L.D.Landay Memorial Conference “Advances in Theoretical Physics”, 22-26/06/08

From Classical to Quantum Integrability In the AdS/CFT correspondence

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AdS/CFT:

superstring on AdS_5xS^5 background **/** N=4 Super-Yang-Mills 4D theory
mutually dual

- N=4 SYM: Integrable conformal 4D gauge theory with calculable conformal dimensions (as functions of YM coupling, in planar limit).
- Metsaev-Tseytlin Superstring : integrable 2D sigma model
  Lax pair and finite gap eqs. – continous Bethe Ansatz eqs.
- Quantization of superstring (and N=4 SYM!): asymptotic Bethe Ansatz.
  SYM dilatation operator as integrable spin chain.
- Exact dimensions of “long” operators (for any YM coupling).
  Example: twist two operator at large spin $S \to \infty$.
- Main problem left: “short” operators and wrapping. Should be solved by Thermodynamical Bethe Ansatz

AdS/CFT Integrability

It looks to be a solvable (=integrable) theory,
at least for non-interacting strings (g_s=0),
or planar SU($N_c \to \infty$) N=4 SYM!

Many modern means of 2d integrability applied:
- Bethe ansatz
- Finite gap method
- Factorizable S-matrix in 2d,
- TBA, etc

Lipatov'00
Frolov,Tseytlin'02
Minahan,Zarembo'02
Bena,Polchinski,Roiban'02
Beisert,Kristjansen,Staudacher'03
Beisert,Staudacher'03
V.K.,Marshakov,Minahan,Zarembo'04
Staudacher'04
Arutyunov, Frolov,Staudacher'04
Beisert,V.K.,Sakai,Zarembo'05

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Beisert,Staudacher'05
Hernandez,Lopez'05
Beisert'05
Janik'05
Beisert,Hernandez,Lopez'05
Beisert,Eden,Staudacher'06

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N=4 Supersymmetric Yang-Mills Theory

Gliozzi,Scherk,Olive'77

- Conformal 4D gauge theory. Action:

$$S=\frac{1}{\lambda}\int d^4x\ \mathrm{tr}\left\{\frac{1}{4}F^2+\frac{1}{2}\left(\nabla\Phi\right)^2-\frac{1}{4}[\Phi,\Phi]^2\right.$$
$$\left.+\Psi\,\nabla\!\!\!/\,\Psi-\frac{1}{2}\Psi[\,\Phi\!\!\!\backslash,\Psi]-\frac{1}{2}\dot\Psi[\,\Phi\!\!\!\backslash,\dot\Psi]\right\}$$

Generators of global superconformal psu(2,2|4) symmetry:

R-symmetry so(6) (scalars)

Poincare

spec. conf.

Dilatation ("Energy"=Dim.)

$$\mathcal{J}=\{R^a_b,\underbrace{\overbrace{L^\alpha_\beta,\dot L^{\dot\alpha}_{\dot\beta},P_{\dot\alpha\beta}}^{\text{Poincare}},K^{\alpha\dot b},D}_{\text{so(2,4)}}|\underbrace{Q^a_{\dot\beta},\dot Q_{\dot a\beta}}_{\text{SUSY}},\underbrace{S^\alpha_b,\dot S^{\alpha\dot\beta}}_{\text{superconf.}}\}$$

so(2,4)

SUSY

superconf.

Operators and Dilatation Hamiltonian

- Local single trace operators form a representation of *psu*(2,2|4):

$$\mathcal{O}(x) = \text{tr}\ [\chi_1(x)\chi_2(x)\ldots\chi_L(x)]$$

where $\chi \in \{\mathcal{D}_{\dot\alpha\beta}, \Phi_{ab}, \Psi_{\alpha b}, \dot\Psi^{b}_{\dot\alpha} \quad \}$

- In general, the action of dilatation gives a mixing matrix:

$$\hat{D}\mathcal{O}_j(x) = \sum_j D_{ij}\mathcal{O}_j$$

- Operators-eigenvectors $\mathcal{O}^{(n)} = \sum_i C_i^{(n)}\mathcal{O}_i$ are characterized by dimensions

