L.D. Landau Memorial Conference
"Advances in Theoretical Physics"

The problem of initial conditions in cosmology

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Based on:

A.O. Barvinsky and A.Yu. Kamenshchik,
Cosmological landscape from nothing: Some like it hot, JCAP 0609:014, 2006;
Thermodynamics via Creation from Nothing: Limiting the Cosmological Constant Landscape, Phys.Rev.D74:121502(R), 2006.

A.O. Barvinsky,
Why there is something rather than nothing (out of everything)?, Phys.Rev. Lett. 99:071301, 2007.

A.O. Barvinsky, C. Deffayet and A.Yu. Kamenshchik,
Anomaly Driven Cosmology: Big Boost Scenario and AdS/CFT Correspondence, JCAP 0805: 020, 2008.

Instead of Introduction

"The future physical theory should contain not only the basic equations but also the initial conditions for them".

L.D. Landau according to I.M. Khalatnikov.

The quantum cosmology predicts the initial conditions for the cosmological evolution of the universe, defining its quantum state - the wave function of the universe. The connection between the Euclidean quantum theory and the quantum tunneling is used. We generalize the traditional scheme.

Goals

1. Description of the birth of the universe from nothing in a mixed state. Density matrix instead of the wave function of the universe.
2. The predicition of initial conditions for the cosmological evolution. Cosmological landscape.
3. The elimination of the “infrared catastrophe” in the Hartle-Hawking prescription.
4. Connections with the string theory.

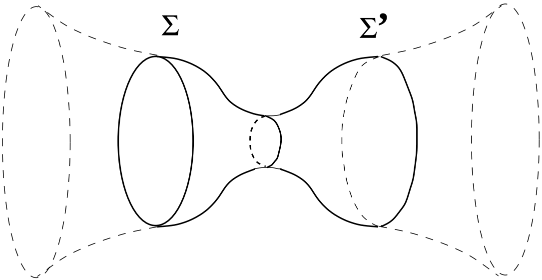
Tools

1. Classical cosmology.
2. Quantum theory of tunneling : Euclidean quantum gravity.
3. Quantum field theory: renormalization in curved spacetime; effective action formalism.
4. Taking into account of non-local quantum effects due to the presence of quantum fields (radiation).

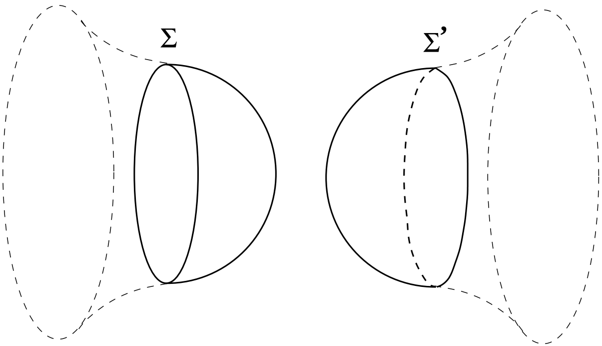
Results

1. The system of self-consistency equations describing the quantum birth of the universe is derived: the generalized Friedmann equation and the “bootstrap” equation.
2. The solution of these equations gives the families of acceptable parameters, characterizing initial conditions for cosmological evolution - “cosmological landscape”.
3. The problem of the infinite initial radius of the universe in the Hartle-Hawking prescription is resolved.
4. Scenarios of the future development of the Universe are studied.

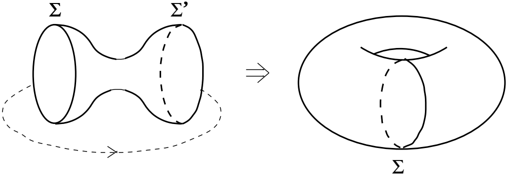
Basic Ideas



The presence of radiation implies a statistical ensemble, rather than a pure state. Density matrix in Euclidean quantum gravity (Page, 1986) originates from an instanton with two disjoint boundaries.



For the pure quantum state (Hartle,Hawking, 1983) the instanton bridge between Σ and Σ' breaks down. The radiation stress tensor prevents these half instantons from closure.



The partition function follows from integrating out the field φ in the coincidence limit $\varphi' = \varphi$ which corresponds to the identification of Σ' and Σ, so that the underlying instanton acquires toroidal topology.

