# Low frequency dynamics of quantum spins in the absence of dissipation. Application to the flux noise in superconducting devices.

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1. Introduction: a general theoretical question of entanglement.
2. Puzzle of low frequency noise in superconducting devices.
3. Dynamics of 2D RKKY quantum spin system at $T=\infty$ (model & “solution”)
4. Conclusions

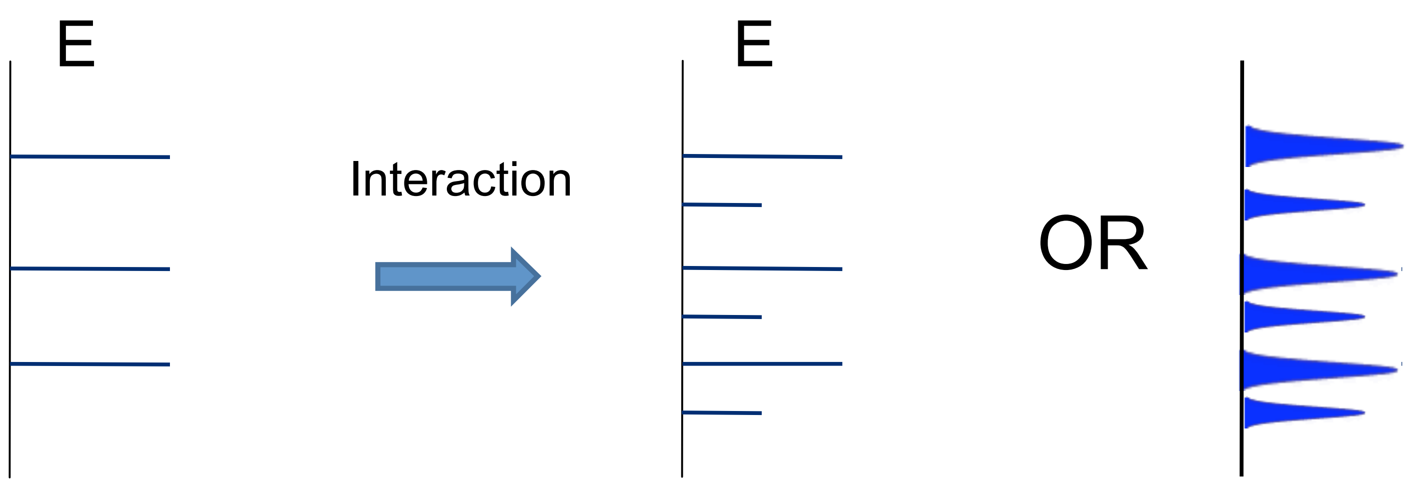
*Collaborators: L. Faoro, A. Kitaev.*

# Main question

Localized modes + interaction between them

Energy delocalization

Local spectrum



# Heisenberg magnet

$$H = \sum_{ij} J_{ij} S_i S_j \qquad S = 1/2 \qquad T = \infty$$

What is dynamics (e.g. spin correlator )?

'Naïve' answer: total magnetization is conserved, so magnetization density

$$\frac{dm}{dt} = D\nabla^2 m \qquad D \approx Ja^2$$

Magnetization and energy diffusion.
Generally accepted as truth without a proof.
Theoretical problem: dissipation and line broadening does not appear in any finite order of the perturbation theory → infinite resummation needed

*Most fundamental unsolved problem in condensed matter physics. (~1980)*

A.I. Larkin

# Heisenberg magnet

$$H = \sum_{ij} J_{ij} S_i S_j \qquad S = 1/2 \qquad T = \infty$$

What is dynamics (e.g. spin correlator )?