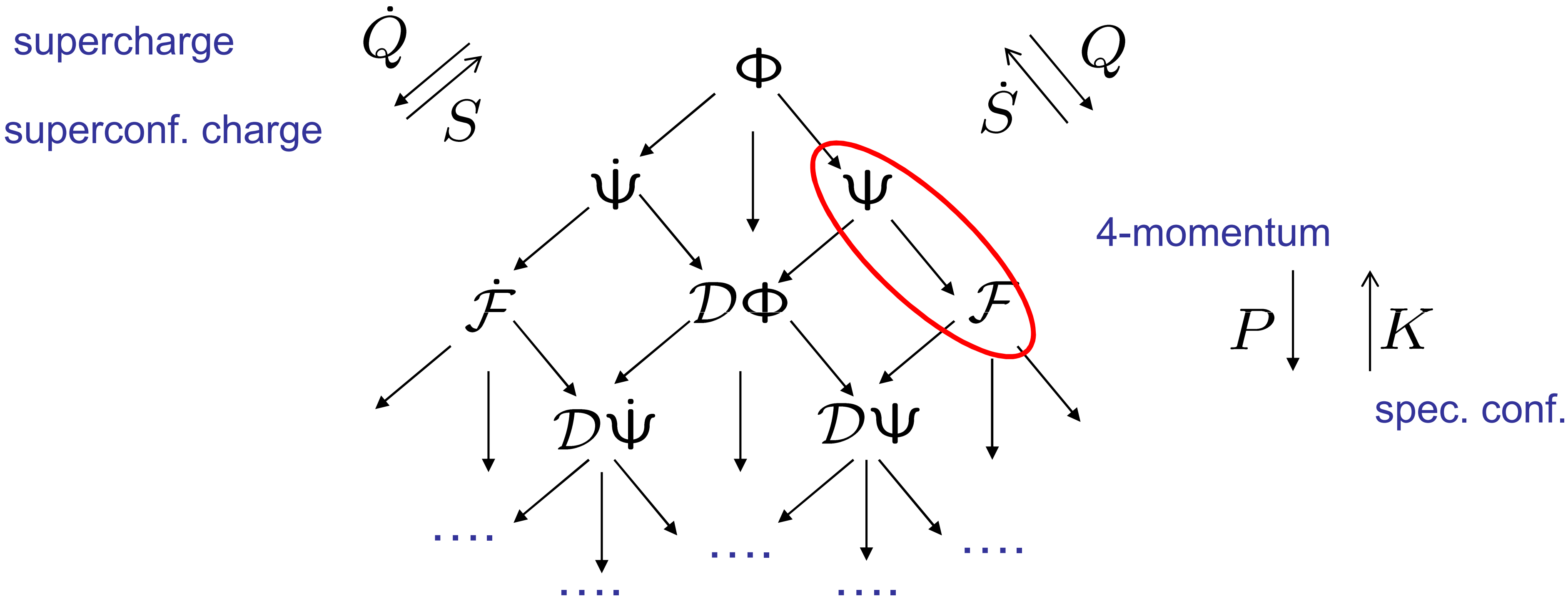
$$\langle \mathcal{O}^{(m)}(x)\mathcal{O}^{(n)}(0)\rangle = \frac{\delta_{mn}}{|x|^{2\Delta_n}}$$

Δ_n is an eigenvalue of dilatation operator $\hat{D} \subset$ *psu*(2,2|4)

It can be computed perturbatively. $\Delta(\lambda) = \Delta^{(0)} + \lambda\Delta^{(2)} + \lambda^2\Delta^{(4)} + \ldots$

Transformation of fields

SYM symmetric under global superconformal symmetry: PSU(2,2|4)



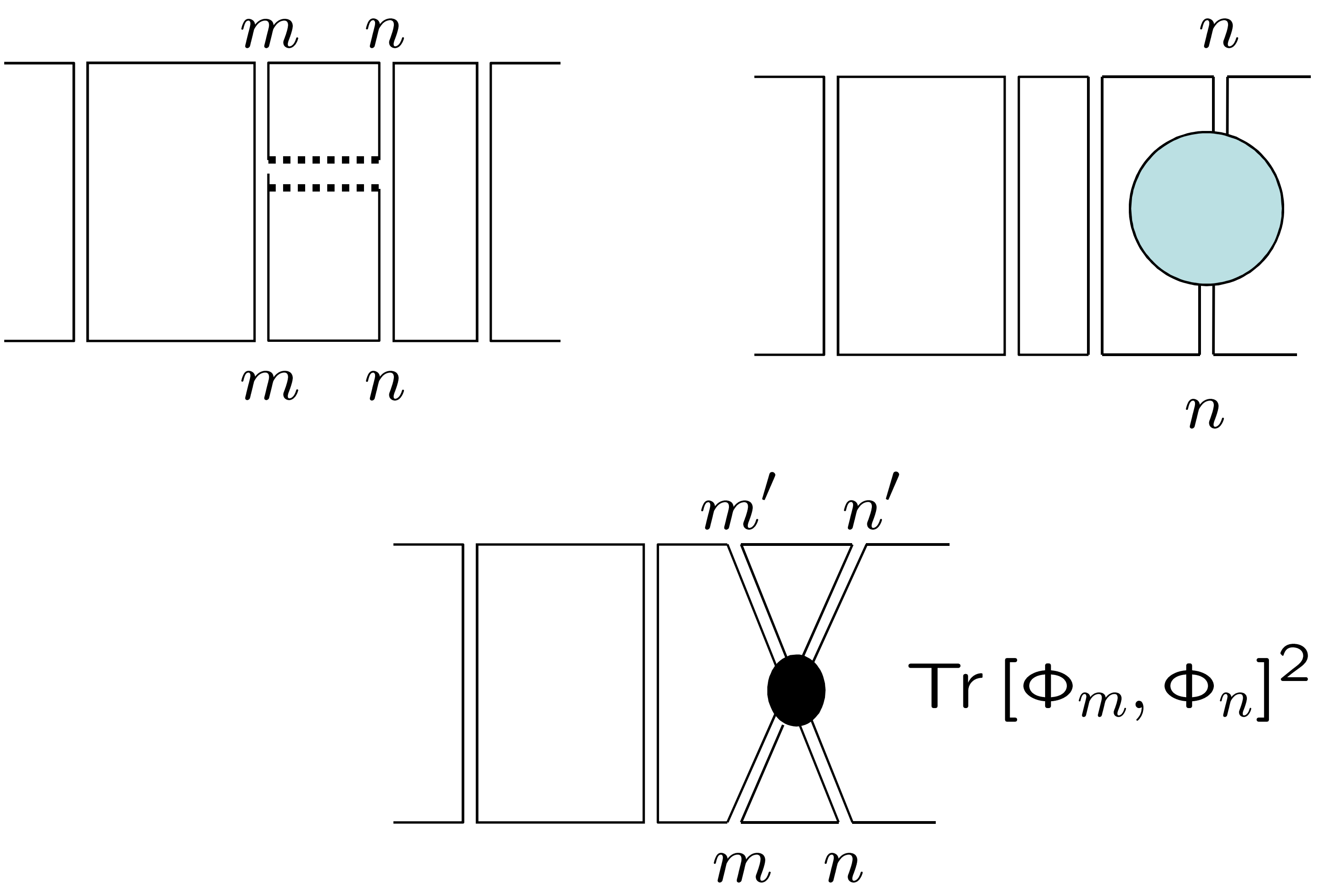
Example:

$$Q\,\Psi_{\alpha a} \;=\; -\frac{1}{2}\sigma^{\mu}_{\alpha\dot{\beta}}\varepsilon^{\dot{\beta}\dot{\gamma}}\sigma^{\nu}_{\dot{\gamma}\gamma}\epsilon^{\gamma}_{a}\mathcal{F}_{\mu\nu} + \frac{i}{2}\sqrt{\lambda}\sigma^{m}_{ab}\sigma^{bc}_{n}\varepsilon_{\alpha\beta}\epsilon^{\beta}_{c}\,[\Phi_m,\Phi^n]$$

