Metal-Insulator Transition in disordered two-dimensional (2d) Fermi liquid

(Anderson transition in the presence of e-e interactions)

(no "Mottness")

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thanks to Vladimir Pudalov from
the Lebedev Physics Institute of RAS

Alex Punnoose & A.F. ,
Phys. Rev. Lett. **88** (2002);
Science **310**, 429 (2005);

S.Anissimova, S Kravchenko,
AP, AF and TM Klapwijk,
Nature Physics, **3** (2007).

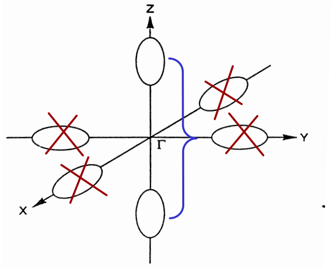
- scaling ideas in the Metal-Insulator Transition (MIT):
  why "in two-dimensions, there is no **true** metallic behavior"

- transport in low and high mobility 2d systems:
  why **high** mobility systems are so interesting**?**

- why: "highly controversial issue of the nature of **apparent** MIT in 2d"

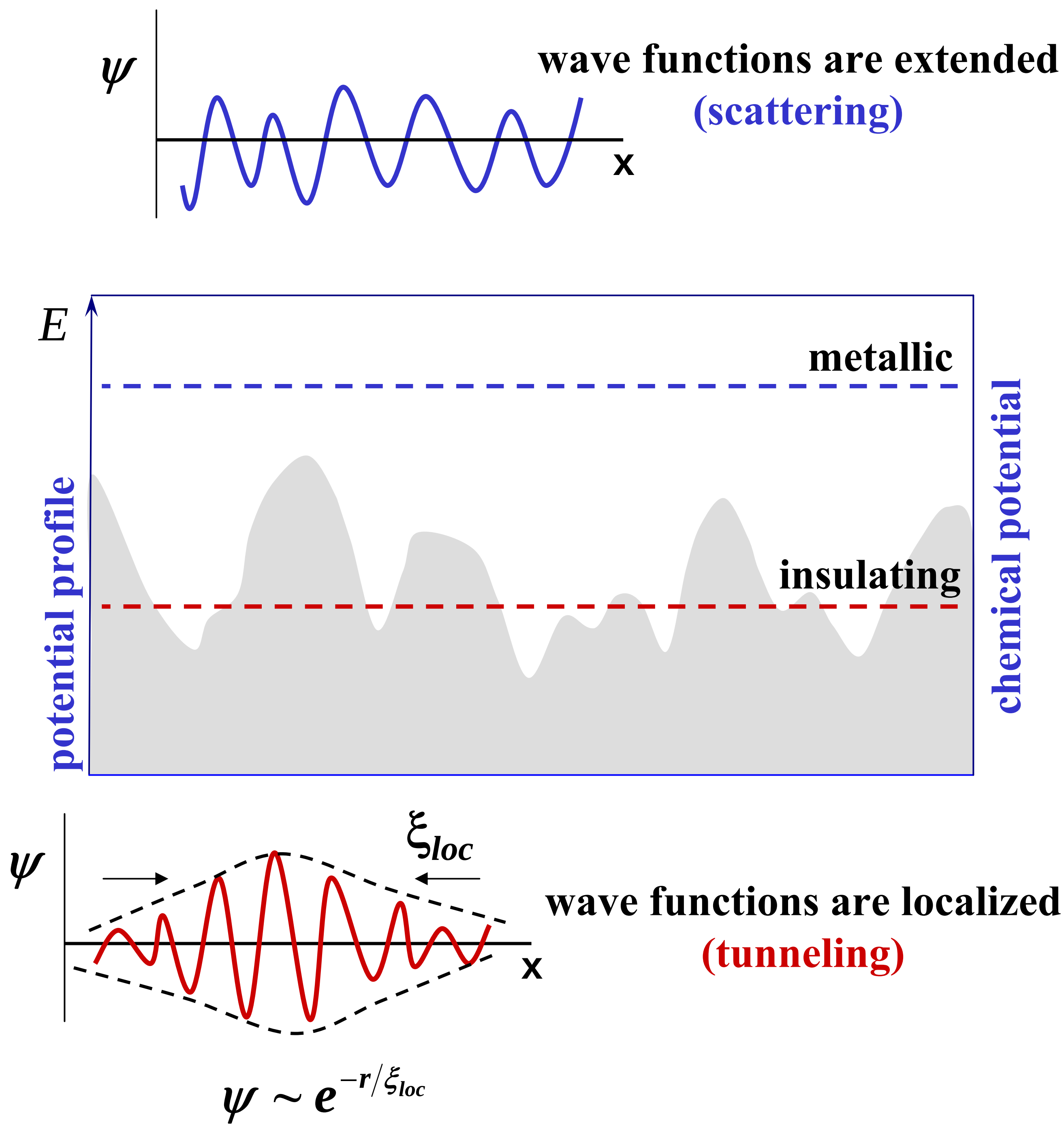
- disordered Fermi-liquid, the **two parameter** RG scheme

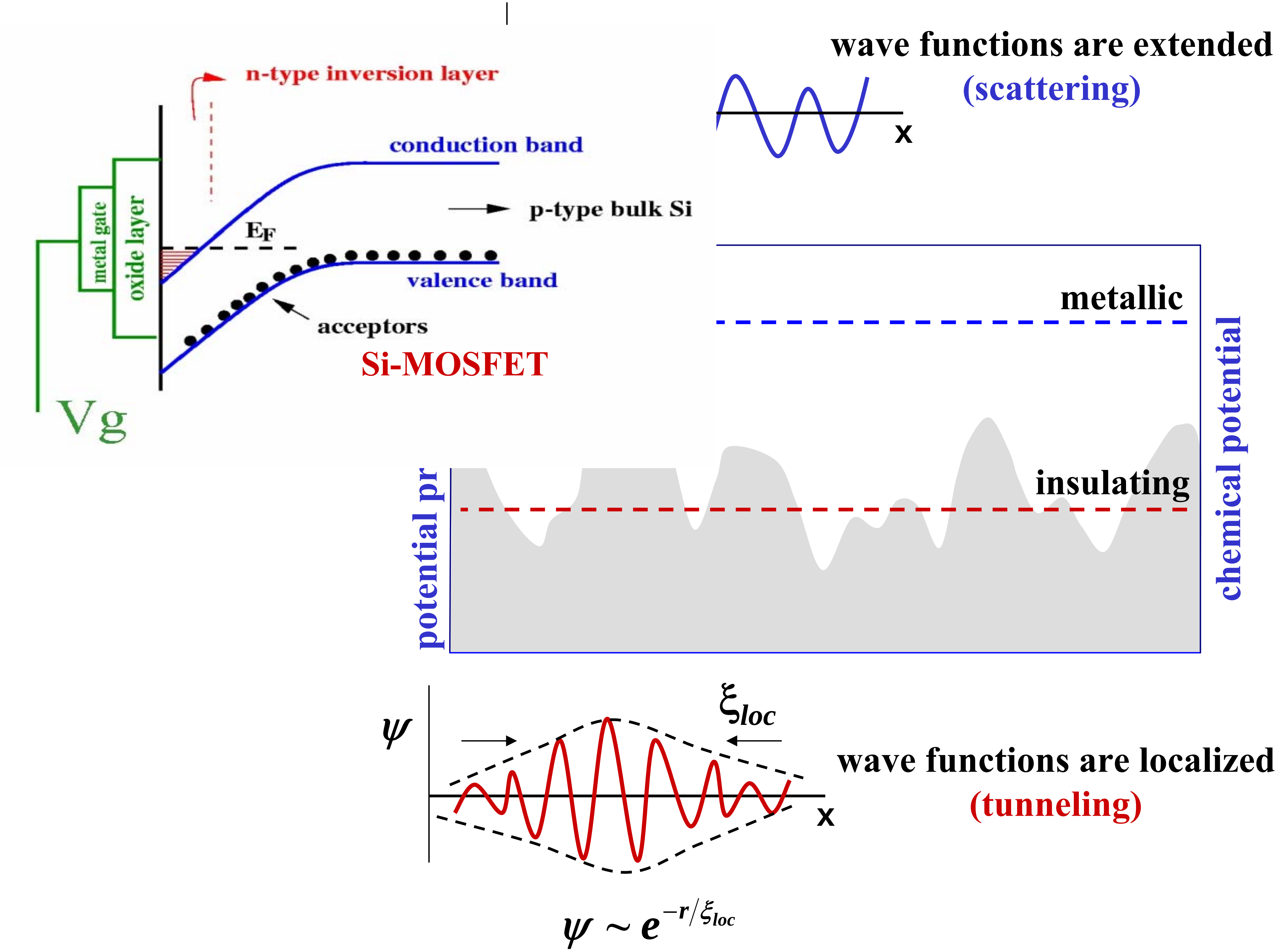
- **valleys** (flavors) in Si-MOSFET; **large-n_v** limit



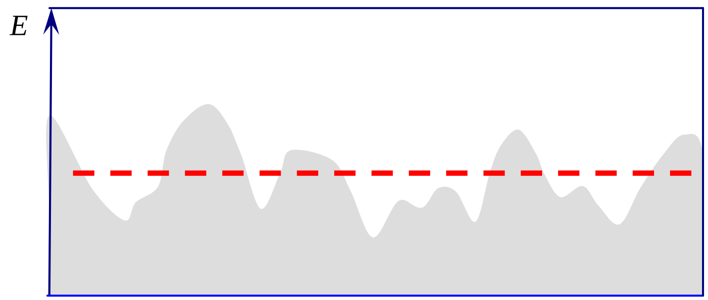
- phase diagram in the **two-loop** solution

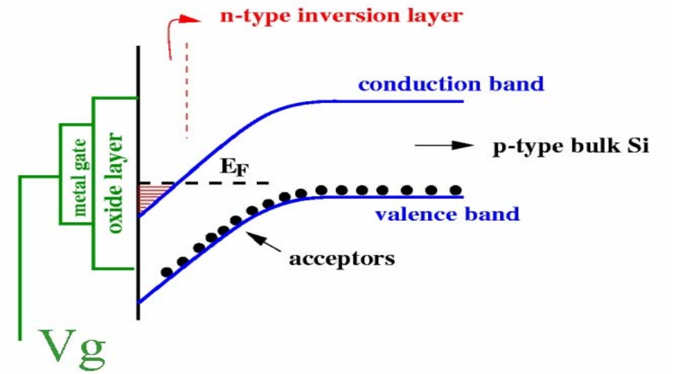
- experimental confirmation (fan-like magnetoresistance; **flow diagram**)


