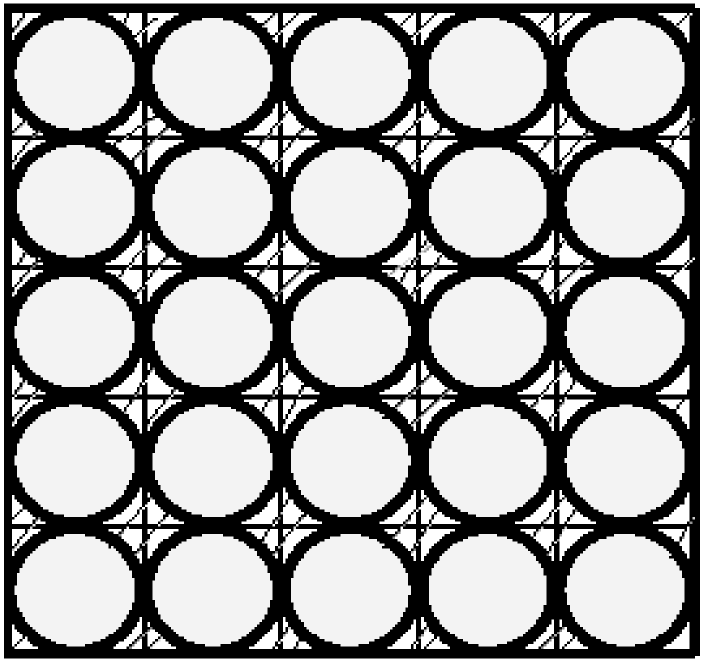
Дау сказал........

Effect of Magnetic field on Transport in Granular Materials

M.Yu. Kharitonov, K.B. Efetov

Typical structure of a granular metal: d=50-200A



Coupling between the grains can vary: possibility of both macroscopically metallic and insulating states.

Nanoscience with numerous applications!

Longitudinal conductivity: Effects of Coulomb interaction

Coauthors in several works: I.S. Beloborodov, A. Tschersich, A.V. Lopatin, V.M Vinokur
Recent review: I.S. Beloborodov, A.V. Lopatin, V.M. Vinokur, K.B. Efetov,
Rev. Mod. Phys. (2007)

Classical expression (without taking into account Coulomb blockade and interference effects)

$$\sigma = e^2 g(T) a^{2-d}$$

$$g = 2\pi \nu_0^2 t_{\mathrm{ij}}^2$$

$g(T)$ -Tunneling conductance between the grains

a Size of the grains

σ -depends on $g(T)$ and brings information about the coupling between the grains. The same for $\rho_{xx} = \sigma^{-1}$

Coulomb interaction and quantum interference lead to a non-trivial temperature dependence of the longitudinal conductivity!

Q. What about σ_{xy} or $\rho_{xy} = \dfrac{\sigma_{xy}}{\sigma_{xx}^2 + \sigma_{xy}^2}$?

A. ρ_{xy} gives information about the *interior* of the grains in the classical limit but is also temperature dependent if Coulomb interaction and interference effects are taken into account.

In the classical limit the Hall resistivity is $R_H^{(0)} = \dfrac{H}{n^* ec}, \qquad n^* = An$

Description of the Hall transport in granular systems is considerably more difficult than that of the longitudinal one!

What was interesting about the longitudinal conductivity

Experimental puzzles:

1. Sufficiently strong coupling between the grains

$$\sigma = \sigma_0 + \alpha \ln T$$

2. Weak coupling between the grains

$$\sigma = a \exp(-b / \sqrt{T})$$

The dimensionality of the array does not seem to play any important role for both 1) and 2)!

Metal-Insulator transition?

If so, what is the reason for such a behavior?

