \mathcal{O}_q | 0 \rangle |^2$$

$$\frac{\tilde{A}(q,\omega)}{A(q,\epsilon_0(q)+\omega)} \to 1, \qquad \omega \to 0$$

Thus the problem of threshold exponent is mapped onto a IR problem for a pseudo-hamiltonian

Effective field theory: Hamiltonian

$$\mathcal{H} = \sum_r \frac{v_r}{4\pi} \int_0^L dx \, \left\{ [\partial_x \varphi_r(x)]^2 + 2\beta_r [\partial_x \varphi_r(x)] \rho_\downarrow(x) \right\}$$

From the analysis of finite-size scaling of spectrum:

$$\beta_+ = 2\pi F(\Lambda|\xi) \qquad \beta_-(q) = -2\pi F(-\Lambda|\xi)$$

$$F(\lambda|\xi) - \frac{1}{2\pi} \int_{-\Lambda}^{\Lambda} d\nu \, K(\lambda,\nu) F(\nu|\xi) = -\frac{\pi + \theta(2\lambda - 2\xi)}{2\pi}$$

R. G. Pereira, S. R. White, I. Affleck (2007);
V. Cheianov, M. Pustilnik (2007)
A. Imambekov and L. Glazman (2007)

Solution to the effective theory

Result for the low-energy scaling

$$\tilde{A}(q,\omega) = c(q)\omega^{\Delta(q)}$$

The exponent in terms of scattering phases

$$\Delta(q) = \frac{1}{4\pi^2}(\beta_+^2 + \beta_-^2) - 1$$

Small q expansion:

$$\Delta = -1 + \frac{Kq^2}{2(\pi\rho_0)^2} + o(q), \qquad q \to 0$$

Unresolved Issues

1. Perturbation theory around the TG regime
2. Relation to thermodynamics in the non-integrable case
3. The effective theory
4. Adding local spin exchange terms to the Hamiltonian
5. Increasing dimensionality