ψ
wave functions are extended
(scattering)
x
E
metallic
insulating
potential profile
chemical potential
ψ
ξloc
wave functions are localized
(tunneling)
x
$\psi \sim e^{-r/\xi_{loc}}$



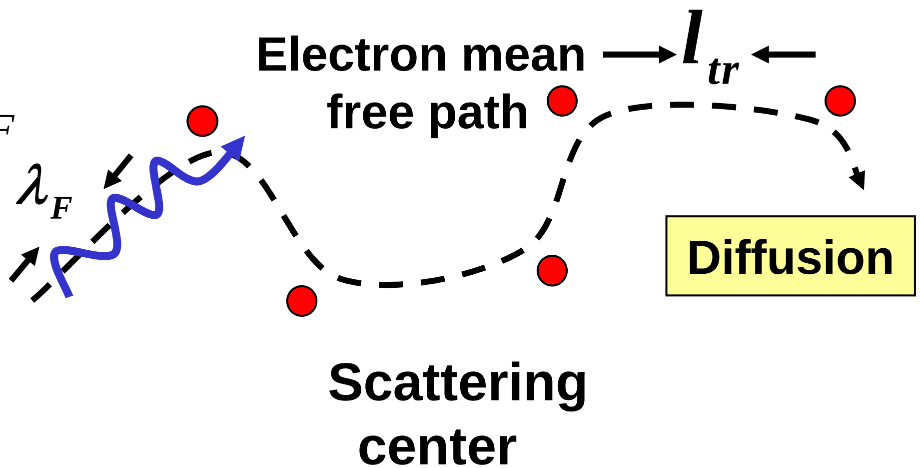
n-type inversion layer
conduction band
p-type bulk Si
E_F
valence band
metal gate
oxide layer
acceptors
Si-MOSFET
Vg
wave functions are extended
(scattering)
x
metallic
insulating
chemical potential
potential pr
ξ_{loc}
ψ
x
wave functions are localized
(tunneling)
$\psi \sim e^{-r/\xi_{loc}}$





Metal:

$$l_{tr} >> \lambda_F$$



$$\tau_{tr} = l_{tr} / v_F$$

Transition: $p_F l_{tr} \sim E_F \tau_{tr} \sim \hbar$ Ioffe – Regel ; Mott

Insulator: area of quantum physics; ξ_{loc}

Nature of the M-I transition?
experiment? everything *was* smeared by a finite temperature

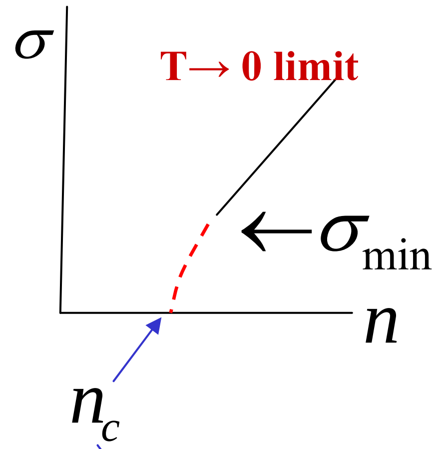
PHYSICAL REVIEW VOLUME 109, NUMBER 5 MARCH 1, 1958

Nature of the M-I transition

Absence of Diffusion in Certain Random Lattices

P. W. ANDERSON
Bell Telephone Laboratories, Murray Hill, New Jersey

M-I Transition at zero temperature in 3d
(σ denotes conductivity, n – electron density)



Quantum Critical Point:

$$\xi_{loc} \to \infty$$

Anderson localization:
importance of large scales

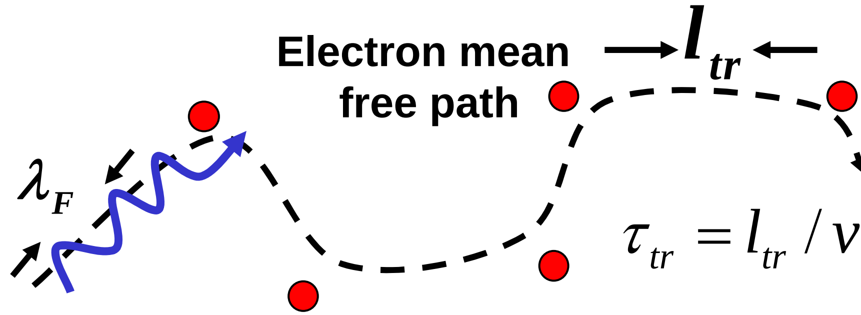
$$\xi_{loc} \gg l_{tr}$$

➡ **scaling;**

What is the role of dimension?

$$\sigma_{Drude} = \frac{ne^2\tau_{tr}}{m}$$

Electron wave length



$$\tau_{tr} = l_{tr} / v_F$$

M-I Transition: $p_F l_{tr} \sim E_F \tau_{tr} \sim \hbar$ Ioffe – Regel criterion

$$\sigma_{d=2} = e^2 \frac{(p_F{}^2 / 2\pi\hbar^2)(l_{tr} / v_F)}{m} \sim e^2/(2\pi\hbar)\ ; \quad 2\pi\hbar / e^2 \approx 25.8\ k\Omega$$

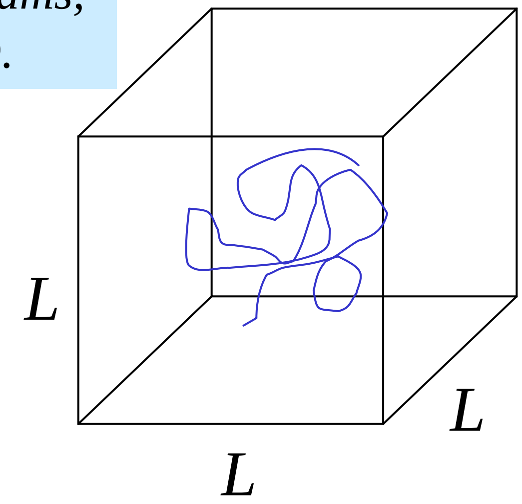
What makes **d=2** so special? $R_{\mathrm{Ohm's Law}} = \frac{L}{\sigma_d L^{d-1}} = \frac{L^{2-d}}{\sigma_d}$

in d=2 conductivity coincides with ***conductance*** (i.e., R^{-1}).

Message from d=2:

forget about conductivity – study ***conductance*** of a "cubic" sample measured in the universal unit $\mathbf{g} = R_{\mathrm{esistance}}^{-1} / (e^2 / h)$

Scaling ideas in transport : *Thouless (74,77); Abrahams, Anderson, Licciardello, Ramakrishnan (79); F. Wegner (79).*



Renormalization Group transformation:
increasing of the blocks size from l_{tr} *to* L

dimension *d=2* is of special importance;
the geometrical factor in the Ohm's law,
$g = \sigma L^{d-2}/(e^2/h)$, disappears at *d=2*.

g(*L*)– dimensionless ***conductance of a sample*** of the size *L*:

$$g(L) = R^{-1}(L)/(e^2/h)$$

$$\frac{dg}{d\xi} = \beta(g)\ ;\ \xi = \ln\ (L/l_{tr}).$$

this equation has a very specific form: $\beta(g; L/l_{tr})$

does not depend explicitly on *L*

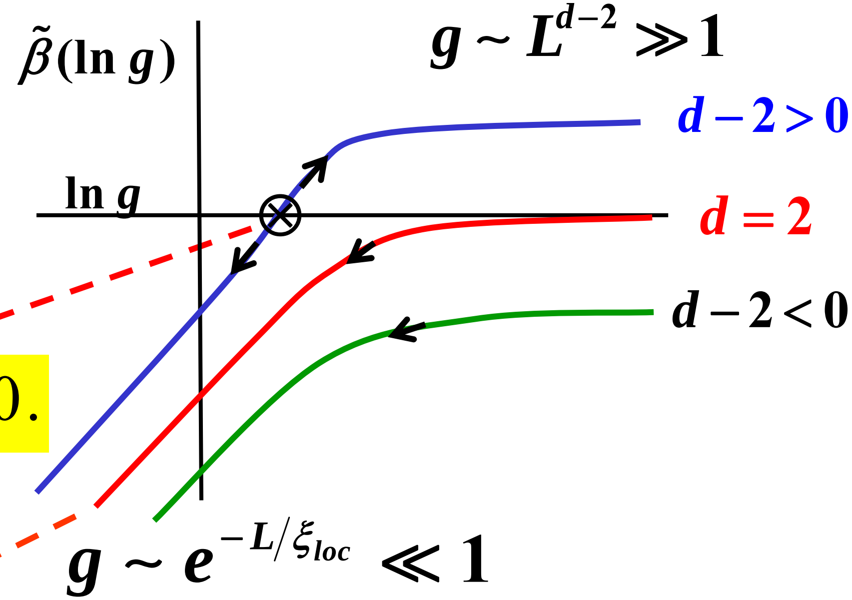
M-I transition: $\beta(g = g_{crit}) = 0.$

Origin of the common believe that
"in two-dimensions, there is no true metallic behavior"

Abrahams, Anderson, Licciardello, Ramakrishnan (79)

$$\frac{d\ln g}{d\xi} = \tilde{\beta}(\ln g)\ ;\ \xi = \ln(L/l_{tr}).$$

one-parameter scaling theory of the MIT: a competition between the Ohm's law factor L^{d-2} and the QM interference