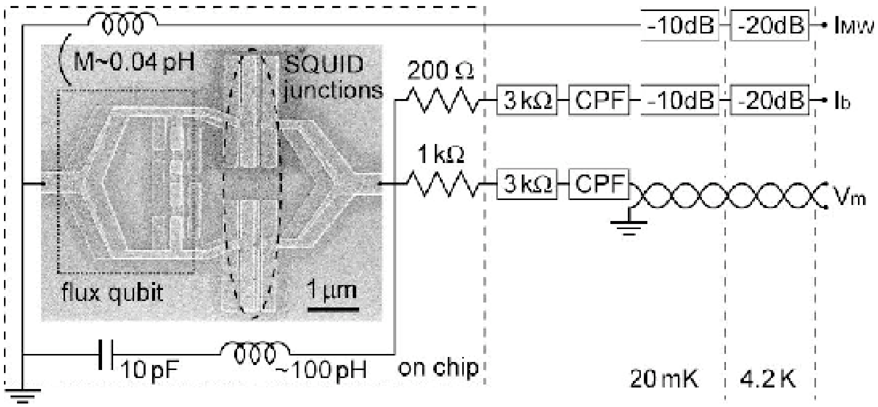
'Naïve' answer (which is very likely to describe the bulk of the spectrum):

$$\langle s(t)s(0) \rangle_{\omega,q} = \frac{1}{i\omega + Dq^2}$$

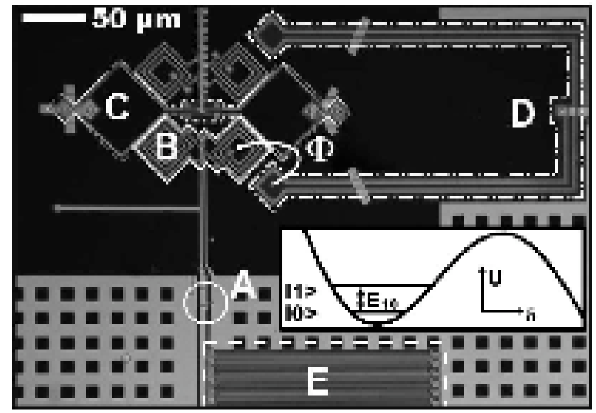
Implies that spin noise has no high frequencies (no $\omega > D/a^2$) and no low frequencies ($\omega < D/L^2$)

This work:
- Large Z (number of neighbors) → dissipative dynamics, dephasing
- RKKY interaction in realistic model → very low frequency noise (as well as any other broad distribution of couplings
- High frequency asymptotic (and thus low frequency in RKKY system) of the spin-spin correlator can be computed exactly.

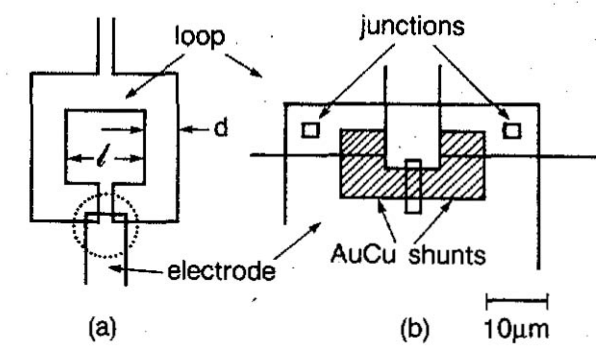
# Important Application: flux noise in small superconducting devices (qubits)




Flux qubit (Nakamura). Area: 1 μm$^2$




Phase qubit (McDermott, Martinis). Area: 100 μm$^2$




Conventional SQUID loop. Area: $10^4$-$10^6$ μm$^2$
Wellstood 1987

# Flux noise: summary of data

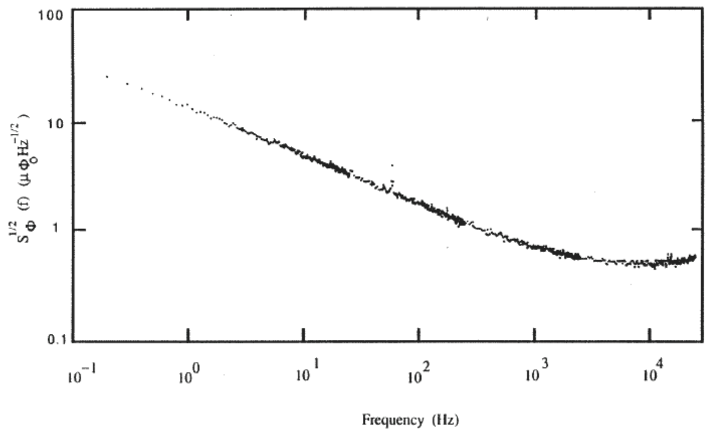



Fig. 8.5 Equivalent Flux noise spectrum from SQUID M2 (5-8-88): at T = 25 mK, V = 2 μV, Φ = 0.25.