Some experimental curves (after A. Gerber *et al*, PRL (1997))

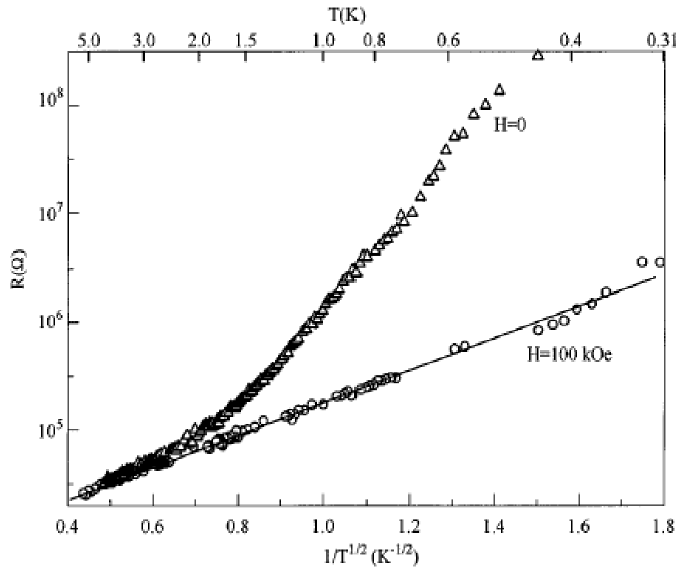


FIG. 1. Resistance of sample 1 measured at zero (triangles) and 100 kOe field (circles) as a function of the inverse square root of the temperature. Sample 1 room temperature resistance is $2 \times 10^3\ \Omega$.

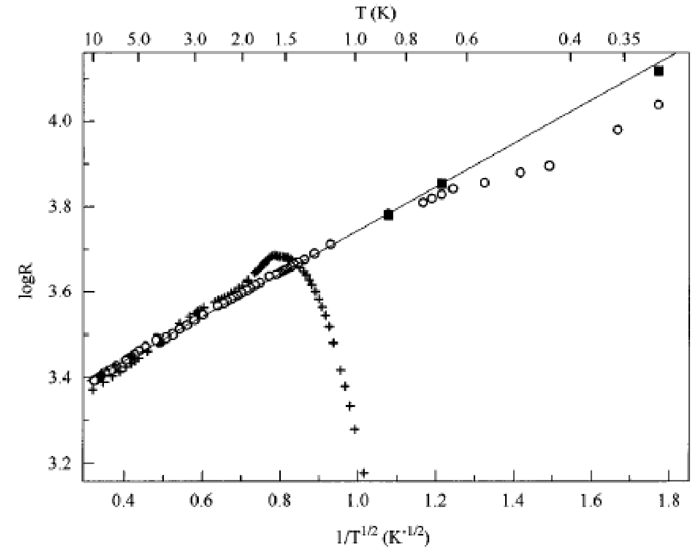


FIG. 2. Resistance of sample 2 measured at zero (crosses) and 100 kOe field (open circles) as a function of the inverse square root of the temperature. Open circles indicate resistance measured with a constant dc current $I = 10^{-6}$ A. Solid squares are zero bias resistances approximated from I-V measurements. Sample 2 room temperature resistance is $800\ \Omega$.

The weak coupling limit (insulator)