+ quantum corrections

One loop: point splitting

Tree level: $\Delta_0 = L$ (degeneracy)

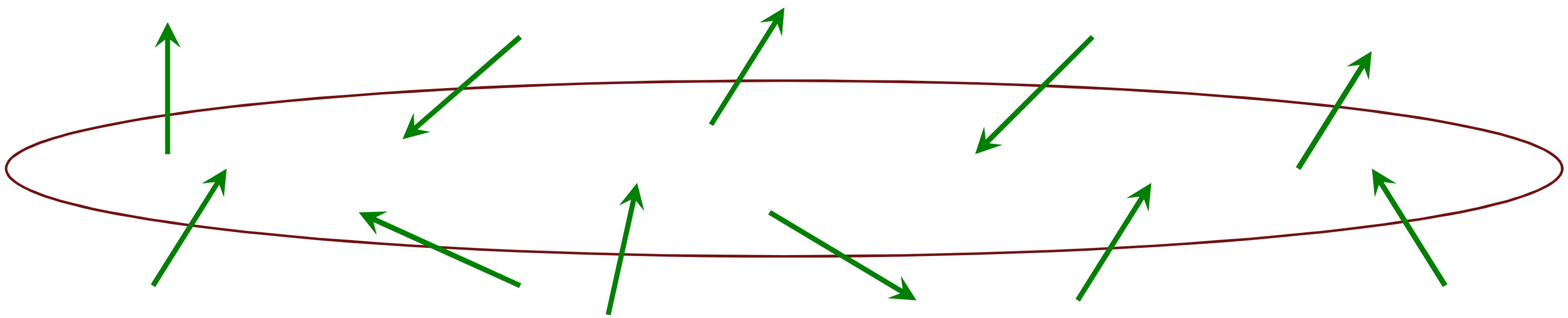


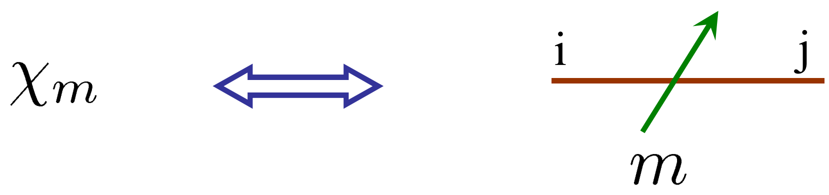
Only the last diagram acts nontrivially on R-indices.

SYM as Integrable psu(2,2|4) Spin Chain

Minahan,Zarembo'02

$$\mathcal{O}_{m_1 m_2 \ldots m_L} = \mathrm{Tr}\left(\chi_{m_1}\chi_{m_2}\cdots\chi_{m_L}\right)$$





SU(2) Sector of SYM and Integrability

- From 3 comples scalars

$$X = \Phi_1 + i\Phi_2, \qquad Y = \Phi_3 + i\Phi_4, \qquad Z = \Phi_5 + i\Phi_6$$

take only X and Z: an operator looks like XXX-chain with two state spins: $\begin{pmatrix} X \\ Z \end{pmatrix}$

$$\mathcal{O} = \mathrm{Tr}\ (ZZZZXXZXXXZZZZZXXXZXZXX) + \text{perm.}$$

- From 2-loop Feynman diagrams of N=4 SYM the chain Hamiltonian:

$$\hat{D} = L + \underbrace{\frac{\lambda}{16\pi^2}\sum_{l=1}^{L}\left(1 - \sigma_l \cdot \sigma_{l+1}\right) +}_{\text{Heisenberg (XXX) chain}}$$

$$\left(\frac{\lambda}{16\pi^2}\right)^2 \sum_{l=1}^{L}\left(\left(1 - \sigma_l \cdot \sigma_{l+2}\right) - 4\left(1 - \sigma_l \cdot \sigma_{l+1}\right)\right) + O(\lambda^3)$$

Beisert,Kristijansen,Staudacher'02

- Perturbatively integrable, at least up to 4-loops!
- Same is true for the full theory, for all loops and any coupling!