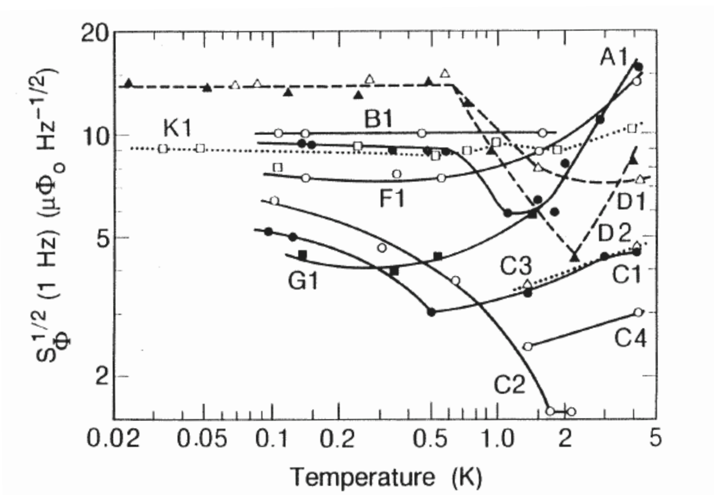



Fig. 8.7 Excess equivalent rms flux noise at 1 Hz vs. temperature for 11 SQUIDs. All devices are biased near 0.25 or 0.75 $\Phi_0$.

T and Area noise independence

**Main conclusions from the data:**

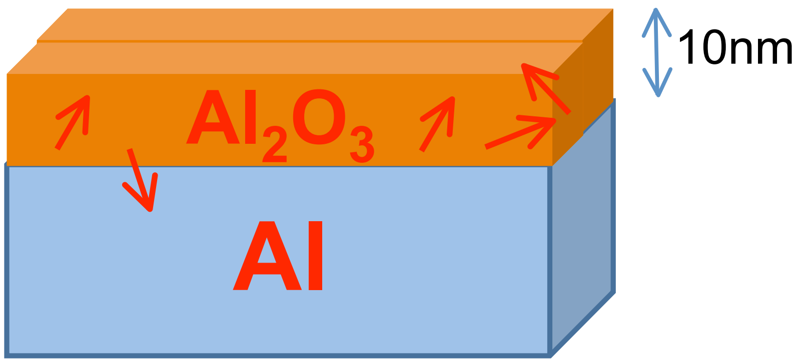
1. Area independence of the flux noise
2. Roughly 1/f frequency dependence
3. "Universal" magnitude: $S_\Phi \approx A/f$ with $A \approx 1\text{-}5\ (\mu\Phi_0)^2$
4. Temperature independence to low T 20mK.
5. Noise extends from mHz to 10 MHz (indirect qubit dephasing data by Nakamura).

**Microscopic origin:**

1. Area independence → spins on the surface: each spin gives $L^2/L^3=1/L$ flux, $L^2$ spins area independent flux noise.
2. Large energy scales ~10 MHz exclude nuclear spins (dipole-dipole interaction only ~1 kHz)
3. Temperature independence down to 20mK → spin-spin interaction
   10 MHz < J <200 MHz
4. Value of A → Area spin density $n \sim 100\ nm^{-2}$

# Microscopic model

- Spins in insulator $\rightarrow$ dipole-dipole interaction $\rightarrow$ J<0.1 MHz for n=100 $nm^2$
- Need spins with stronger interaction.
- Natural candidate – spins on the surface of superconductor