Radiation and instantons

$$ds^2 = N^2(\tau)\, d\tau^2 + a^2(\tau)\, d^2\Omega^{(3)},$$

$$\frac{\dot{a}^2}{a^2} = \frac{1}{a^2} - H^2 - \frac{C}{a^4},$$

$$a(\tau) = \frac{1}{\sqrt{2}H}\sqrt{1 - (1 - 4CH^2)^{1/2}\cos 2H\tau}.$$

$$a_\pm = \frac{1}{\sqrt{2}H}\sqrt{1 \pm (1 - 4CH^2)^{1/2}}, \quad 4H^2C \leq 1$$

Density matrix

$$\rho[\varphi,\varphi'] = e^{\Gamma} \int\limits_{g,\phi|_{\Sigma,\Sigma'}=(\varphi,\varphi')} D[g,\phi]\,\exp\big(-S_{\rm E}[g,\phi]\big).$$

Γ is the Euclidean effective action.

$$e^{-\Gamma} = \int\limits_{g,\phi|_{\Sigma}=g,\phi|_{\Sigma'}} D[g,\phi]\,\exp\big(-S_{\rm E}[g,\phi]\big)$$

$$\Gamma[g] = S_{\rm E}[g] + \Gamma_{1-{\rm loop}}[g].$$

Conformal anomaly and ghosts

We consider conformally-invariant fields.

$$ds^2 = a^2(\eta)(d\eta^2 + d^2\Omega^{(3)}),$$

$$d\bar{s}^2 = d\eta^2 + d^2\Omega^{(3)}.$$

Euclidean Einstein static universe.

Quantum effective action has a conformal anomaly.

$$g_{\mu\nu}\frac{\delta\Gamma_{1-\text{loop}}}{\delta g_{\mu\nu}} = \frac{1}{4(4\pi)^2}g^{1/2}\left(\alpha\Box R + \beta E + \gamma C^2_{\mu\nu\alpha\beta}\right),$$

$$E = R^2_{\mu\nu\alpha\gamma} - 4R^2_{\mu\nu} + R^2.$$

This anomaly, when integrated functionally along the orbit of the conformal group, gives the relation between the actions on conformally related backgrounds

$$\Gamma_{1-\mathrm{loop}}[\,g\,] = \Gamma_{1-\mathrm{loop}}[\,\bar{g}\,] + \Delta\Gamma[\,g, \bar{g}\,],$$
$$g_{\mu\nu}(x) = e^{\sigma(x)}\bar{g}_{\mu\nu}(x),$$

where

$$\Delta\Gamma[\,g, \bar{g}\,]$$
$$= \frac{1}{2(4\pi)^2}\int d^4x\bar{g}^{1/2}\left\{\frac{1}{2}\left[\gamma\,\bar{C}^2_{\mu\nu\alpha\beta} + \beta\left(\bar{E} - \frac{2}{3}\bar{\Box}\bar{R}\right)\right]\sigma\right.$$
$$\left. + \frac{\beta}{2}\left[(\bar{\Box}\sigma)^2 + \frac{2}{3}\bar{R}\,(\bar{\nabla}_\mu\sigma)^2\right]\right\}$$
$$-\frac{1}{2(4\pi)^2}\left(\frac{\alpha}{12} + \frac{\beta}{18}\right)\int d^4x\left(g^{1/2}R^2(g) - \bar{g}^{1/2}R^2(\bar{g})\right)$$

(Barvinsky,Mirzabekian,Zhytnikov,1995)

Higher-derivative terms are all proportional to the coefficient α. The α- term can be arbitrarily changed by adding a local counterterm $\sim g^{1/2}R^2$. We fix this local renormalization ambiguity by an additional criterion of the absence of ghosts. The conformal contribution to the renormalized action on the minisuperspace background equals

$$\Delta\Gamma[\,g,\bar{g}\,] \equiv \Gamma_R[\,g\,] - \Gamma_R[\,\bar{g}\,]$$
$$= m_P^2\,B\int d\tau\left(\frac{\dot{a}^2}{a} - \frac{1}{6}\,\frac{\dot{a}^4}{a}\right),$$
$$m_P^2\,B = \frac{3}{4}\,\beta,$$

with the constant $m_P^2\,B$ which for scalars, two-component spinors and vectors equals respectively $1/240$, $11/480$ and $31/120$.