Exact spectrum at one loop (XXX)

Rapidity parametrization: $$e^{ip} = \frac{u + i/2}{u - i/2}$$

$$\left(\frac{u_k + i/2}{u_k - i/2}\right)^L = \prod_{(k\neq)j=1}^{J} \frac{u_k - u_j + i}{u_k - u_j - i}$$

Bethe’31

J – numer of flipped spins

Anomalous dimension:

$$\Delta - L = \frac{\lambda}{8\pi^2} \sum_{k=1}^{J} \frac{1}{u_k^2 + 1/4}$$

« Classics »: Riemann surface

$u \sim L \to \infty$

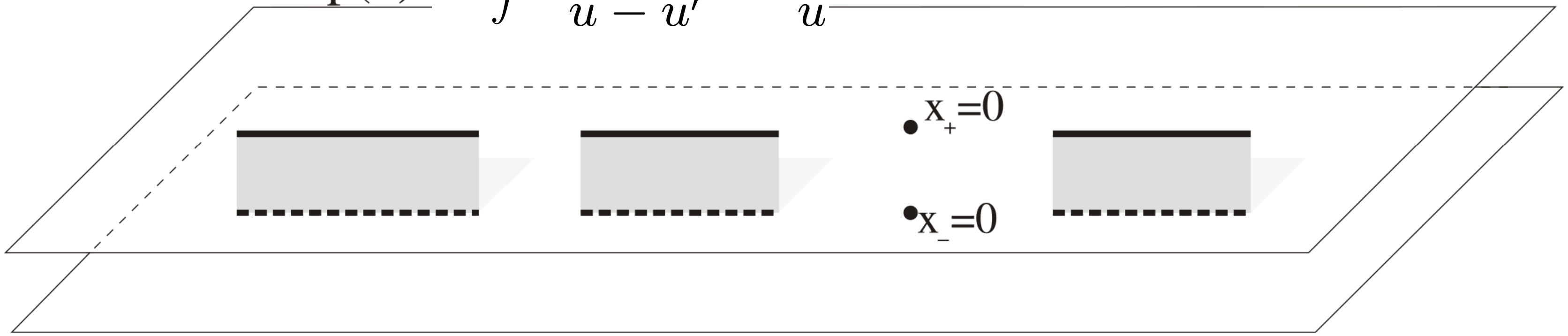
$$2\pi i m_k + L \log\left(\frac{u_k + i/2}{u_k - i/2}\right) = \sum_{(k\neq)j=1}^{J} \log\frac{u_k - u_j + i}{u_k - u_j - i}$$

$$2\pi m_k + \frac{L}{u_k} = \sum_{(k\neq)j=1}^{J} \frac{1}{u_k - u_j}$$

$$2\pi m + \frac{1}{u} = \mathcal{P}\int \frac{du'\rho(u')}{u - u'}$$

p(x+i0) + p(x-i0)=2 π m

Qusimomentum $\mathrm{p(x)} = \int \frac{du'\rho(u')}{u-u'} - \frac{1}{u}$



• Similar to the Riemann surface of classical finite gap solution of string

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AdS/CFT correspondence

Maldacena'97
Gubser,Klebanov,Polyakov'98
Witten'98

$\mathcal{N}=4$ SYM	Strings on $AdS_5 \times S^5$
't Hooft coupling: $\lambda = g_{YM}^2 N$	String tension: $T = \frac{1}{2\pi\alpha'} = \frac{\sqrt{\lambda}}{2\pi}$
Number of colors: N	String coupling: $g_s = \frac{\lambda}{4\pi N}$
Large-N limit	Free strings
Strong coupling	Classical strings
Local operators Scaling dimension: $\Delta(\lambda)$	String states Energy: $E(\lambda)$

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Metsaev-Tseytlin superstring

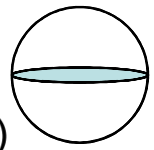
- It is a sigma model on the coset

$$AdS_5 \times S^5 \sim \mathrm{PSU}(2,2|4)/\left(\mathrm{Sp}(2,2)\times \mathrm{Sp}(4)\right)$$

- Supergroup element *g* : (4|4)×(4|4) supermatrix of SU(2,2|4)

$$\frac{SU(4)}{Sp(4)} \sim \frac{SO(6)}{SO(5)} = S^5$$

Isometry of sphere



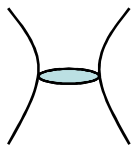
$SU(4) \sim SO(6)$

$B = C^\dagger \cdot \mathrm{diag}(1,1,-1,-1)$

$$g = \left(\begin{array}{c|c} A & B \\ \hline C & D \end{array}\right) \in \mathrm{PSU}(2,2|4)$$

16 complex fermions

$SU(2,2) \sim SO(2,4)$



Isometry of Anti de Sitter (conformal group)

$$\frac{SU(2{,}2)}{Sp(2{,}2)} \sim \frac{SO(2{,}4)}{SO(1{,}5)} = AdS_5$$

- Same isometry group *psu*(2,2|4) as for N=4 SYM theory!

Subsector: σ-model on $S^3 x R_1$

[Frolov,Tseytlin'02]

- Classical motion can be limited by a subset $S^3 \times R_1 \subset S^5 x AdS_5$

AdS time direction

- Polyakov string action in conformal gauge

$$S = \frac{\sqrt{\lambda}}{4\pi} \int d\sigma d\tau \left[(\nabla X_a)^2 - (\nabla t)^2 \right], \qquad X_1^2 + \ldots + X_4^2 = 1$$

- Gauge for AdS "time": $t(\sigma, \tau) = \kappa \tau$

- We get the O(4) sigma model: Integrable, with a Lax pair.

[Zakharov,Mikhailov'70's]

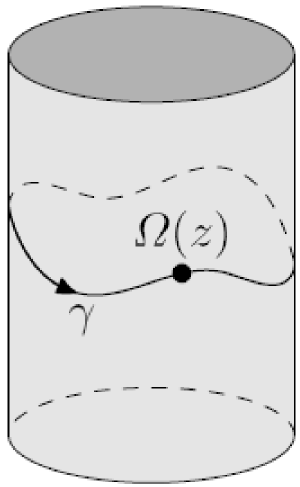
- Solvable by finite gap method.