RKKY interaction between spins

in a close proximty of the superconductor:

$$J = \frac{\sqrt{2}}{\pi \nu r^3} \left( \frac{1}{\ln(v_F / r T_K)} \right)^2 \exp(-2r/\xi) \cos\phi$$

For n~100 $nm^2$ and $T_K \sim 1$ mK $J \sim 10 - 100$ MHz $(r \ll \xi)$

# Recent independent experimental evidence for electron spin origin

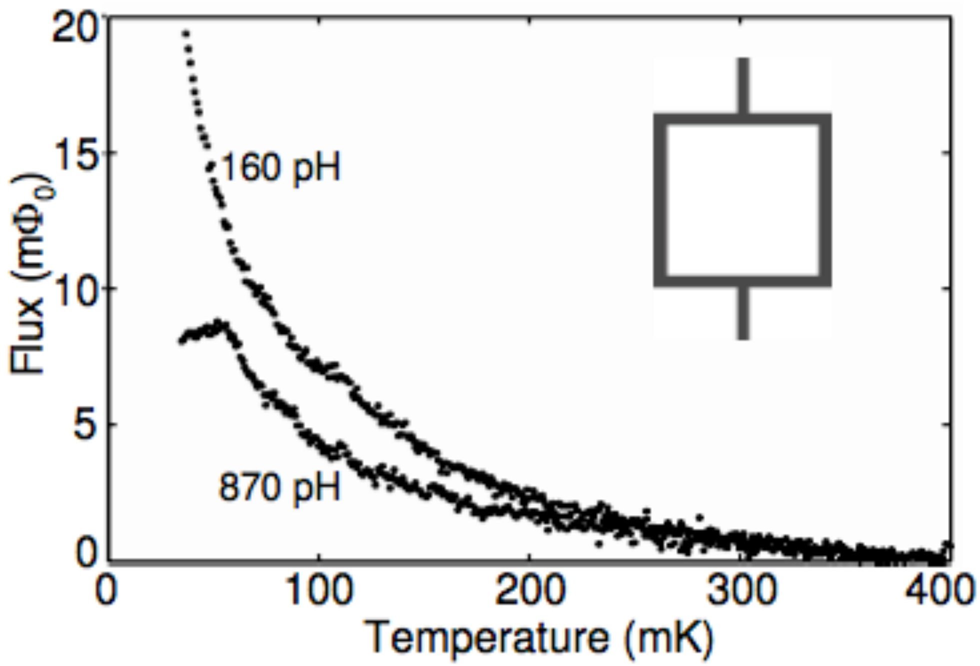



FIG. 4: Temperature dependence of the flux threading narrow (2 $\mu$m) linewidth Nb/AlOx/Nb SQUIDs with inductances 160 pH and 870 pH. Device geometry is shown in the inset.

McDermott et al

1. Observation of a small Curie susceptibility of the right order of magnitude for surface electrons spins.
2. In some sample(s) the susceptibility has a cusp indicating spin freezing at lowest temperatures.

# Microscopic model: summary

Spins on a 2D plane with RKKY interaction:

$$H = \frac{1}{2}\sum_{ij} J_{ij} S_i S_j \qquad J_{ij} = \frac{J_0 \cos\phi}{(ar_{ij})^3} \qquad na^2 = 1$$

with a non-dissipative dynamics

Questions:

•Assume that average dynamics is indeed spin diffusion, $\langle s(t)s(0)\rangle_{\omega,q} = n\frac{Dq^2}{\omega^2 + (Dq^2)^2}$ , what type of noise spectrum one gets?

•Minimal frequency corresponding to spin diffusion is $f_{min} = J_0(a/W)^2$, $f_{min} \sim 1kHz$ for small (W~1 μm) devices. What is spectrum at $f \ll f_{min}$?