$$3d:\ \beta(g_{crit}) = 0.$$

2d electrons eventually become localized

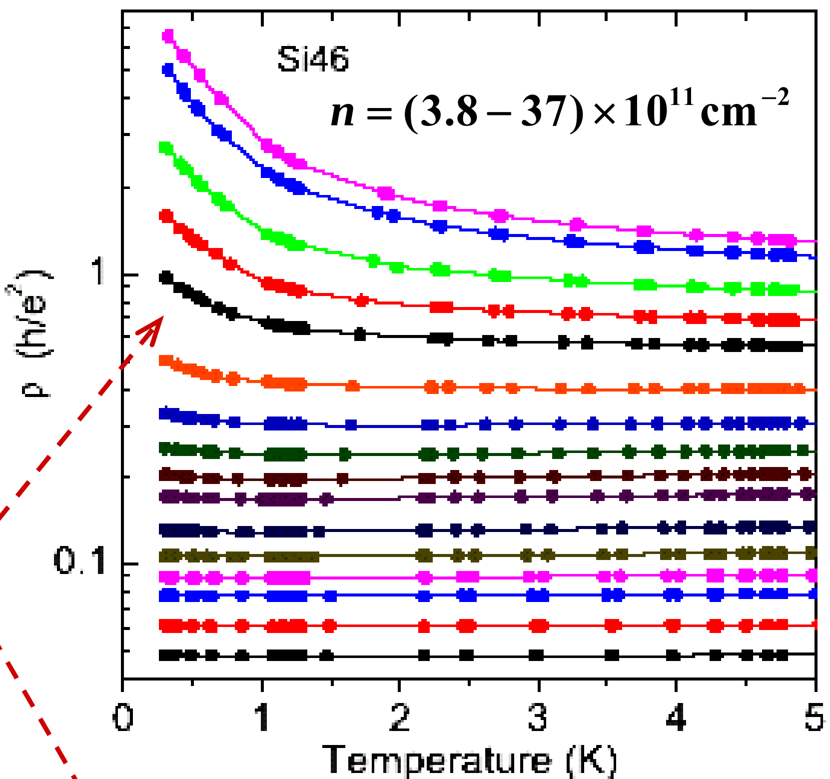
$$\ln(1/T\tau_{tr}) > g(l_{tr})$$

scale is controlled by the temperature

here conductance g is assumed to be the only relevant scaling parameter

Si-MOSFET
low mobility $\mu=1{,}500\,\mathrm{cm^2/Vs}$

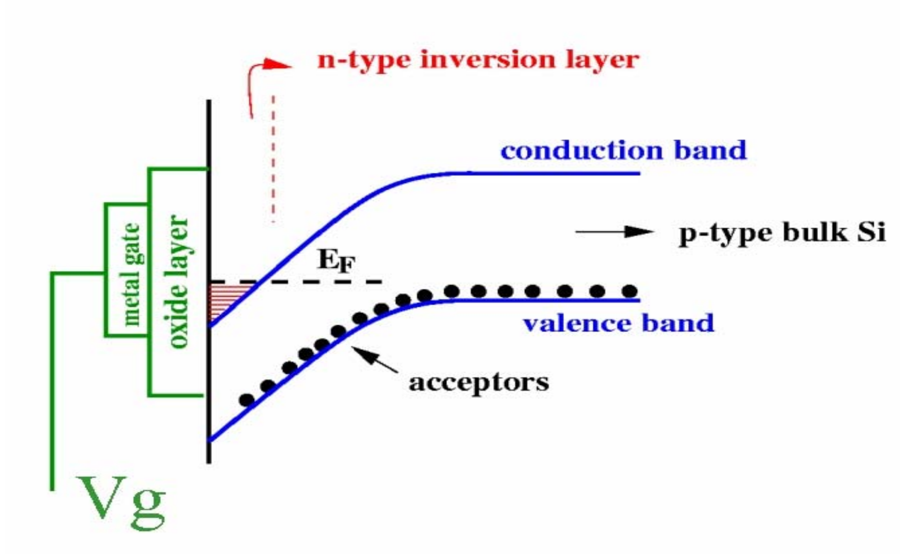
ρ **, not** σ **!**



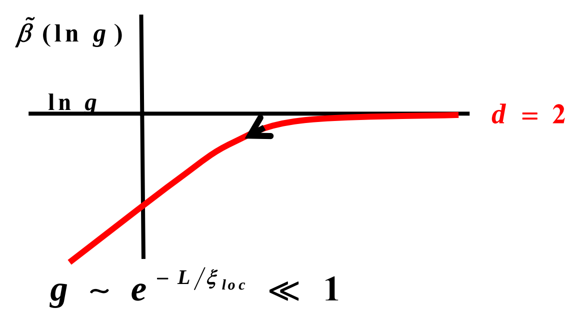
2D electron states
eventually become localized

$$\ln(1/T\tau_{tr}) > g(l_{tr}) = 1/\rho$$

2DEG in Si-MOSFETs



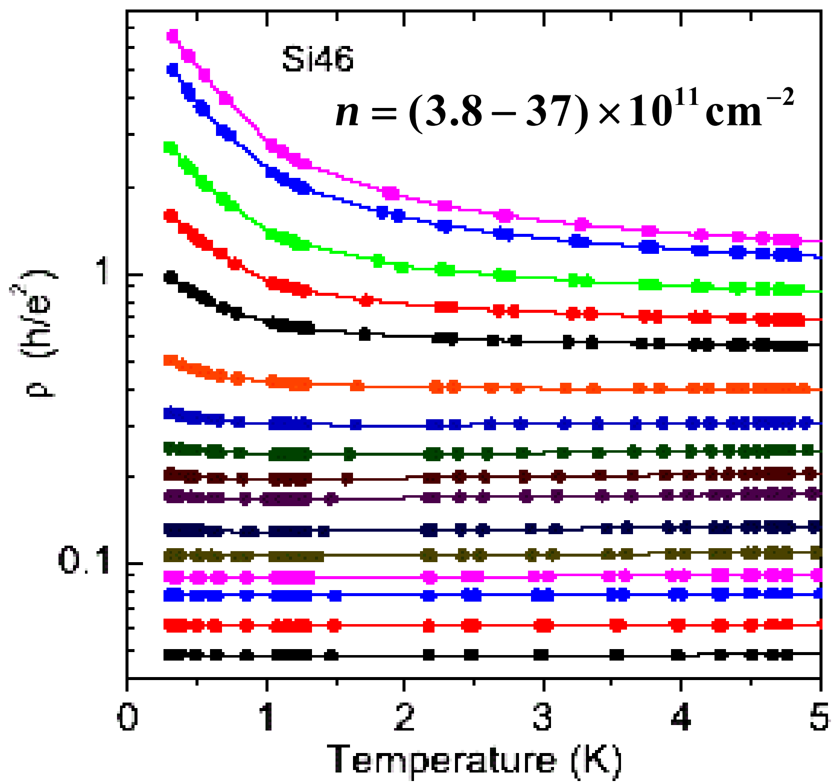
Energy $\left(\frac{k_z^{\,2}}{2m_z}\right)$ is quantized in the z-direction



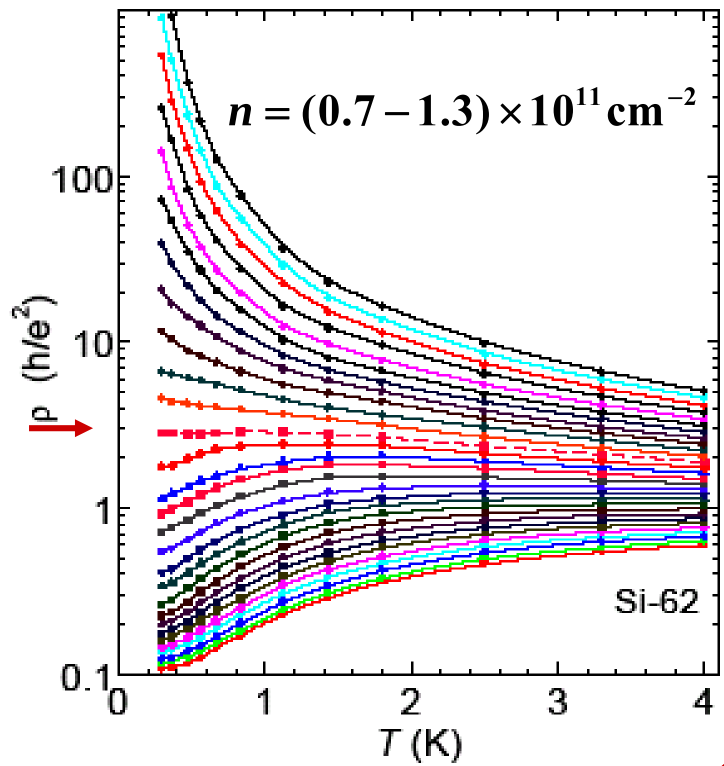
apparent Metal Insulator Transition in 2d (Si-MOSFET)

S. Kravchenko et al., 1994 (Pudalov's group)

low mobility μ=1,500 cm²/Vs



high mobility μ=39,000 cm²/Vs



2D electron systems eventually become insulating

$$\ln(1/T\tau) > 1/\rho$$

$$n_c \sim 10^{11}\,cm^{-2} \qquad \rho_c \sim \frac{\pi h}{e^2}$$

apparent MIT transition

apparent:

1 clearly seen or understood: *It was apparent for all of us*

2 appearing but not necessary true or genuine: *the apparent cause but not the real one*

A.S. Hornby, Oxford Student’s Dictionary of Current English

what is specific in high mobility samples?

$$\mu \propto \sigma / n$$

$$\mu^{-1} = \frac{m}{e} \tau_{tr}^{-1}$$

at the metal-insulator transition: $l_{tr} \sim \lambda \quad \Rightarrow \quad E_F \sim 1/\tau_{tr}$

the higher mobility, the **lower** density can be reached **remaining still in the metallic phase**

$$E_F \propto n$$

MOSFETs: $n = 10^{11} cm^{-2}$

$$E_F = \frac{\pi \hbar^2 n}{2m} \approx 7K$$

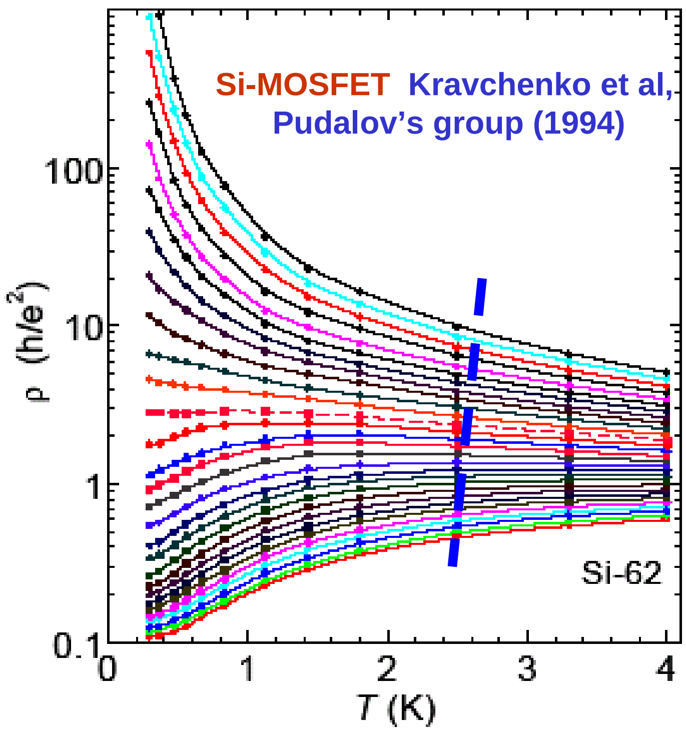
$$r_s \equiv \frac{E_{ee}}{E_F} \simeq 10$$

$$E_{ee} = \frac{e^2}{\varepsilon} (\pi n)^{1/2} \approx 70K$$

naïve and incorrect!