[Novikov,Dubrovin,Its,Matveev,Krichever'70-80's]

- KMMZ solution: Bethe ansatz eqs. In classical limit

V.K.,Marshakov,Minahan,Zarembo'04

Classical Integrability of string



- All Bianchi identities and eqs. of motion (current conserv.) are packed into a Lax eq.:

$$(d+\mathcal{A}(z))\wedge(d+\mathcal{A}(z))=0,$$

Bena,Roiban,Polchinski'02

Where $\mathcal{A}(z)\in psu(2,2|4)$ depends on the string fields g and on free parameter z

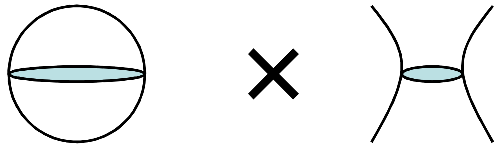
- Monodromy matrix: $\Omega(z)=P\exp\oint\mathcal{A}(z)d\sigma$

Conserved quantities: eigenvalues of $\Omega(z)\subset PSU(2,2|4)$

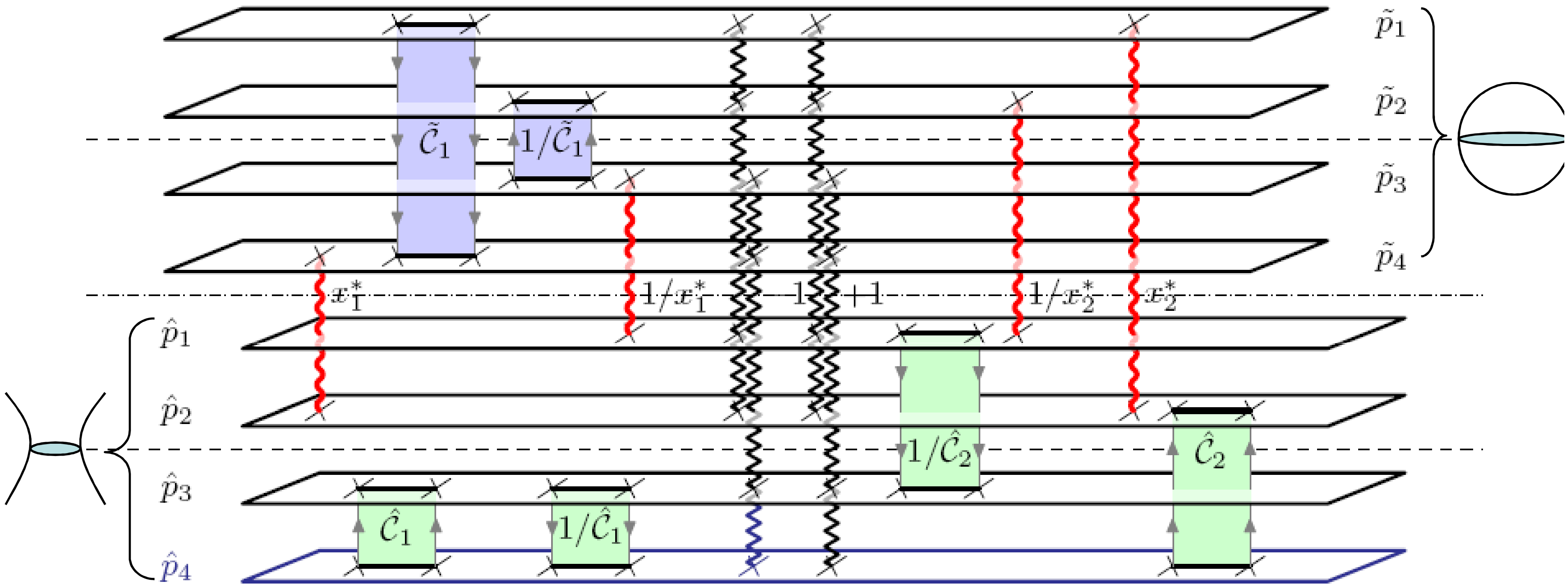
$$\{e^{i\tilde{p}_1(z)},e^{i\tilde{p}_2(z)},e^{i\tilde{p}_3(z)},e^{i\tilde{p}_4(z)}||e^{i\hat{p}_1(z)},e^{i\hat{p}_2(z)},e^{i\hat{p}_3(z)},e^{i\hat{p}_4(z)}\}$$

Eigenvalues are found by solving a characteristic equation for Ω . They define Riemann surface.

Riemann surface classical string



Beisert, V.K., Sakai,Zarembo'05



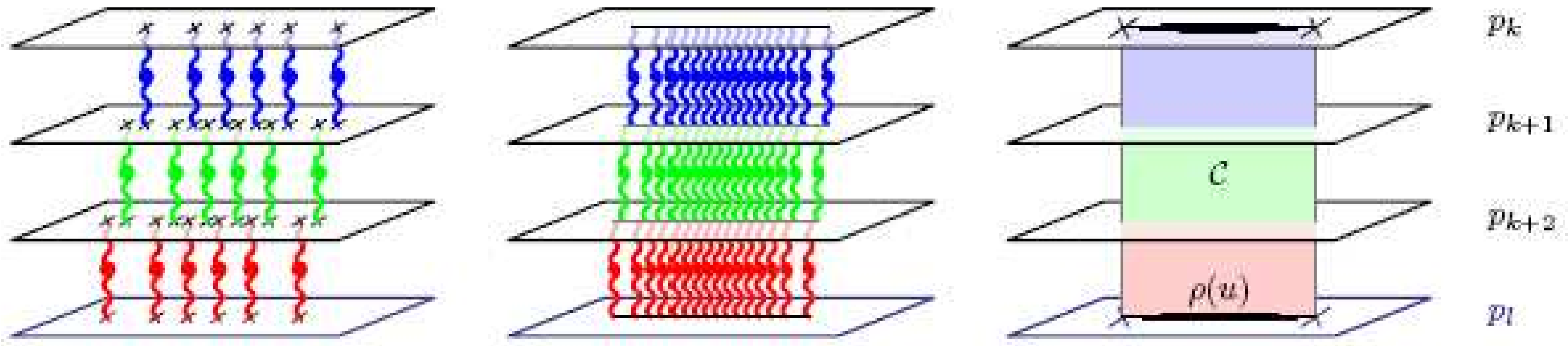
- Algebraic curve encodes all "action" variables;
- "Angle" variables defined by inclomplete holomorphic integrals. (possible to restore corresponding classical string motion).
- Good start for quantization (non-pert. symmetry $x \to 1/x$ important!)
- Finite gap eqs. (classical Bethe eqs.): $p(x+i0) + p(x-i0) = 2\pi m$

How to quantize this superstring?