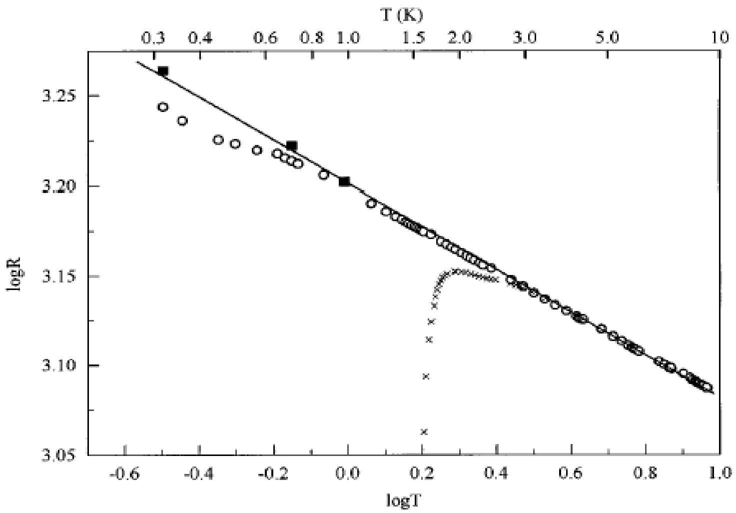


FIG. 3. Resistance of sample 3 as a function of temperature on a log-log scale, as measured at (zero) (×) and 100 kOe field (open circles). Open circles indicate resistance measured with a constant dc current $I = 10^{-5}$ A. Solid squares are zero bias resistances approximated from I-V measurements. Sample 3 room temperature resistance is 500 Ω.

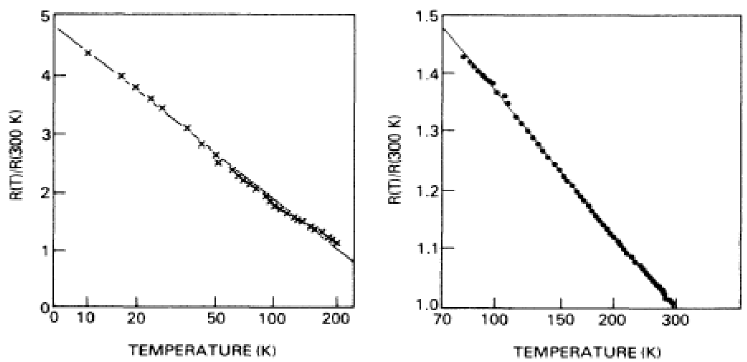


FIG. 5. The resistance normalized to the room-temperature value vs $\ln T$ for two different superconducting samples. The graph on the left is for a sample with $R_\square(300\text{ K}) = 2000\ \Omega/\square$, while that on the right is for $R_\square(300\text{ K}) = 100\ \Omega/\square$. The lines are guides to the eye.

$$R = AT^{-\alpha}$$

or

$$R = A(1 - \alpha \ln T)$$

$$\alpha = 0.117$$

R.W. Simon *et al*,
Phys. Rev. B (1987)

Strong coupling limit. Metal?

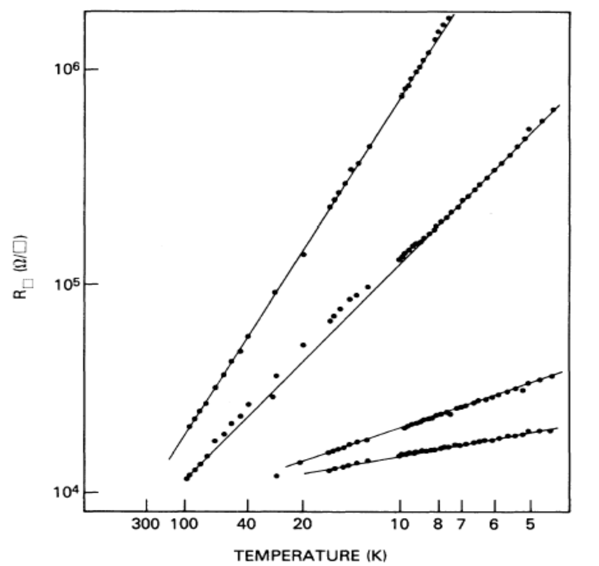


FIG. 4. The logarithm of the sheet resistance as a function of $1/\sqrt{T}$ for a variety of the nonsuperconducting samples. The lines are guides to the eye.

High resistivity sample again.

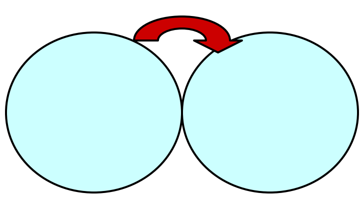
Both 1) and 2) are typical for many experiments!