is the phenomenon universal?
Si-MOSFET, p-GaAS, n-GaAs

$$1/\tau = (e/m)\mu^{-1}$$



$1/\tau_{tr} \sim 2 \div 3K$

mobility $\mu \sim 3\times10^4 cm^2/Vs$

$[R_{max}(T)-R(0)]/R(0)\sim7$ **!**

$r_s\sim10$

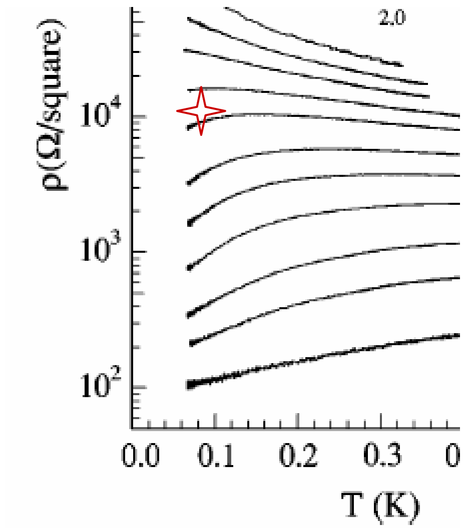


FIG. 2. ρ vs T for p = 3.2, 2.2, 1.7, 1.2, 0.9, 0.7, 0.5, 0.4, 0.3, 0.23, and 0.15×10¹⁰ cm⁻² from the bottom. The inset shows ρ

p-GaAs Noh et al,
Dan Tsui's group (2003)

$[R_{max}(T)-R(0)]/R(0)\sim2$

$r_s>20$

$\mu \sim 3\times10^5 cm^2/Vs \quad 1/\tau_{tr} \approx 0.1K$

n-GaAs Lilly et al,
Eisenstein's group (2003)

$R_{max}(T)-R(0)\sim25\%$ $r_s\sim10$

Referee : “ ***highly controversial*** issue of the nature of ***apparent*** MIT in two-dimensions.”

p-GaAs

1. Ultra high mobility: diffusive regime is hard to reach; $1/\tau$ ~100 mK
2. Long range scattering: transport time >> quantum life time (quasiclassical motion)
3. Very low density: large-r_s , but also effects of non-degeneracy at $T \sim E_F$
4. Theoretical possibilities: percolation, screening (but also WC, stripes, bubbles, etc)

At low enough temperatures, diffusive regime will unavoidably be reached, but at finite T, one is observing a crossover – not the MIT (a la Anderson).

Si-MOSFET: advantages

1. Moderately “high” mobility: diffusive window $T < 1/\tau_{tr} < E_F$; $1/\tau_{tr}$ = 2-3 K
2. Short range scattering: transport time is comparable to quantum life time (quantum-mechanical scattering)
3. E-e nteraction: nearly optimal value of the bare values of the interaction
4. Multi-valley system: effectiveness of the fluctuations induced by e-e interactions is enhanced

$(1/\varepsilon_F\tau_{tr}) << 1$

"Probing the …Quantum Phase Transition in the …localization"

Phys. Rev. Lett. 84, 1567 (2000)

Amsterdam University group

InGaAs/InP mobility $\mu = 1.6\times 10^4 cm^2/Vs$

$Si-MOSFET$ **mobility** $\mu \sim 3\times 10^4 cm^2/Vs$

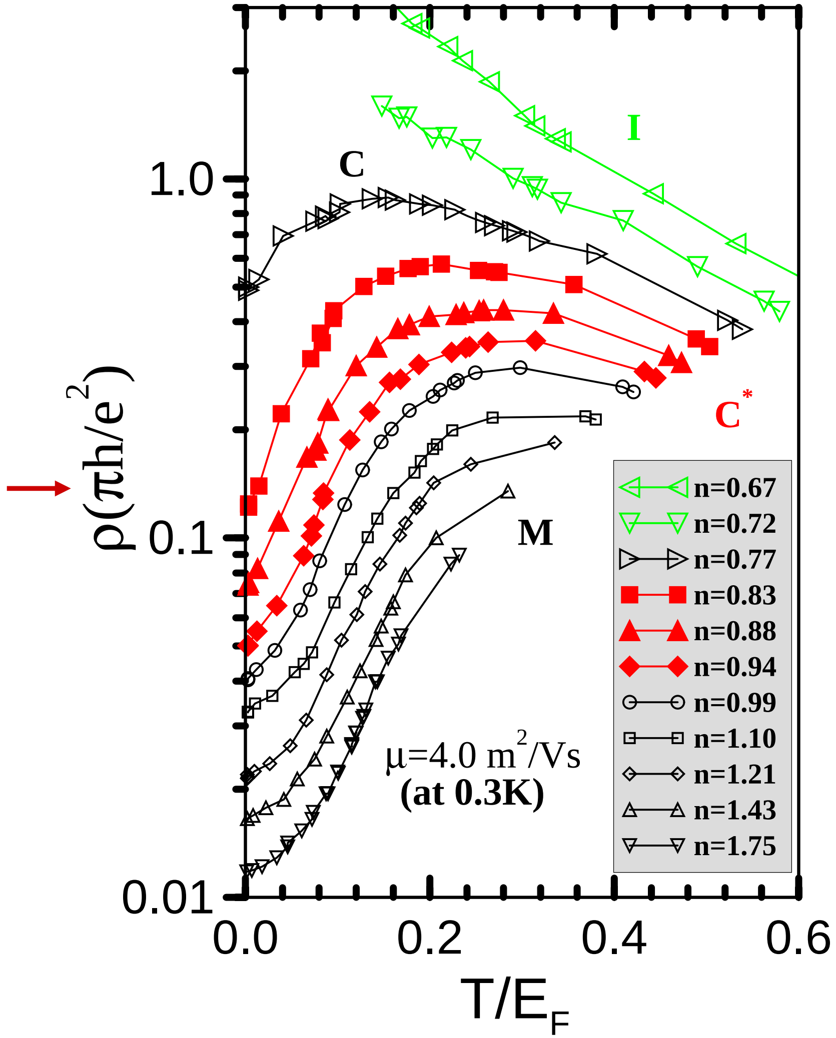
relatively

Because of the short range random alloy potential scattering, the low mobility $In_xGa_{1-x}As/InP$ structure has proven to be exceptionally important for studying scaling phenomena. This produces a wide range in T where the transport is dominated by Anderson (de)localization effects. This is in sharp contrast to high mobility $GaAs/Al_xGa_{1-x}As$ heterostructures where the long range potential fluctuations dramatically complicate the observability of the critical phenomenon, given the limitations of the experiment [3,4].

$T \sim 0.1 \div 5K$

non-monotonic $\rho(T)$ in a high mobility sample

(first most important slide)



Pudalov, et al., ('98)

similar data but replotted as T/E_F

the regions of my interest (C and C*):
strongly interacting electrons in the diffusive regime

$$T < \hbar/\tau \leq E_F$$

A universal picture with a single parameter scaling?

$$\frac{d\rho}{d\xi} = \beta_{"\infty"}(\rho)$$

starting point: **disordered Fermi-Liquid;**

in Si-MOSFET *e-e* interaction effectively is not too strong!

MOSFETs: $n = 10^{11} cm^{-2}$ $\qquad r_s \equiv \frac{E_{ee}}{E_F} \simeq 10$

no noticeable anomalies on the metallic side of the MI transition!
Shubnikov oscillations, Hall coefficient,
weak localization corrections

$$\mathbf{g^*/g_0 = \frac{1}{1+F_0^\sigma} \approx 1.5 \qquad m^*/m \approx 3}$$

singlet amplitude γ_1
is ***universal "1"***
after screening

$\beta_{"\infty"}(\rho)$

warning: in the following
the case of Coulomb interaction only;
no free-electron limit in RG equations.

"triplet" amplitude γ_2

$$1+\gamma_2 = \frac{1}{1+F_0^\sigma}$$

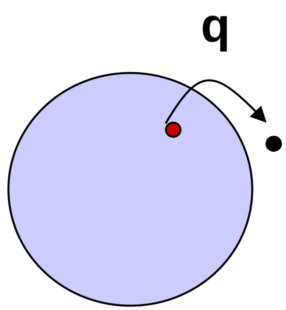
***this* is a driving force of**
MIT transition in 2D !

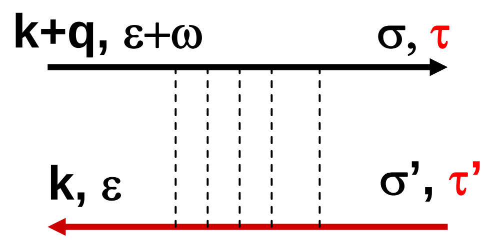
low energy physics is described by two-particle propagators
(rather than single-particle Green's functions)
fluctuations of charge, spin and valley **densities**

Disorder-averaged two-particle propagators

semiconductors: spin + valleys

$$\left(\frac{\partial}{\partial t} - \nabla^2\right)$$

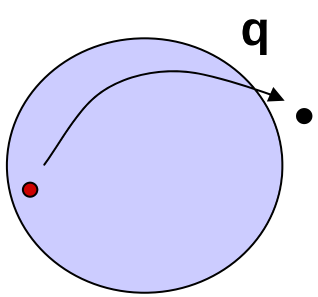


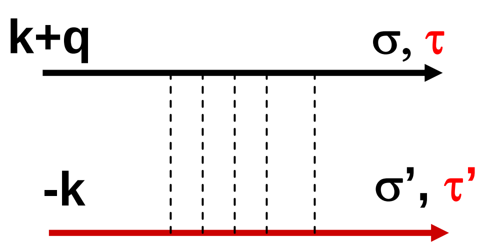


$$\approx \frac{1}{Dq^2 - i\omega}$$

particle-hole propagator
(diffuson)

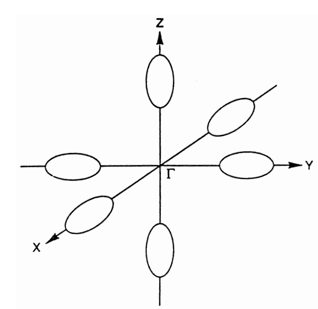
diffusive pole





$$\approx \frac{1}{Dq^2 - i\omega}$$

particle-particle propagator
(cooperon)