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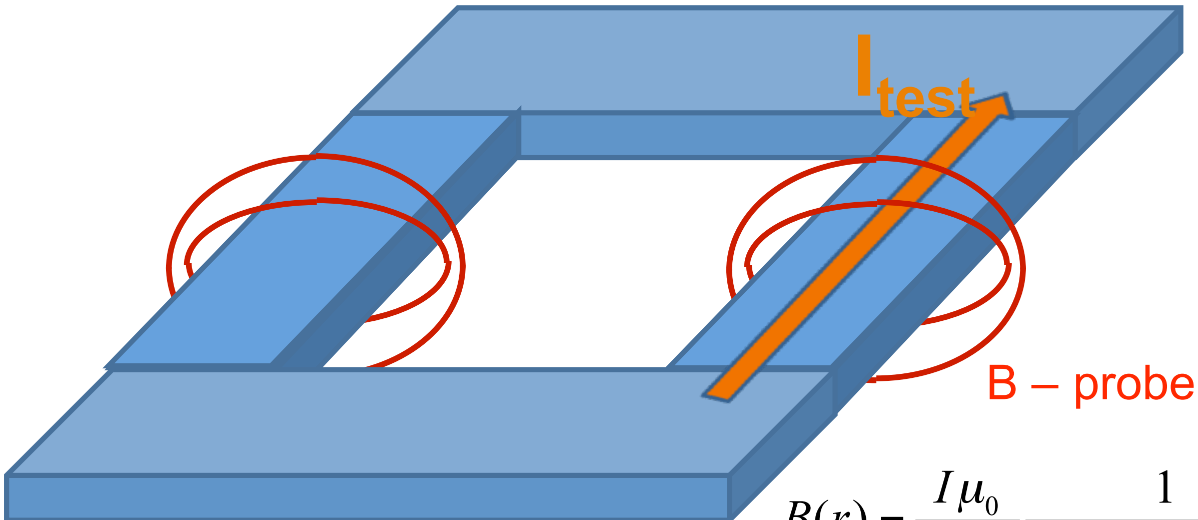
*1/f* noise for $f_{min} \ll f \ll J_0$ due to field form-factor.

•Minimal frequency corresponding to spin diffusion is $f_{min} = J_0(a/W)^2$, $f_{min} \sim 1kHz$ for small (W~1 μm) devices. What is spectrum at $f \ll f_{min}$?

Also *1/f* due to close pairs of spins that are strongly coupled to each other.

# Noise spectrum in diffusive regime

What is the effective flux produced by the spins at the superconducting surface when this flux is measured by the current loop formed by the same superconductor?




The flux is measured by the supercurrent that it induces:
$\Phi_{eff} = dE/dI_{test}$
→ We need current-spin coupling
→ Compute effective field produced by test current.

B – probe magnetic field: $H(I)=-g\mu_B B_I(r) S_j$

$$B(r) = \frac{I\mu_0}{2\pi W}\frac{1}{\sqrt{1-(2x/W)^2}}$$ - due to edge singularity

spin diffusion changes the effective flux: $B(q) = \sqrt{\frac{\pi}{2}}\frac{I\mu_0}{q}$

By coincidence (?), the spin diffusion gives 1/f dependence of the flux noise!

$$\left\langle \Phi^2 \right\rangle_\omega = \sqrt{\frac{2}{\pi}}\sum_q B^2(q)\frac{Dq^2}{\omega^2+(Dq^2)^2} \sim \left(\frac{R}{W}\right)\frac{n_s(\mu_0\mu_B)^2}{\omega}$$

# Low frequency noise

$$\text{At } \omega \ll D/L^2 \quad \left\langle \Phi^2 \right\rangle_\omega = \sqrt{\frac{2}{\pi}} \sum_q B^2(q) \frac{Dq^2}{\omega^2 + (Dq^2)^2} \sim const$$

Diffusion approximation neglects the rare pairs of spins at distances smaller than *a:*

$$H = J_{ij} S_i S_j - h_i S_i - h_j S_i \qquad J_{ij} = \frac{J_0 \cos\phi}{(ar_{ij})^3} \gg J_0 \sim h$$

Transition between singlet and triplet states of the spin pair is due to high-frequency tail of the effective field spectrum: $\Gamma = \langle h(t)h(0) \rangle_{\omega=J}$

We need high frequency asymptotic of the spin-spin correlator in the absence of close pairs (close pairs are rare, they do not interact)

# High frequency noise in a system of quantum spins.

$S(\omega\to\infty) = \int <S(0)S(t)> e^{i\omega t}dt$ is determined by the position of the closest singularity in the complex plane of t.