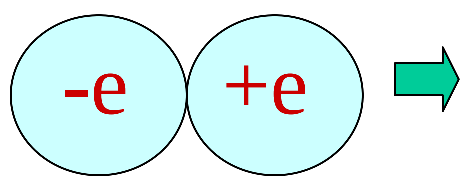
Why?

It is always so!

How to describe the granular metals?

Coulomb interaction plays a crucial role!





$$E_c = e^2 / 2C$$

E_c -charging energy

Other energies in the system:

$\delta = (\nu V)^{-1}$ -mean level spacing in a grain

$g\,\delta$ -tunneling energy between the grains

g-tunneling conductance

The Hamiltonian

$$\hat{H} = \hat{H}_0 + \hat{H}_t + \hat{H}_c$$

$$\hat{H}_0 = \int \psi^+(\mathbf{r}) \left(-\frac{\nabla^2}{2m} + U(\mathbf{r}) \right) \psi(\mathbf{r})\, d\mathbf{r}$$

$$\hat{H}_t = \sum_{\mathbf{i},\mathbf{j},\alpha,\alpha'} t_{\mathbf{ij}} \hat{\psi}^+_{\alpha \mathbf{i}} \hat{\psi}_{\alpha' \mathbf{j}}$$

$$\hat{H}_c = \frac{e^2}{2} \sum_{\mathbf{ij}} \hat{N}_{\mathbf{i}} C^{-1}_{\mathbf{ij}} \hat{N}_{\mathbf{j}}$$

$$\hat{N}_{\mathbf{i}} = \int \hat{\psi}^+(\mathbf{r}_{\mathbf{i}})\, \hat{\psi}(\mathbf{r}_{\mathbf{i}})\, d\mathbf{r}_{\mathbf{i}} - \bar{N}$$

t_{ij}-tunneling amplitude from grain to grain, C_{ij} - capacitance matrix

Methods of calculation:

1. Phase Hamiltonian
2. Perturbation theory in the limit $g \geq 1$
(strong coupling between the grains).

Phase Hamiltonian

Study of Coulomb interaction via bosonization is well known in superconductors where $\Delta_i(\tau) = \exp(2\phi_i(\tau))$

$\phi(\tau)$ is the phase.

One can reduce the electron Hamiltonian to an effective phase Hamiltonian H_{eff} (Efetov (1980))

$$H_{eff} = \sum_{ij} [B_{ij}\rho_i\rho_j - J_{ij}\cos(2(\phi_i - \phi_j))]$$

$$B_{ij} = \frac{1}{2}(2e)^2\left(C^{-1}\right)_{ij}$$

Where $\rho_i = -i\partial / \partial\phi_i$ (eigenvalues are integers)

How can one write a normal metal in terms of phases?

Final action: $S = S_c + S_t,$

S_c is the charging energy

$$S_c = \frac{1}{2e^2} \sum_{\mathbf{ij}} \int_0^\beta d\tau C_{\mathbf{ij}} \frac{d\tilde{\phi}_{\mathbf{i}}(\tau)}{d\tau} \frac{d\tilde{\phi}_{\mathbf{j}}(\tau)}{d\tau}$$

S_t describes the tunneling between the grains

$$S_t = \pi g \sum_{|\mathbf{i}-\mathbf{j}|=a} \int_0^\beta d\tau d\tau' \alpha(\tau - \tau') \sin^2\left(\frac{\tilde{\phi}_{\mathbf{ij}}(\tau) - \tilde{\phi}_{\mathbf{ij}}(\tau')}{2}\right)$$

0D limit for a single grain!

where $$\alpha(\tau) = T^2 \left(\mathrm{Re}\left(\sin\left(\pi T \tau + i\delta\right)\right)^{-1}\right)^2$$

$$g = 2\pi\nu_0^2 t_{\mathbf{ij}}^2$$

This form is analogous to the Ambegaokar, Eckern and Schon (1982) action written to describe quantum dissipation.

However, it is applicable only for $T \geq \max(\delta, g\delta)$

↓

No quantum dissipation and no dephasing at T=0!