Elements of disordered Fermi-Liquid :

leading order in $\dfrac{1}{\varepsilon_F \tau_{tr}} \ll 1$

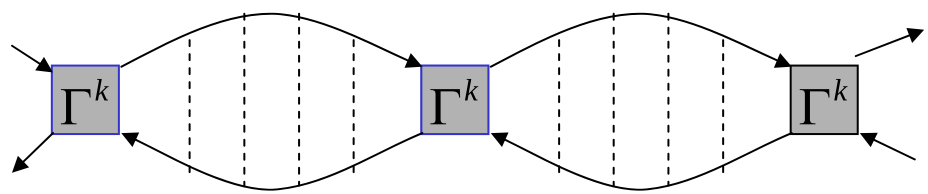
$\tau_{tr}^{-1} \gg \varepsilon, T$ (plane waves are not eigenstates anymore)

multiple rescattering of pairs of quasiparticles
(each time two into two) should be modified by **diffusion**:

$$\left(\frac{\omega}{\omega - \vec{v}\,\vec{k}} \right)_{clean} \Rightarrow \left(\frac{\omega_n}{\omega_n + Dk^2} \right)_{disorder}$$

Static amplitudes remain unchanged in the leading order $1/\varepsilon_F \tau_{tr} \ll 1$

dynamic amplitude of scattering in the particle-hole channel; only $l = 0$ harmonic is relevant



$\Rightarrow$

$$\Rightarrow \Gamma^k \frac{\omega_n}{\omega_n + Dk^2} \Gamma^k \frac{\omega_n}{\omega_n + Dk^2} \Gamma^k$$

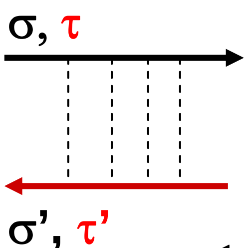
valley in high-mobility Si-MOSFET is a good quantum number (second most important slide)

- quantum numbers (pseudo spin)

$$|\sigma,\tau\rangle \equiv |\{\uparrow\downarrow\},\{\pm\}\rangle$$

2 spin $\otimes$ *2 valleys = 4 states*

- total number of electron-hole states



$= 4 \otimes 4 = 16$

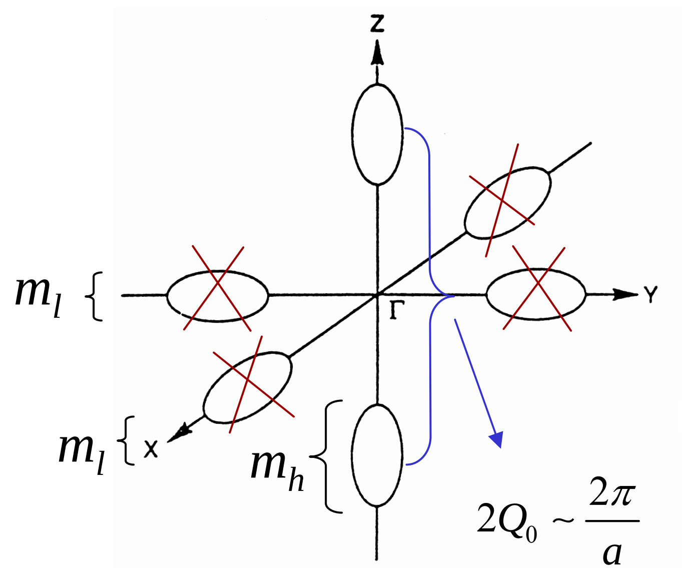
1 singlet + 15 "triplets"

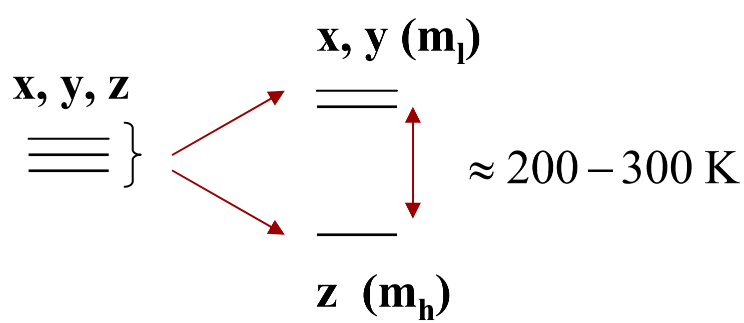
$$(4n_v^2 - 1)$$

Specifics of high mobility samples:
no inter-valley scattering

$$k_z \ll Q_0 \; !$$

Lifting of the valley degeneracy in a (001) layer





energy is quantized in the z-direction (perpendicular to the interface)

valley in high-mobility Si-MOSFET is a good quantum number

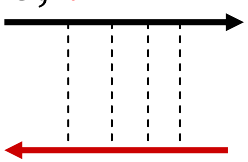
- quantum numbers (pseudo spin)

$$|\sigma,\tau\rangle \equiv \left|\{\uparrow\downarrow\},\{\pm\}\right\rangle$$

2 spin ⊗ 2 valleys = 4 states

- total number of electron-hole states

σ, τ

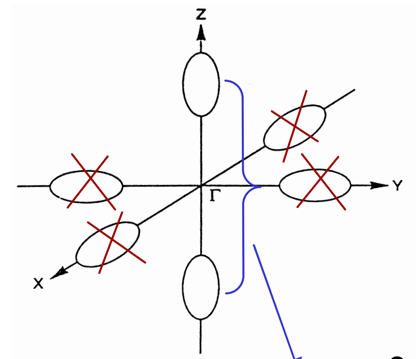


σ’, τ’

$= 4 \otimes 4 = 16$

1 singlet + 15 “triplets”

$(4n_v^2 - 1)$



$$2Q_0 \sim \frac{2\pi}{a}$$

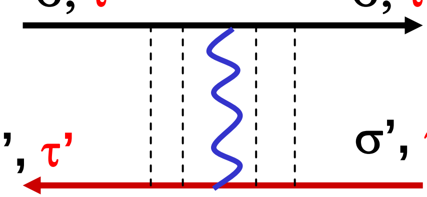
e-e interaction: couples all spin/valleys species together but does not mix them (!)

valleys: much more fluctuation degrees of freedom are involved

Specifics of high mobility samples:
no intervalley scattering

$$k_z \ll Q_0 \; !$$

σ, τ σ, τ

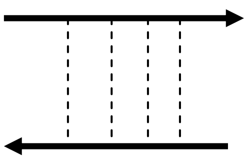


σ’, τ’ σ’, τ’

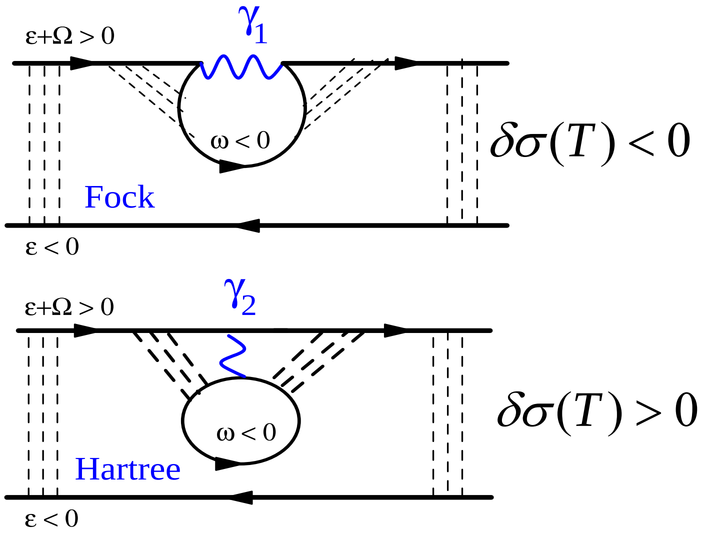
beyond the disordered Fermi-liquid:
interplay of the electron-electron interactions and disorder

1) ***e-e*** interactions activate virtual transitions; because of the slowness (softness) of the diffusion modes this leads to non-analytical in temperature corrections to the resistances

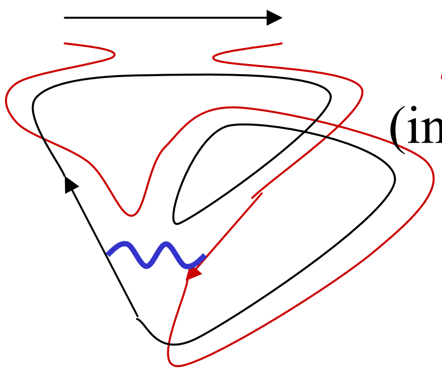
Altshuler, Aronov (79); Altshuler, Aronov and Lee (80)



$$\sim \frac{\mathbf{1}}{\boldsymbol{Dq^2 - i\,\omega}}$$



two contributions
of opposite signs (a la Hartree-Fock);
"Hartree" contribution is anti-localizing
(eventually causes the MI transition in 2D)



2) a source of non-linearity
(in this example 3 modes to 1)

3) renormalization
of the *e-e* interaction amplitudes

a unified scheme which includes:
disorder and e-e interaction,
charge, spin (and valleys)
RG-flow

non-linear σ– model with *e-e* interactions

A. F. (83)

$$S[Q]=\frac{\pi\nu}{4}\int \mathrm{Tr}\left[D\left(\vec{\nabla}Q\right)^2-4z\mathrm{Tr}\left(\hat{\varepsilon}Q\right)+Q(\hat{\Gamma}_s+\hat{\Gamma}_t+\hat{\Gamma}_c)Q\right]d^2r$$

$$Q_{nm}^{\alpha\beta\ ij} \qquad TrQ=0 \qquad Q^2=I$$

parameter z describes renormalization of the DOS of the diffusion modes

1) RG flow ⟶ **2) physical quantities**

Two parameter scaling

AF (1983-84)

finite ρ increases γ_2

while, γ_2 reduces ρ

to all orders in e-e interactions

cooperon *singlet* *"triplet"*

$$\frac{d\ln\rho}{\rho d\xi} = \left[n_v + 1 - (4n_v^2 - 1)\left((\frac{1+\gamma_2}{\gamma_2})\ln(1+\gamma_2) - 1 \right) \right]$$

$$\frac{d\gamma_2}{\rho d\xi} = \frac{(1+\gamma_2)^2}{2}$$

$$\xi = \ln(1/T\tau_{tr}) \ , \ T\tau_{tr} << 1$$

RG-equations has been adjusted for $\mathbf{n_v}$ *valleys by A.Punnoose and AF, PRL (2002) ;*
Owing to the factor $(4\mathbf{n_v}^2 - 1)$ *anti-localization trend increases dramatically!*

the interplay of disorder (i.e. ρ) and interaction γ_2 changes the trend and gives non-monotonic resistance