Effective action on a static Einstein instanton

For a conformal scalar field

$$S[\bar{g},\phi] = \frac{1}{2}\sum_n \int_0^{\eta_0} d\eta \left(\left(\frac{d\phi_n}{d\eta}\right)^2 + \omega_n^2\,\phi_n^2 \right),$$

$$\begin{aligned} & e^{-\Gamma_{1-\mathrm{loop}}[\bar{g}]} \\ &= \int \prod_n d\varphi_n \int\limits_{\phi_n(\eta_0)=\phi_n(0)=\varphi_n} D[\phi]\,\exp\big(-S[\bar{g},\phi]\big) \\ &= \mathrm{const}\,\prod_n \left(\sinh\frac{\omega_n\eta_0}{2}\right)^{-1}. \end{aligned}$$

The effective action equals the sum of contributions of the vacuum energy E_0 and free energy $F(\eta_0)$,

$$\varGamma_{1-\mathrm{loop}}[\,\bar{g}\,] = \sum_n \Big[\,\eta_0\,\frac{\omega_n}{2} + \ln\big(1 - e^{-\omega_n \eta_0}\big)\,\Big]$$

$$= m_P^2\,E_0\,\eta_0 + F(\eta_0),$$

$$m_P^2\,E_0 = \sum_n \frac{\omega_n}{2} = \sum_{n=1}^{\infty} \frac{n^3}{2},$$

$$F(\eta_0) = \sum_n \ln\big(1 - e^{-\omega_n \eta_0}\big)$$

$$= \sum_{n=1}^{\infty} n^2\,\ln\big(1 - e^{-n\eta_0}\big).$$

The vacuum energy (an analog of the Casimir energy) on Einstein static spacetime is

$$m_P^2\, E_0 = \frac{1}{960} \times \begin{cases} 4 \\ 17 \\ 88 \end{cases}$$

respectively for scalar, spinor and vector fields.
We should take into account the effect of the finite ghost-avoidance renormalization:

$$\Gamma_R[\,\bar{g}\,] = m_P^2\, C_0\, \eta_0 + F(\eta_0),$$
$$m_P^2\, C_0 = m_P^2\, E_0 + \frac{3}{16}\,\alpha.$$

Direct observation: for all conformal fields of low spins

$$m_P^2\, C_0 = \frac{1}{2}\, m_P^2\, B.$$

Effective Friedmann and bootstrap equations

Full conformal time

$$\eta_0 = 2\int_{\tau_-}^{\tau_+} \frac{d\tau\, N(\tau)}{a(\tau)},$$

The effective action

$$\begin{aligned}
&\Gamma[\, a(\tau), N(\tau)\,] \\
&= 2m_P^2 \int_{\tau_-}^{\tau_+} d\tau \left(-\frac{a\dot{a}^2}{N} - Na + NH^2 a^3\right) \\
&+2Bm_P^2 \int_{\tau_-}^{\tau_+} d\tau \left(\frac{\dot{a}^2}{Na} - \frac{1}{6}\,\frac{\dot{a}^4}{N^3 a}\right) \\
&+F\left(2\int_{\tau_-}^{\tau_+} \frac{d\tau\, N}{a}\right) + Bm_P^2 \int_{\tau_-}^{\tau_+} \frac{d\tau\, N}{a}\,. \quad \text{nonlocal}
\end{aligned}$$

Effective Friedmann equation

$$\frac{\delta\Gamma}{\delta N} = 2m_P^2\left(\frac{a\dot{a}^2}{N^2} - a + H^2a^3\right)$$
$$+2Bm_P^2\left(-\frac{\dot{a}^2}{N^2a} + \frac{1}{2}\,\frac{\dot{a}^4}{N^4a}\right)$$
$$+\frac{2}{a}\left(\frac{dF(\eta_0)}{d\eta_0} + \frac{B}{2}m_P^2\right) = 0.$$

In the gauge $N = 1$:

$$\frac{\dot{a}^2}{a^2} + B\left(\frac{1}{2}\,\frac{\dot{a}^4}{a^4} - \frac{\dot{a}^2}{a^4}\right) = \frac{1}{a^2} - H^2 - \frac{C}{a^4},$$

where the amount of radiation constant C is given by the bootstrap equation

$$m_P^2C = m_P^2\frac{B}{2} + \frac{dF(\eta_0)}{d\eta_0} \equiv \frac{B}{2}m_P^2 + \sum_{n=1}^{\infty}\frac{n^3}{e^{n\eta_0} - 1}.$$

The on-shell effective action

$$\Gamma_0 = F(\eta_0) - \eta_0 \frac{dF(\eta_0)}{d\eta_0}$$
$$+4m_P^2 \int_{a_-}^{a_+} \frac{da\dot{a}}{a} \left(B - a^2 - \frac{B\dot{a}^2}{3} \right).$$