• Condensation of SYM Bethe roots gives cuts

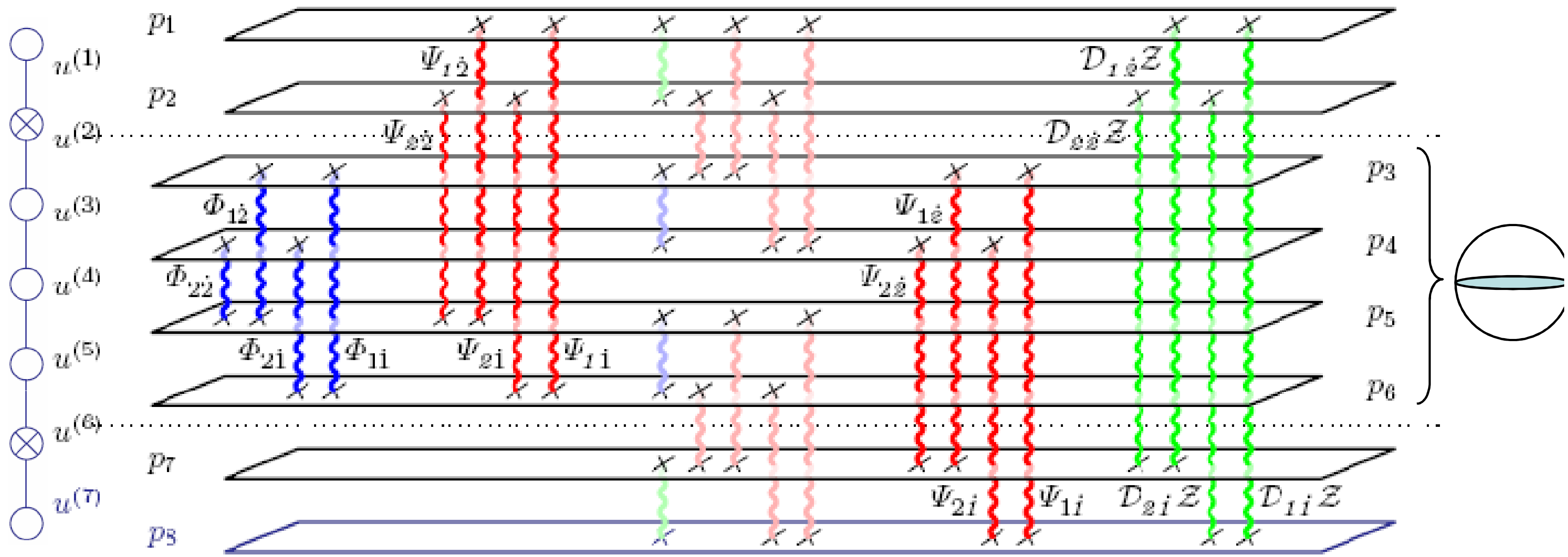
Beisert, V.K., Sakai,Zarembo'05

• Formation of cuts from strings of stacks:



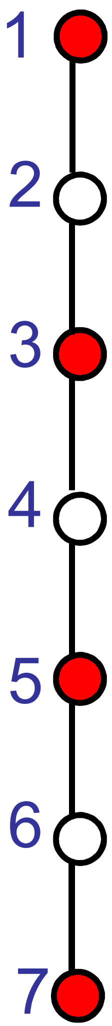
As we saw in detail for the su(2) sector of SYM

Dictionary: Stacks-Fields



- To each field of SYM corresponds a Bethe root or stack of roots.
- Bosonic roots with the same mode number n_k condense into cuts in the scaling limit of long operators.
- Fermionic roots stay apart.

- Algebraic curves of string and SYM coincide by the appropriate identification of parameters: AdS/CFT correspondence!



psu(2,2|4)

Bethe Ansatz eqs.

Beisert,Staudacher'05

$$1 = \prod_k \; e_{-2}\left(u_j^{(2)} - u_k^{(2)}\right) e_{+1}\left(u_j^{(2)} - u_k^{(3)}\right)$$

$$1 = \prod_k e_{-1}\left(u_j^{(3)} - u_k^{(2)}\right) \; r_{+}\left(u_j^{(3)}, u_k^{(4)}\right)$$

$$\left(\frac{x_j^{(4)+}}{x_j^{(4)-}}\right)^L = \prod_k \sigma^2\left(x_j^{(4)}|x_k^{(4)}\right) r_{-}\left(u_j^{(5)}, u_k^{(4)}\right) e_{+2}\left(u_j^{(4)} - u_k^{(4)}\right) r_{-}\left(u_j^{(3)}, u_k^{(4)}\right)$$

$$1 = \prod_k r_{+}\left(u_j^{(5)}, u_k^{(4)}\right) \; e_{-1}\left(u_j^{(5)} - u_k^{(6)}\right)$$

$$1 = \prod_k e_{+1}\left(u_j^{(6)} - u_k^{(5)}\right) e_{-2}\left(u_j^{(6)} - u_k^{(6)}\right)$$