singlet amplitude
is ***universal "1"*** *after screening*

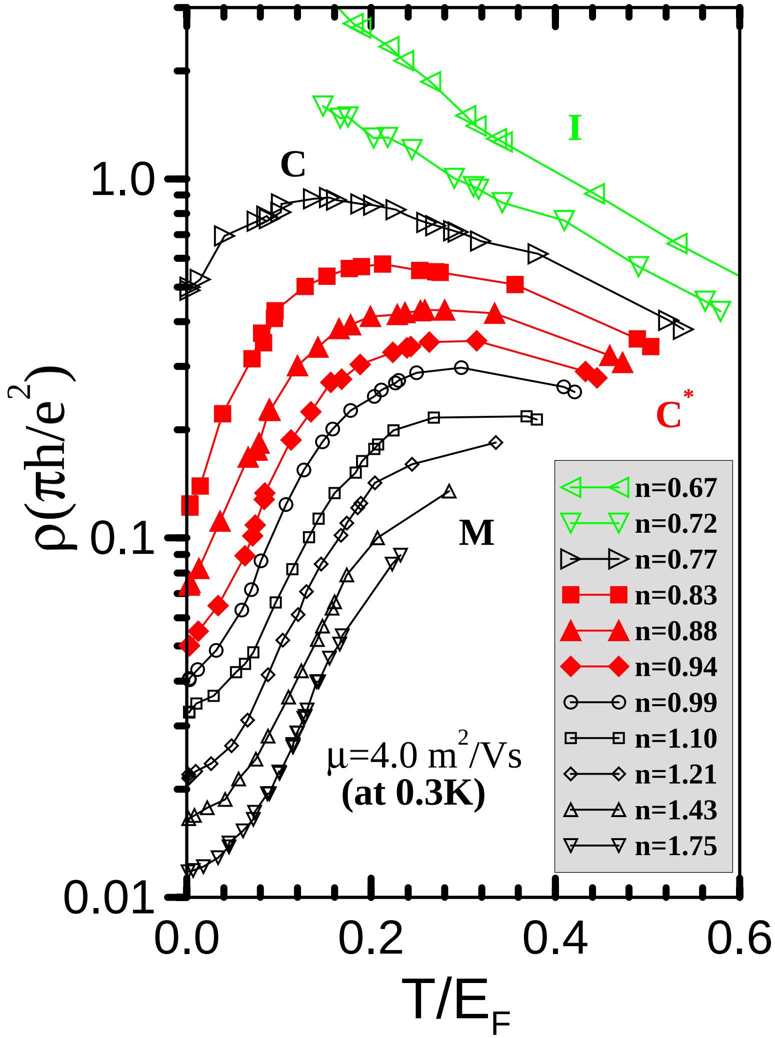
$$\beta_{"\infty"}(\rho)$$

Solutions of the RG-equations:
a series of non-monotonic curves ρ(T) .
After rescaling the solutions are described by a *single* curve

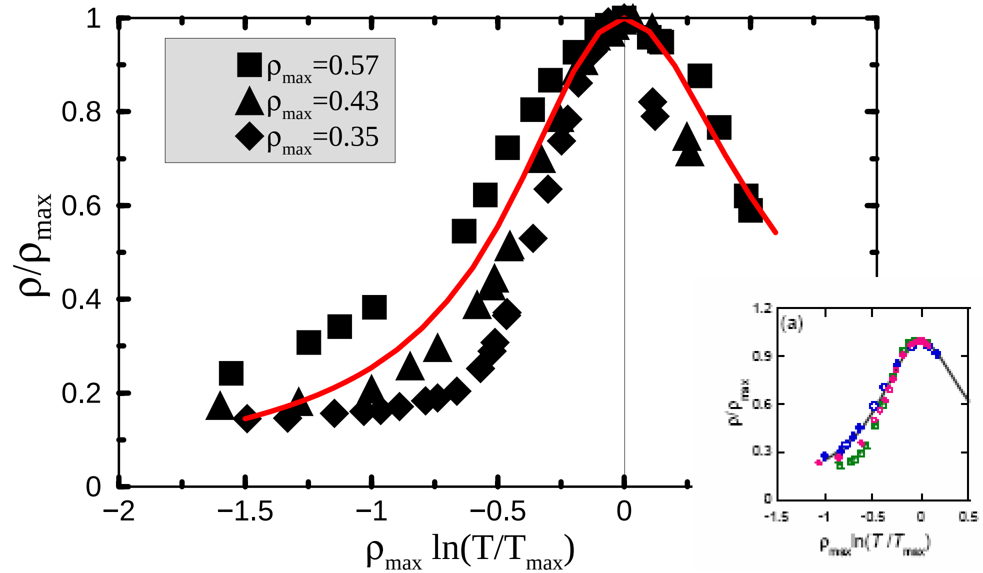
$$\rho(T) = \rho_{max} R(\eta)$$

$$\eta = \rho_{max} \ln(T_{max}/T)$$

Analysis of the region C* in a high-mobility sample with RG for two valleys



Pudalov, et al., ('98)



Data from the region **C*** in a high-mobility samples (Pudalov's and Klapwijk's)

- the drop of five times in $\rho(T)$ and its slowing down has been captured in the correct temperature interval
- **no adjustable parameters** are used

A.Punnoose and AF, PRL (2002)

S. Anissimova et al cond mat/0609181

what is specific in high mobility MOSFET samples?

amount of degrees of freedom available for fluctuations!

in-plane magnetic field: two valleys

$$\hbar/\tau > g\mu_B H_{||} > kT$$

$$\frac{d\rho}{d\xi} = \rho^2\left[n_v + 1 - (2n_v^2 - 1)\left(\frac{1+\gamma_2}{\gamma_2}\ln(1+\gamma_2) - 1\right)\right]$$

instead of $(4n_v^2 - 1)$ **only** $(2n_v^2 - 1)$

$$at\ T \sim 1K\quad \gamma_2 \approx 0.5 \div 0.7$$

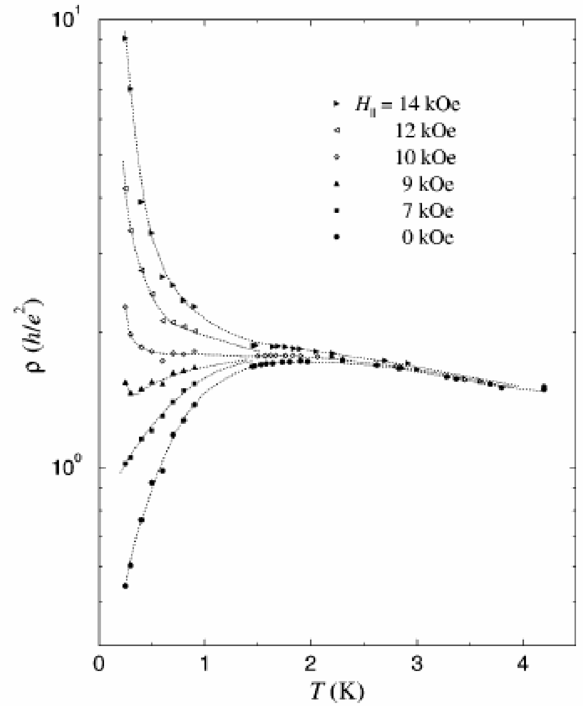


FIG. 4. Resistivity of sample 2 versus temperature in the absence of a field (bottom curve) and in five different parallel magnetic fields. The electron density corresponds to $\delta = 0.10$.

in-plane magnetic field

M.Sarachik group 1997-98

is not enough for *anti* localization in the presence of magnetic field

$at\ T \sim 1K$ **a magnetic field** $\sim 1 Tesla$ **blocks the drop in the resistance;**

see Burmistrov and Chtchelkatchev (2008) for more accurate analysis

Two-parameter scaling: first lessons

Internal problem of the one loop theory:
divergence in the interaction amplitudes at a finite temperatures

$$\gamma_2(T^*) \to \infty$$

$$D_s(T^*) \to 0$$

spin diffusion coefficient

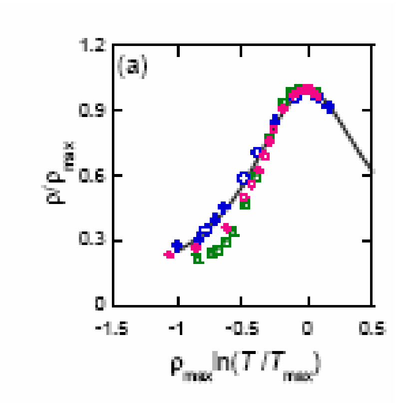
creation of localized moments and
two-stage route (1984) to the MIT transition
(first spins, next charges),
AF & Castellani, Di Castro, P.A Lee et al

- **for** $n_v = 2$ the theory is consistent down to $T^* \approx T_{\max}\, e^{-30000}$

$$\ln(\ln 1/T^*) \sim (2n_v)^2$$

- in the limit $n_v \to \infty$ the theory is internally consistent including T=>0

Two-parameter scaling



- **one-loop conclusion of the two-parameter scaling:**

  for 2-valleys, the existence of
  the ***Metal to Insulator*** transition in 2d is
  (~~“apparently”~~) **logically unavoidable,**
  provided that the Anderson localization at strong disorder is undisputable

---

- Why two-loop?
  because in one-loop the solutions after rescaling become
  a single curve.

$$\rho(T) = \rho_{max} R(\eta)$$
$$\eta = \rho_{max} \ln(T_{max}/T)$$

Valleys (flavors): the large-n_v limit
not more than one interaction per loop
in the large n_v limit

Alex Punnoose & A.F. , Science **310**, 429 (2005)

screening makes the bare value of γ_2 to scale as $1/(2n_v)$

Natural variables: $t = n_v \rho$ and $\theta_2 = 2n_v\gamma_2$

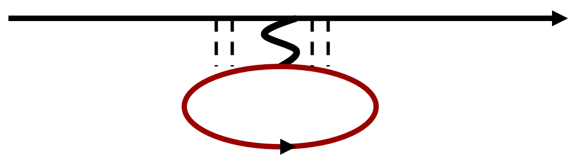
resistance per species

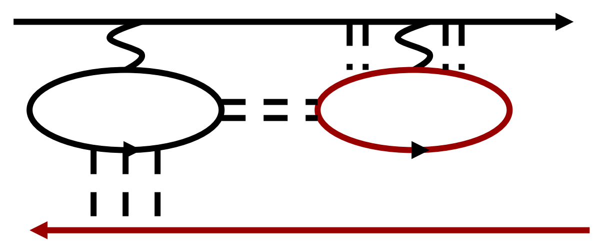
screened interaction has to be compensated by the loop summation; each loop involves a sum over the spin and valley indices

The limit $n_v \to \infty$ is taken keeping t and θ_2 finite

one loop $\Rightarrow \sum_{spin+valleys} \Rightarrow 2n_v t\gamma_2 = t\theta_2$