The Friedmann equation

$$\dot{a}^2 = \sqrt{\frac{(a^2 - B)^2}{B^2} + \frac{2H^2}{B}(a_+^2 - a^2)(a^2 - a_-^2)} - \frac{(a^2 - B)}{B}$$

has the same two turning points $a_\pm$ as in the classical case provided

$$a_-^2 \geq B.$$

This requirement is equivalent to

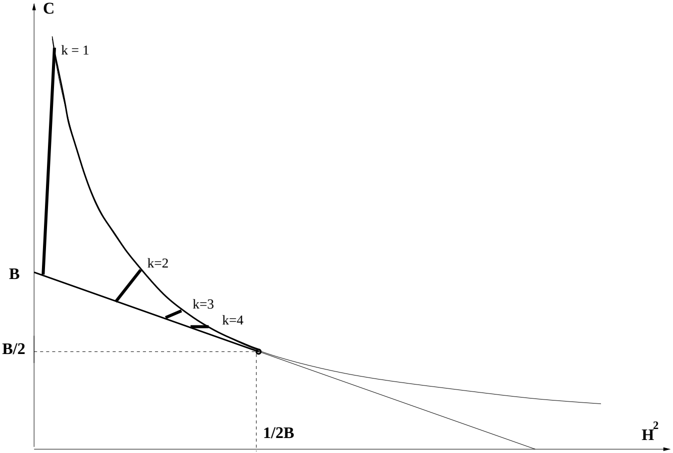
$$C \geq B - B^2 H^2, \quad BH^2 \leq \frac{1}{2}.$$

Together with

$$CH^2 \leq \frac{1}{4},$$

the admissible domain for instantons reduces to the curvilinear wedge below the hyperbola and above the straight line to the left of the critical point

$$C = \frac{B}{2}, \quad H^2 = \frac{1}{2B}.$$


C
k = 1
k=2
k=3
k=4
B
B/2
1/2B
H
2

Numerical analysis of the Friedmann and bootstrap equations shows that the one-parameter family of instantons interpolates between the point on the lower line boundary with parameters

$$H^2 \approx 2.997\, m_P^2, \;\; C \approx 0.004\, m_P^{-2}, \;\; \Gamma_0 \approx -0.1559,$$

and the point on the upper hyperbolic boundary

$$H^2 \approx 12.968\, m_P^2, \;\; C \approx 0.0193\, m_P^{-2}, \;\; \Gamma_0 \approx -0.0883.$$

The last instanton describes the creation of a static Einstein Universe of the constant size

$$a = a_+ = a_- = 1/(\sqrt{2}H)$$

with the hot gas of conformal invariant field in the equilibrium state with the temperature

$$T = \frac{1}{a\eta_0} = \frac{H}{\pi\sqrt{1 - 2BH^2}}.$$

The Infrared catastrophe is eliminated.

Outside of the admissible domain for the instantons with two turning points one can construct instantons with one turning point which close at $a = 0$. Such instantons correspond to the Hartle-Hawking pure quantum states. However, in this case

$$a \to 0$$

$$\eta_0 = \infty$$

$$\Gamma_0 \sim 4m_P^2 B \int_0^{a_+} \frac{da\dot{a}}{a}\left(1 - \frac{\dot{a}^2}{3}\right)$$

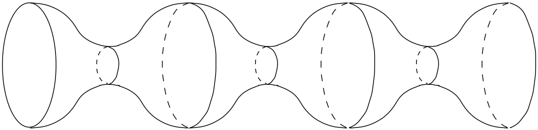
and diverges at the lower limit. Thus,

$$\Gamma_0 = +\infty, \quad \exp(-\Gamma_0) = 0$$

completely rules out all pure-state instantons.