$\to$ Need asymptotic of large orders ($k\to\infty$) in Taylor expansion $<S(0)S(t)>=\sum s_k t^k$

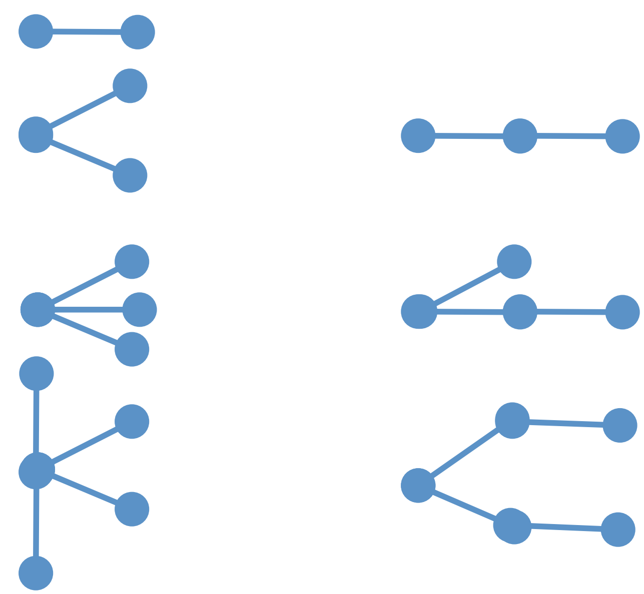
$$\frac{\partial S_i}{\partial t} = h_i \times S_i \quad h_i = \sum_j J_{ij} S_j$$

$$\frac{\partial^2 S_i}{\partial t^2} = h_i \times h_i \times S_i + \frac{\partial h_i}{\partial t} \times S_i$$

$$\frac{\partial^3 S_i}{\partial t^3} = h_i \times h_i \times h_i \times S_i + h_i \times \frac{\partial h_i}{\partial t} \times S_i + \ldots$$

$$\frac{\partial^4 S_i}{\partial t^4} = h_i \times h_i \times h_i \times h_i \times S_i + 2\frac{\partial h_i}{\partial t} \times \frac{\partial h_i}{\partial t} \times S_i + \ldots$$

Diagrammatically:



# High frequency noise in a system of quantum spins.

$S(\omega\to\infty) = \int <S(0)S(t)>e^{i\omega t}dt$ is determined by the position of the closest singularity in the complex plane of t.

→ Need asymptotic of large orders (k→∞) in Taylor expansion $<S(0)S(t)>=\sum s_k t^k$

$$\frac{\partial S_i}{\partial t} = h_i \times S_i \quad h_i = \sum_j J_{ij} S_j$$

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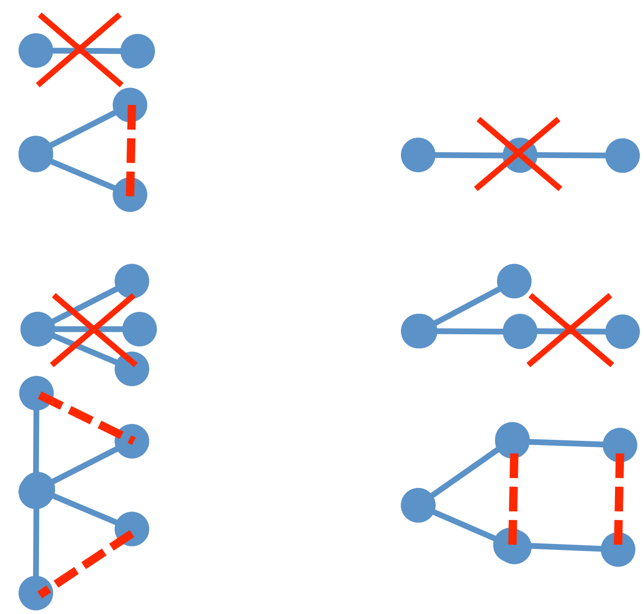
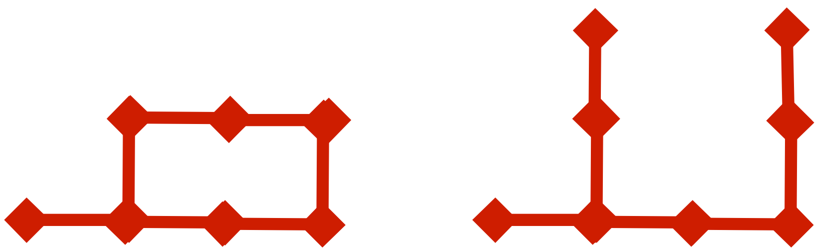
Averaging over J and taking trace over spins:



# Leading diagrams in higher orders

All lines are doubled

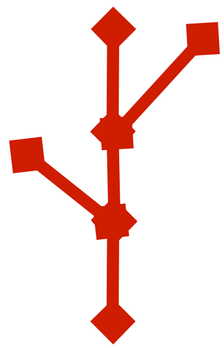


In the same order in $t^k$ loop diagrams contain less summations and thus are subdominant : they cannot change the asymptotic behavior of the coefficients in high orders.

→ To obtain high frequency behavior it is sufficient to count only tree-like diagrams, i.e. solve the problem on Caylee tree.

# Leading diagrams in higher orders

**What gives the dominant contribution in high orders?**



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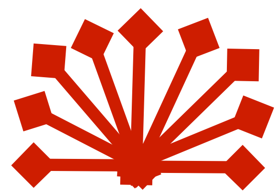

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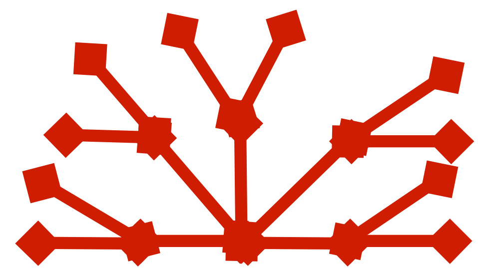


Trees?
Non-linearity due to a chain of spins is dominant.

Bushes?
Non-linearity of a single spin is the most important.

Shrubs?
Optimal diagram structure is intermediate

Larger number of pairings in bush-like diagrams

Larger combinatorial factor makes shrubs dominant.

# How to sum shrubs?

$$D^n = \left\langle S \frac{\partial^n S}{\partial t^n} \right\rangle = \sum_{k_1,k_2...k_m} C_{k_1,k_2...k_m} \left\langle S\ h^{k_1-1} \times (h^{k_2-1} \times (h^{k_3-1} \times (... \times S))) \right\rangle \qquad \sum k = n \quad h^k = \frac{d^k h}{dt^k}$$

Ignoring the non-commutativity of the vector product of different fields would make everything much simpler. The combinatorial coefficient is does not depend on the order of $k_1$, $k_2$...

$$C_{k_1,k_2...k_m} = \frac{n!}{\prod k_i!} \frac{1}{m!}$$

Averaging over the fields is also straightforward
Gaussian average:

$$\left\langle h^a h^b \right\rangle = (-)^a (J^2 Z)^{(a+b)/2} D^{a+b}$$

Finally:

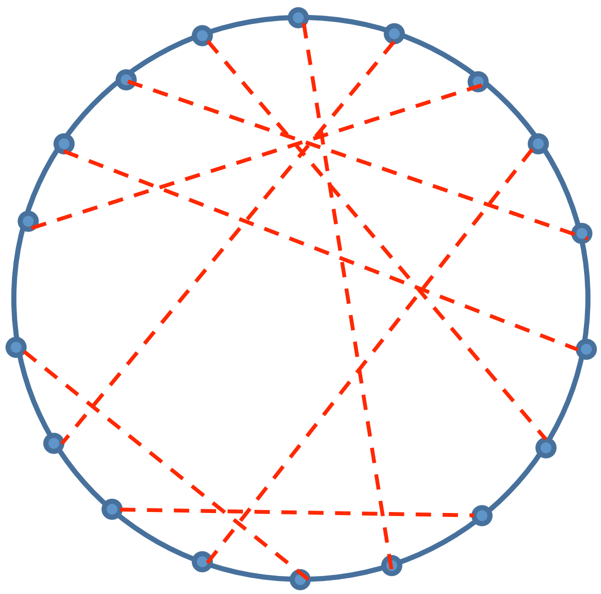
$$\frac{1}{n!} D^{(n)} = \frac{2}{3} \sum_{q_1..q_m} (2g)^m \frac{(2m+1)!!}{(2m)!} \left( \frac{1}{q_1!} D^{(q_1)} \right) .. \left( \frac{1}{q_m!} D^{(q_m)} \right)$$

# Simplification of high orders

Need to compute average

$$\left\langle S\ h^{k_1-1}\times(h^{k_2-1}\times(h^{k_3-1}\times(\ldots\times S)))\right\rangle = \left\langle \mathrm{Tr}\ \hat{h}^{k_1-1}\hat{h}^{k_2-1}\hat{h}^{k_3-1}\ldots\right\rangle \quad \hat{h} = \varepsilon_{\alpha\beta\gamma}h_\gamma$$

Typical high order term has many cross averages so that only one sum over vector indices remains free. The coefficients are thus the same as for the single Gaussian field.



Comparing the 'commutative' approximation with the direct computation. They are equivalent in all low orders and in all terms that contain only single type of the correlator on the right. First difference appears in the sixth order:

$$D^{(6)} = 2gD^{(4)} + 50g^2D^{(2)}D^{(0)} + 70g^3(D^{(0)})^3$$

$$D^{(6)} = 2gD^{(4)} + 48g^2D^{(2)}D^{(0)} + 70g^3(D^{(0)})^3$$

Conclusion: neglect of vector stucture is a very good approximation.

# Solution for D(it)

$$\frac{1}{n!}D^{(n)} = \frac{2}{3}\sum_{q_1..q_m}(2g)^m \frac{(2m+1)!!}{(2m)!}\left(\frac{1}{q_1!}D^{(q_1-2)}\right)..\left(\frac{1}{q_m!}D^{(q_m-2)}\right)$$

Introduce generating function $F(t) = \sum_{q\geq 2}\frac{1}{q!}D^{(q-2)}t^q \quad (F'' = D)$

$$F''(t) = \frac{2}{3}\sum_m \frac{2m+1}{m!}(gF)^m = \frac{4gF+2}{3}e^{gF}$$

Solution has a simple pole at $t=0.3951\ g^{-1/2}$

High frequency asymptotic of the spin-spin correlation function:

$\langle S^2 \rangle_\omega = \exp(-0.3951\ g^{-1/2}\ \omega)$

# Consequencies for the low frequency spectrum

Exponential dependence $\langle S^2 \rangle_\omega = \exp(-0.3951\ g^{-1/2}\ \omega)$ implies that transition rate of pairs coupled with $J \gg J_0$ is exponentially small.

Probability to find such pair

$$P(J)\, dJ = (J_0/J)^{5/3}\, dJ/J_0$$

The number of fluctuators with rate $\Gamma$:

$$P(\Gamma)d\Gamma = \frac{1}{\ln^{5/3}(J_0/\Gamma)}\frac{d\Gamma}{\Gamma}$$

# Conclusions

1.Localized spins on the surface of superconductor provide a 'clean' example of the quantum system of localized interacting modes that form a highly entangled state with a self-induced decoherence.
2.The low frequency dynamics of these spins is due to the close pairs that change rarely their state from singlet to triplet.
3.The rate of this process is controlled by the high frequency end of the spectrum of the spins surrounding a given pair.
4.The high frequency asymptotic of the spin-spin correlation function can be computed exactly.
5.The exponential dependence of the high frequency end results in the 1/f rate of the close pair transitions.
6.This physics is very likely responsible for the low frequency noise that plagues SQUIDs and superconducting qubits.