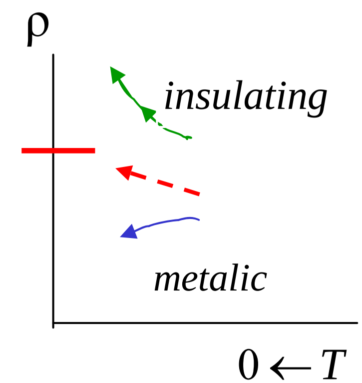
two loops $\Rightarrow t^2(2n_v\gamma)^2 = t^2\theta^2$



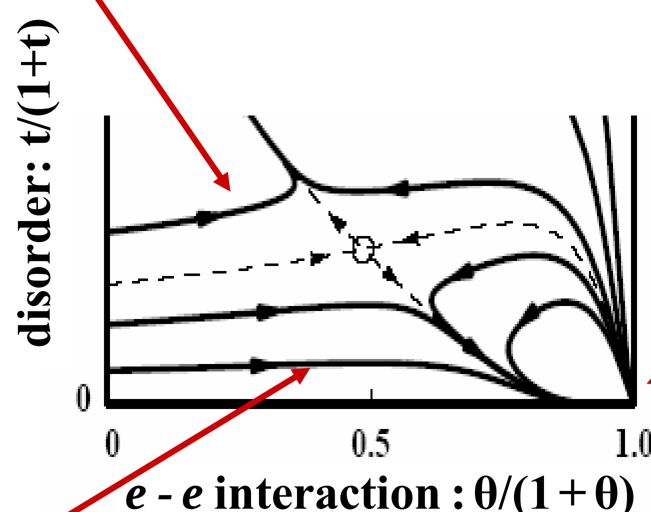


Alex Punnoose & A.F. , Science **310**, 429 (2005)

MIT for interacting electrons in d=2: a competition between disorder and *e-e* interactions



disorder takes over



(the strong coupling corner is not relevant for the MIT)

metallic phase stabilized by *e-e* interaction

Experimental determination of the interaction strength

C_{ee} reflects the strength of the spin-related interactions

$$\Delta\sigma(B_{\|},T) = -\left(\frac{e^2}{\pi h}\right) n_v^2\, C_{ee}(\gamma_2;\rho)\left(\frac{g\mu_B B_{\|}}{k_B T}\right)^2$$

in the small-ρ limit, but still with γ_2 modified by the RG flow:

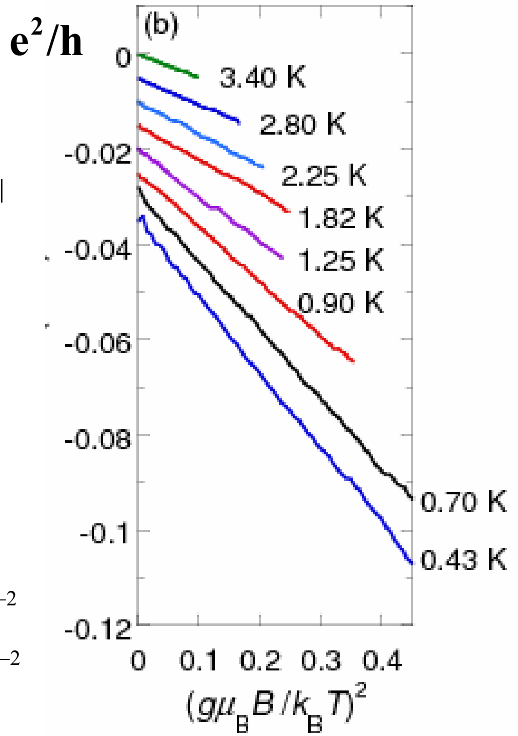
C_{ee} = 0.091 $\gamma_2(1+\gamma_2)$, where

$\gamma_2=\gamma_2(T)$

$$kT \gg g\mu_B B_{\|}$$

$n \simeq 0.9\times10^{11}\,cm^{-2}$

$n_c \approx 0.8\times10^{11}\,cm^{-2}$

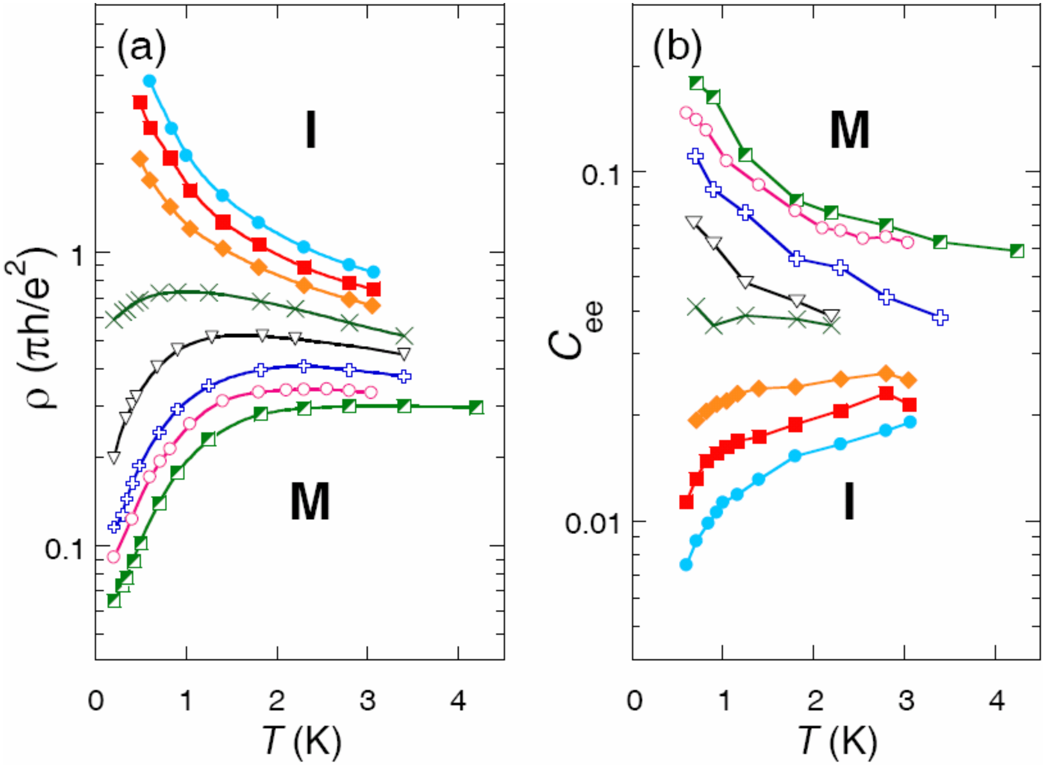


The parameter **C_{ee}**(T) is given by the slope

Anissimova, Kravchenko, Punnoose, Finkel'stein & Klapwijk (2006)

Experimental determination of the interaction strength (cont.)

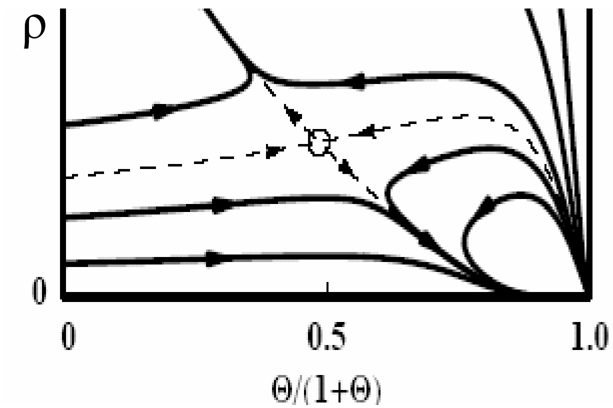
Both $\rho(T)$ and $C_{ee}(T)$ display fan-like spread across the MIT

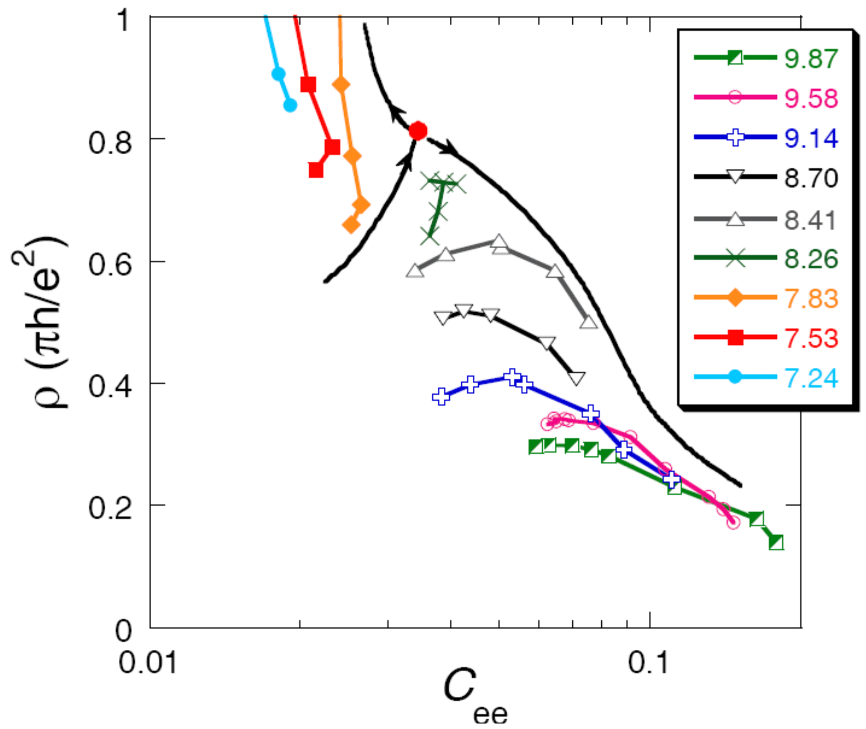


First observation of the scale dependence of the effective interaction strength

Anissimova, Kravchenko, Punnoose, Finkel'stein & Klapwijk (2006)

Flow Diagram and Quantum Critical Behavior of the Metal-Insulator Transition in 2D