Instanton garlands

Multiple instantons



$$\eta_0^{(k)} = 2k \int_{\tau_-}^{\tau_+} \frac{d\tau}{a} = 2k \int_{a_-}^{a_+} \frac{da}{a\dot{a}}.$$

Numerical analysis for $k = 2$ shows the existence of the one-parameter family of instantons similar to the case of $k = 1$. It interpolates between the point on the lower boundary of (C, H^2)-plane

$$H^2_{(2)} \approx 45.89\, m_P^2,\ \ C_{(2)} \approx 0.0034\, m_P^{-2},\ \ \Gamma_0^{(2)} \approx -0.0113,$$

and the point on the upper (hyperbolic) boundary

$$H^2_{(2)} \approx 61.12\, m_P^2,\ \ C_{(2)} \approx 0.0041\, m_P^{-2},\ \ \Gamma_0^{(2)} \approx -0.0145.$$

Such families exist for all $k, 1 \leq k \leq \infty,$ and their infinite sequence is saturated at the critical point.

$$\eta_0^{(k)} \simeq \ln k^2$$

$$H_{(k)}^2 \simeq \frac{1}{2B}\left(1 - \frac{\ln^2 k^2}{2k^2\pi^2}\right),$$

$$C_{(k)} \simeq \frac{B}{2}\left(1 + \frac{\ln^2 k^2}{2k^2\pi^2}\right),$$

$$\Gamma_0^{(k)} \simeq -m_P^2 B \, \frac{\ln^3 k^2}{4k^2\pi^2}.$$

The length of instanton families decreases as $\frac{1}{k^4}$. Infinite garlands ($k \to \infty$) do not dominate the instanton distribution.

Growing spin of a conformal particle decreases the instanton scale and makes its probability weight higher.

For N fields

$$C \to NC,$$
$$B \to NB,$$
$$\eta_0 \to \eta_0,$$
$$F(\eta_0) \to NF(\eta_0),$$
$$H^2 \to \frac{H^2}{N}.$$

The initial radius of the universe grows with the growing spin and number of fields.

Where Euclidean Quantum Gravity and Cosmology come from ?

The answer: from the Lorentzian quantum gravity.

The density matrix of the Universe for the microcanonical ensemble in Lorentzian quantum cosmology of spatially closed universes describes an equipartition in the physical phase space of the theory, but in terms of the observable spacetime geometry this ensemble is peaked about the set of cosmological instantons (solutions of the Euclidean quantum cosmology) limited to a bounded range of the cosmological constant.

Cosmological evolution

Modified Lorentzian Friedmann equation

$$\frac{\dot{a}^2}{a^2} + \frac{1}{a^2} = \frac{1}{B}\left\{1 - \sqrt{1 - 2B\left(H^2 + \frac{\mathcal{C}}{a^4}\right)}\right\},$$
$$\mathcal{C} \equiv C - \frac{B}{2}.$$

The Casimir energy is totally screened, only the termal radiation weigh.

Big Boost singularity

- To have a realistic value for an effective cosmological constant it is necessary to have many conformal fields.
- The mechanisms for growing number of the conformal fields exist in some string inspired cosmological models with extra dimensions.
- If this mechanism works and if the $B(a)$ grows faster than the rate of decrease of the energy density $\varepsilon(a)$ one encounters a new type of the cosmological singularity - Big Boost.

Big Boost singularity

$$
\begin{aligned}
a(t_{BB}) &= a_{BB} < \infty, \\
\dot{a}(t_{BB}) &= \dot{a}_{BB} < \infty, \\
\lim_{t \to t_{BB}} \ddot{a}(t) &= \infty.
\end{aligned}
$$

Other singularities

Big Bang (Crunch)

$$a(t_0) = 0.$$

Big Rip

$$\lim_{t \to t_{BR}} a(t) = \infty,$$
$$\lim_{t \to t_{BR}} \dot{a}(t) = \infty,$$
$$\lim_{t \to t_{BR}} \frac{\dot{a}(t)}{a(t)} = \infty.$$

Big Brake

$$a(t_{BBr}) = a_{BBr} < \infty,$$
$$\dot{a}(t_{BBr}) = 0,$$
$$\lim_{t \to t_{BBr}} \ddot{a}(t) = -\infty.$$

P.S. Initial conditions, singularities, density matrix and Landau's legacy

How all this is connected to the scientific legacy of L.D. Landau ?

1. Initial conditions problem pointed out by Landau.
2. New type of cosmological singularity.
3. Most important: the role of the density matrix in physics.

It was L.D. Landau who has introduced the notion of the density matrix in 1927 in
Das Dämpfungsproblem in der Wellenmechanics, Z. Physik, 45 (1927) 430
parallely with F. Bloch and J. von Neumann.
Landau was the youngest. He was 19 - year old.

In the presented series of works

- It was shown that the density matrix is a fundamental object.
- Apperence of the density matirx is not a result of our ignorance.
- The universe in the framework of quantum cosmology is born in a mixed and not in a pure quantum state.