- Zhukovsky parametrization used: $e_k(u) = \frac{u + ik/2}{u - ik/2}, \quad r_{\pm}(u, \tilde{u}) = \frac{x(u) - x^{\pm}(\tilde{u})}{x(u) - x^{\mp}(\tilde{u})}.$

$$u = x + 1/x, \quad x(u) = \frac{1}{2}\left(u + \sqrt{u^2 - 4}\right), \quad x^{\pm} = x(u \pm i/2)$$

Completely fixes dimensions of long operators of N=4 SYM! $\delta = \sqrt{\lambda}\sum_j \left(\frac{i}{x_j^+} - \frac{i}{x_j^-}\right)$

(by rapidities of the middle node)

Exact dressing factor

Janik'06

Beisert,Hernandez,Lopez'06

Beisert,Eden,Staudacher'06

$$\frac{1}{i}\log\sigma^2(1,2) = \chi(x_1^+,x_2^+)+\chi(x_1^-,x_2^-)-\chi(x_1^-,x_2^+)-\chi(x_1^+,x_2^-)$$

$$\chi(1,2) = -i\oint\frac{dy_1}{2\pi}\frac{1}{x_1-y_1}\oint\frac{dy_2}{2\pi}\frac{1}{x_2-y_2}\log\Gamma\left(1+ig\left(y_1+\frac{1}{y_1}-y_2-\frac{1}{y_2}\right)\right)$$

$$\simeq g\left(x_1+\frac{1}{x_1}-x_2-\frac{1}{x_2}\right)\log\left(1-\frac{1}{x_1x_2}\right) \quad + \quad O(1/g)$$

Arutyunov,Frolov,Staudacher'04

Guessed from finite gap KMMZ solution

V.K.,Marshakov,Minahan,Zarembo'04

Hernandez,Lopez'06

Confirmed by the full string one loop result

Gromov,Vieira'07

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Dispersion relation

- Algebra closure on the state with an operator χ inserted with "momentum" p:

$$\Sigma_k \; \mathrm{Tr}(\ldots\ldots Z\,Z\,Z\,Z\,Z\,Z\,Z\,\overset{k}{\chi}\,Z\,Z\,Z\,Z\,Z\,Z\,Z\,Z\,Z \ldots\ldots) \quad \exp(ikp)$$

fixes

$$\mathcal{P} = \sqrt{\frac{\lambda}{2}}(e^{-ip} - 1)$$

$$E = \sqrt{1 + 8\lambda \sin^2\left(\frac{p}{2}\right)} - 1$$

- eigenvalue of the dilatation operator D

Beisert,Dippel,Staudacher'06

- For multiple insertions of operators:

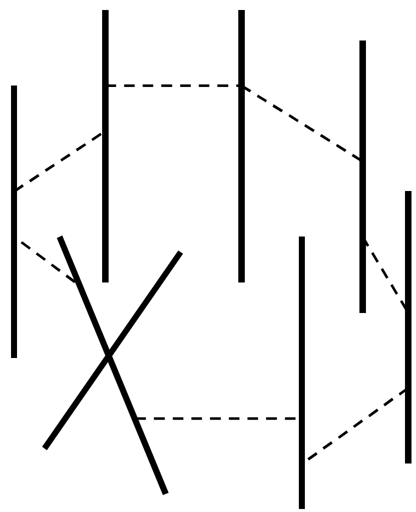
$$E = \sum_{k=1}^{J} \left(\sqrt{1 + 8\lambda \sin^2 \frac{p_k}{2}} - 1\right)$$

BPS spectrum

Problems

- Asymptotic BAE's are not derived from first principles

- Does not work for "short" operators, like Konishi's
  $tr\,(\Phi^a\,\Phi_a)$, due to wrapping problem:



Hope for the full solution: finite size effects from Thermodynamical Bethe Ansatz (TBA)

Janik, Lukowski'07
Frolov,Arutyunov'07
Gromov,Schaefer-Nameki,Vieira'07

END

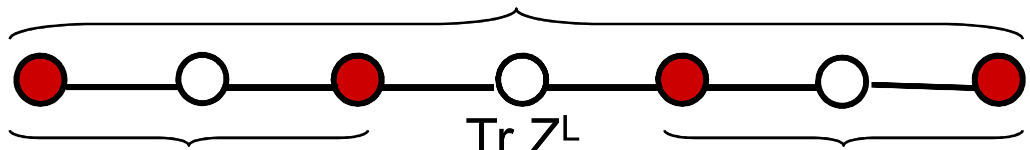
AdS/CFT scattering matrix

- Choice of vacuum: BPS operator

Beisert'06

$$\mathrm{Tr}(\ldots\ldots Z\,Z\,Z\,Z\,Z\,Z\,Z\,Z\,Z\,Z\,Z\,Z\,Z\,Z\,Z\,Z\,Z\,Z\ldots\ldots)$$

It breaks the symmetry:



Algebra relations: $su(2|2)_{ext} \rightarrow su(2|2) \times R^3$

$$\{Q^\alpha_a, S^b_\beta\} = \delta^b_a \mathcal{L}^\alpha_\beta + \delta^\alpha_\beta \mathcal{R} + \delta^b_a \delta^\alpha_\beta E$$

$$\{Q^\alpha_a, Q^\beta_b\} = \epsilon^{\alpha\beta}\epsilon_{ab}\mathcal{P}$$

$$\{S^a_\alpha, S^\beta_b\} = \epsilon^{ab}\epsilon_{\alpha\beta}\mathcal{P}^*$$

central charges

Action on States

- State : a supervector $(\phi_1, \phi_2|\psi_1, \psi_2)$

$$
\begin{aligned}
Q_a^\alpha|\phi^b\rangle &= A\,\delta_a^b|\psi^\alpha\rangle \\
Q_a^\alpha|\psi^\beta\rangle &= B\,\epsilon^{ab}\epsilon_{\alpha\beta}|\phi^b\mathcal{Z}^+\rangle \\
S_\alpha^a|\phi^b\rangle &= C\,\epsilon^{ab}\epsilon_{\alpha\beta}|\psi^\beta\mathcal{Z}^-\rangle \\
S_\alpha^a|\psi^\beta\rangle &= D\,\epsilon^{ab}\delta_\alpha^\beta|\phi^a\rangle
\end{aligned}
$$