This is possible in the limit $T \geq \delta$ ⟹ Integration over the phases $\tilde{\phi}_i(\tau)$

$$\tilde{\phi}_i(\tau) = \phi_i(\tau) + 2\pi T k_i \tau,$$

where $-\infty < \phi_i(\tau) < \infty$, $\phi_i(0) = \phi_i(\beta)$.

k_i are integers (winding numbers)

Efetov, Tschersich (2003)

$$H_{eff} = \frac{1}{4} \sum_{ij} B_{ij} \rho_i \rho_j$$

The electrostatic energy:

$$B_{ij} = \frac{1}{2} (2e)^2 \left(C^{-1}\right)_{ij}$$

Where $\rho_i = -i\partial / \partial\phi_i$ (eigenvalues are integers)

For certain forms of C_{ij} the inverse matrix B_{ij} can be very large

Example: $C_{ij} = C_0 \delta_{ij} + C l_{ij}$

$C_0 << C$

$$l_{ij} = \begin{cases} -1, & \text{nearest neighbors} \\ 1, & i=j \\ 0, & \text{otherwise} \end{cases}$$

⇓

Superconducting grains:
superinsulator

Vinokur et al 2008

Normal metal grains:
quarter superinsulator (!?)

The phase functional can be studied without difficulties in the limits $g \geq 1$ and $g \leq 1$

$g \geq 1$ One can use renormalization group integrating over fast variations of ϕ

RG Equation: $$\frac{\partial g(\xi)}{\partial \xi} = -\frac{1}{2\pi d}$$

$$\sigma = e^2 g(T) a^{2-d}$$

Result: $$g(T) = g - \frac{1}{2\pi d} \ln \frac{g E_c}{T}$$

Valid as long as $g \geq 1$

$g \leq 1$ Expansion in g.

$B \propto E_c$ -is the energy of the lowest excitation

Result: $$\sigma = 2\sigma_0 \exp(-B/T)$$

At $T \geq E_c$ one has $\sigma = \sigma_0$

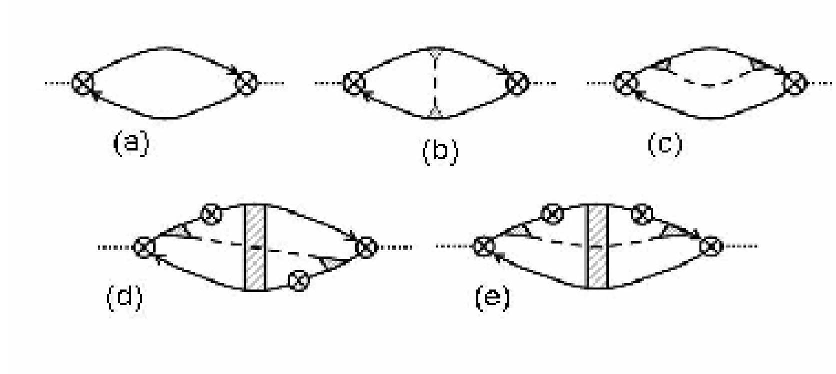
Perturbation theory in the limit $g \geq 1$ (applicable for any low T):

Relevant diagrams:

a) Classical conductivity

b-c) The “bosonic” contribution

d-e) Altshuler-Aronov contribution



Result:

$$\sigma = \sigma_0 + \delta\sigma_1 + \delta\sigma_2.$$

$$\frac{\delta\sigma_1}{\sigma_0} = -\frac{1}{2\pi d g_T} \ln\left[\frac{g_T E_C}{\max(T, g_T\delta)}\right]$$

$$\frac{\delta\sigma_2}{\sigma_0} = \begin{cases} \frac{\alpha}{12\pi^2 g_T}\sqrt{\frac{T}{g_T\delta}} & D = 3, \\ -\frac{1}{4\pi^2 g_T}\ln\frac{g_T\delta}{T} & D = 2, \\ -\frac{\beta}{4\pi}\sqrt{\frac{\delta}{T g_T}} & D = 1. \end{cases}$$

Altshuler-Aronov corrections, the same as in a homogeneous metal.