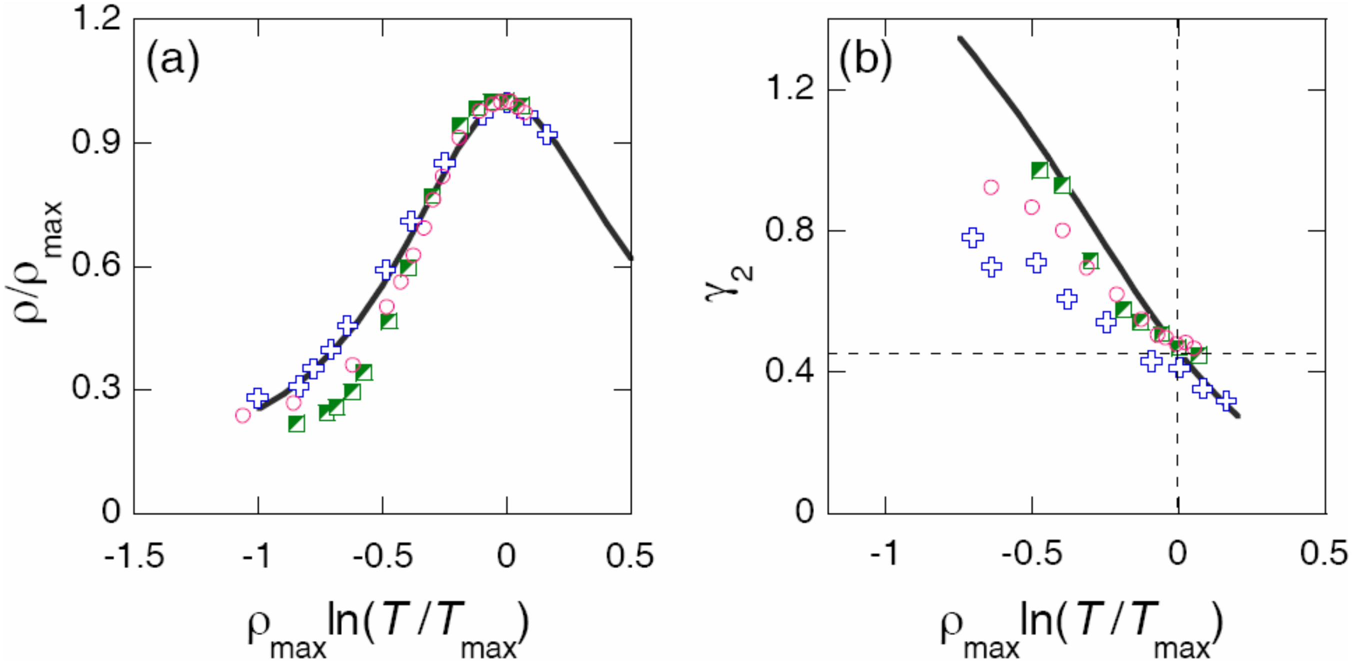


The disorder-interaction flow diagram of the 2D electron liquid in Si-MOSFETs. The red dot at ρ_c=*0.8* and C_{ee} =*0.035* indicates the location of the QCP from which the three separatrices (black lines) emanate. Arrows indicate the direction of the flow as the temperature is lowered. Only T>0.5 data have been used in order to preserve the valley degeneracy.

Anissimova, Kravchenko, Punnoose, Finkel'stein & Klapwijk (2006)

Quantitative comparison in the region of small resistance (one-loop)

$C_{ee} = 0.091\ \gamma_2(1+\gamma_2)$



1. Data from region $\rho_{max} < 0.4$ $(\pi h/e^2)$
2. $\gamma_2(T_{max}) = 0.45$ ($\gamma_2^* = 2.08$ for the case of a single valley)
3. No adjustable parameters

nearly optimal value of the **bare** values of the amplitude of the e-e interaction

Punnoose & Finkel'stein (2002)
Anissimova, Kravchenko, Punnoose, Finkel'stein & Klapwijk (2007)
also Pudalov's group & Burmistrov and Chtchelkatchev

Further Continuation

non-linear σ– model with *e-e* interactions

A. Finkel'stein (83)

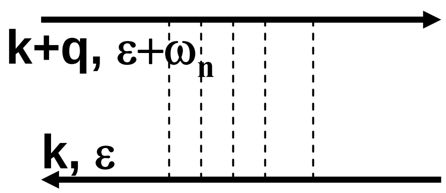
$$S[Q]=\frac{\pi\nu}{4}\int \mathrm{Tr}\left[D\left(\vec{\nabla}Q\right)^2-4z\mathrm{Tr}\left(\hat{\varepsilon}Q\right)+Q(\hat{\Gamma}_s+\hat{\Gamma}_t+\hat{\Gamma}_c)Q\right]d^2r$$

$$Q_{nm}^{\alpha\beta\ ij} \qquad TrQ=0 \qquad Q^2=I$$

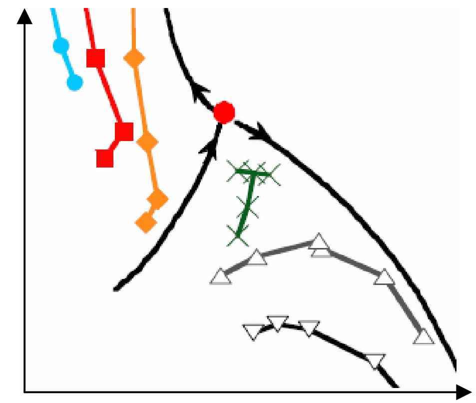
one more RG-equation: $$\frac{d\ln z}{d\xi}=\varsigma(t,\Theta)$$

The parameter z describes
the renormalization of the DOS of the diffusion modes

Physical consequence of the existence of the fixed point: thermodynamics at MIT have a critical behavior



$$\approx \frac{1}{Dq^2 + z\,\omega_n}$$



$$\frac{d \ln z}{d\xi} = \varsigma(t,\Theta)$$

$$z = 1/T^{\varsigma_c}$$

At the transition:

$$\varsigma_c = \varsigma(t_c,\Theta_c) \approx 1/4$$

The parameter z corresponds to the effective DOS of the diffusion modes

Specific heat: $C_v \sim (z\nu)T \sim T^{1-\varsigma_c}$

Spin susceptibility: $\chi/\chi_0 = z(1+\gamma_2) \sim 1/T^{\varsigma_c}$

Disordered Fermi-Liquid : leading order in $(1/\varepsilon_F\tau) << 1$

$$\tau^{-1} \equiv \tau_{elast}^{-1}$$

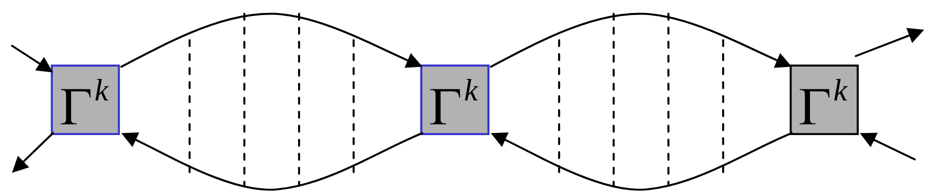
$\tau^{-1} >> \varepsilon$ (plane waves are not eigenstates anymore)

multiple rescattering of pairs of quasiparticles (each time two into two) should be modified by **diffusion**:

$$\left(\frac{\omega}{\omega - \vec{v}\,\vec{k}}\right)_{clean} \Rightarrow \left(\frac{\omega_n}{\omega_n + Dk^2}\right)_{disorder}$$

Static amplitudes remain unchanged in the leading order $(1/\varepsilon_F\tau) << 1$

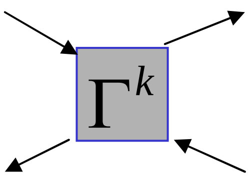
dynamic amplitude of scattering in the particle-hole channel; only $l = 0$ harmonic is relevant



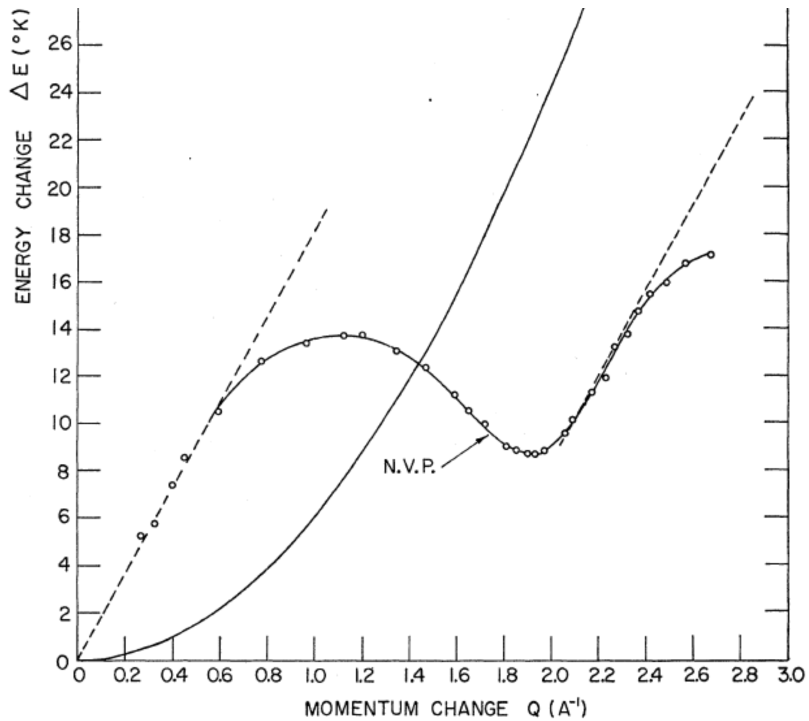
$\Rightarrow$

$$\Rightarrow \Gamma^k \frac{\omega_n}{\omega_n + Dk^2} \Gamma^k \frac{\omega_n}{\omega_n + Dk^2} \Gamma^k$$

$$g^*/g_0 = \frac{1}{1+F_0^\sigma} \approx 1.5 \qquad m^*/m \approx 3$$



$$E_{\mathbf{kin}} >> 1/\tau_{tr}$$



The dispersion curve in liquid He obtained by inelastic neutron scattering, 1961.
(The roton minimum as an illustration of enhancement of the effective mass in the electron liquid with large r_s.)
The enhancement is due to trapping of an electron by its neighbors. Although the mode softens, the energy of the internal motion increases.

Conclusions

➡ **The interplay of interactions and disorder fundamentally revises the common belief that 2D electron systems become insulating at low enough temperatures;**

➡ ***theory:*** **Using a large-N approximation scheme (valleys), we obtained a two-parameter scaling theory that exhibits a metal-insulator transition in 2d. The transition between the metallic and insulating phases is controlled by a finite-resistance unstable fixed point.**
The theory is internally consistent: there are no divergences in the interaction amplitude at a finite temperature.

➡ ***experiment:*** **Both the resistance and the effective interactions are scale (temperature) dependent. Not only resistance but also the interaction amplitude exhibit a fan-like spread as the MIT is crossed. Resistance-interaction flow diagram clearly reveals a quantum critical point.**

➡ **The metallic side of the diagram is accurately described by the theory.**

my personal opinion: **dilute electron liquid in Si-MOSFETs has been predestined to check the RG-theory in the disordered Fermi liquid.**

third most important slide

apparent:

1 clearly seen or understood: *It was apparent for all of us*

2 appearing but not necessary true or genuine: *the apparent cause but not the real one*

A.S. Hornby, Oxford Student's Dictionary of Current **Physics**