Hall resistivity

1. Classical picture of the Hall resistivity of the granular material.

R-longitudinal resistivity:
information about intergrain tunneling

$$R = \left(e^2 g(T) a^{2-d}\right)^{-1}$$

$$g = 2\pi\nu_0^2 t_{\mathbf{ij}}^2$$

Q. What kind of information can one extract from the Hall resistivity R_H ?

A. Carrier density inside the grains (no dependence on the tunneling!)

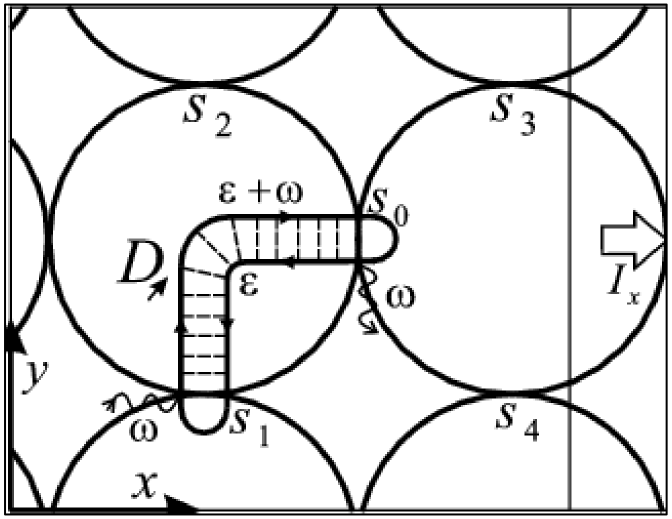
Most important formula:

$$R_H = \frac{\sigma_{xy}}{\sigma_{xx}^2 + \sigma_{xy}^2}$$

Results of an explicit calculation:

$$\sigma_{xx} \propto t_{ij}^2 \qquad \sigma_{xy} \propto t_{ij}^4$$

R_H does not depend on t_{ij} !



The contribution to σ_{xy} comes from the diffuson D with non-zero space harmonics (in contrast to σ_{xx} !)

Classical Hall resistivity: $R_H^{(0)} = \dfrac{H}{n^* ec}, \qquad n^* = An$

A-numerical coefficient, depending on the geometry of the grains (A=1 for cubic grains, $A = \pi / 4$ for spherical grains)

2) Effect of the Coulomb interaction:

Again, logarithmic in temperature corrections δR_H

$$\frac{\delta R_H}{R_H^{(0)}} = \frac{c_d}{4\pi g_T} \ln\left[\frac{g_T E_c}{T}\right]$$

in the region $g_T \delta \leq T \leq g_T E_c$

Lower temperatures: $T \leq g_T \delta$

1. Weak localization correction: $\delta R_H^{WL} = 0$

2. Altshuler-Aronov corrections

$$\frac{\delta\sigma_{xx}^{AA}}{\sigma_{xx}^{(0)}} = -\frac{1}{4\pi^2 g_T} \ln\frac{\Gamma}{T},\ T \ll \Gamma, d = 2$$

$$\Gamma = g_T \delta$$

Insulating regime, $g_T \leq 1$

Activation law: clear explanation in terms of electron hopping from grain to grain.

How to explain $\exp(-a/\sqrt{T})$ law?

Beloborodov et al (2005): elastic and inelastic co-tunneling through many grains and random fluctuation of the Fermi energy in the grains due to charged impurities ⇒ Efros and Shklovskii mechanism

Conclusions:

1. Phase dscription and diagrammatic schemes are developed to study granular metals.

2. A crossover from the logarithmic to the exponential dependence of the conductivity on temperatures at not very low temperatures.

3. Hall resistivity gives information about the carrier density inside the grains and has also logarithmic in temperature corrections.

4. Logarithmic dependence of the conductivity observed in many granular materials is explained.