- Closure of algebra: $AD - BC = 1$

$$
\begin{aligned}
\mathcal{C}|\chi\rangle &= \frac{1}{2}(AD + BC)|\chi\rangle \\
\mathcal{P}|\chi\rangle &= AB|\chi\mathcal{Z}^+\rangle \\
\mathcal{K}|\chi\rangle &= CD|\chi\mathcal{Z}^-\rangle
\end{aligned}
$$

- Without central charges the representation is shortened: AB=CD=0.

Fixing su(2|2) Scattering Matrix

- Action of S-matrix on matrix elements

$$\mathcal{S}_{12}|\phi_2^a\phi_1^b\rangle = A_{12}|\phi_2^{\{a}\phi_1^{b\}}\rangle + B_{12}|\phi_2^{[a}\phi_1^{b]}\rangle + \frac{1}{2}C_{12}\epsilon^{ab}\epsilon_{\alpha\beta}|\psi_2^\alpha\psi_1^\beta\mathcal{Z}^-\rangle$$

$$\mathcal{S}_{12}|\psi_1^\alpha\psi_2^\beta\rangle = D_{12}|\psi_2^{\{\alpha}\psi_1^{\beta\}}\rangle + E_{12}|\psi_2^{[\alpha}\psi_1^{\beta]}\rangle + \frac{1}{2}F_{12}\epsilon^{ab}\epsilon_{\alpha\beta}|\phi_2^a\phi_1^b\mathcal{Z}^+\rangle$$

$$\mathcal{S}_{12}|\phi_1^a\psi_2^\beta\rangle = G_{12}|\psi_2^\beta\phi_1^a\rangle + H_{12}|\phi_2^a\psi_1^{\beta]}\rangle$$

$$\mathcal{S}_{12}|\psi_1^\alpha\phi_2^b\rangle = K_{12}|\psi_2^\alpha\phi_1^b\rangle + L_{12}|\phi_2^b\psi_1^{\alpha]}\rangle$$

- Commutation (with braiding) with any su(2|2) symmetry generator J

$$[J_1 \otimes I\,\mathcal{P}_2 + \mathcal{P}_1^* \, I \otimes J_2,\ S_{12}] = 0$$

fixes the S-matrix completely up to a scalar dressing factor σ

- S-matrix satisfies the Yang-Baxter relations - integrability!

Fixing su(2|2) Scattering Matrix

- Commutation with any su(2|2) symmetry generator J

$$[J_1 \otimes I + I \otimes J_2, S_{12}] = 0$$

fixes the S-matrix completely up to a scalar dressing factor σ

- Crossing equation and extra physical and analytical input fix σ completely!

Janik'05

Beisert,Hernandez,Lopez'06

Beisert,Eden,Staudacher'06

- S-matrix satisfies the Yang-Baxter relations - integrability!

Scattering on the SYM Spin Chain

Staudacher'04

- Scattering of two operators χ_1 , χ_2 and asymptotic S-matrix:

$$\Sigma_{k,m}\ \mathrm{Tr}(\ldots\ Z\ \overset{k}{\chi}\ Z\ Z\ Z\ \ldots\ Z\ Z\ Z\ Z\ \overset{m}{\chi}\ Z\ Z\ \ldots)\quad \exp(ik\,p_1 - im\,p_2)\ +$$

$$+\ S_{12}(p_1,p_2) \times \Sigma_{k,m}\ \mathrm{Tr}(\ldots\ Z\ \overset{k}{\chi}\ Z\ Z\ Z\ \ldots\ Z\ Z\ Z\ Z\ \overset{m}{\chi}\ Z\ Z\ \ldots)\ \exp(ik\,p_2 - im\,p_1)$$

- Usual assumption: full S-matrix of elementary operator insertions factorizes:

Beisert'06

$$S_{PSU(2,2|4}(p_1,p_2)\ =\ \sigma^2(p_1,p_2)\ S_{SU(2|2)}(p_1,p_2) \times S_{SU(2|2)}(p_1,p_2)$$

- It suffices to construct $S_{SU(2|2)}$ and the phase σ^2 (dressing factor)

Global and local charges

- Angular momenta:

$$Q_L = \frac{\sqrt{\lambda}}{4\pi} \int d\sigma \, \mathrm{Tr} \, \left(i\partial_0 g \, g^\dagger \tau^3 \right) , \qquad Q_R = \frac{\sqrt{\lambda}}{4\pi} \int d\sigma \, \mathrm{Tr} \, \left(i g^\dagger \partial_0 g \, \tau^3 \right)$$

- Virasoro conditions: $\mathrm{tr} j_\pm^2(\sigma,\tau) = 2\kappa^2$

- Energy and momentum of sigma model:

$$E^{\mathrm{cl}} \pm P^{\mathrm{cl}} = -\frac{\sqrt{\lambda}}{8\pi} \int \mathrm{tr} \, [j_0 \pm j_1]^2 d\sigma = \frac{\sqrt{\lambda}}{2} \kappa^2$$

- AdS "Energy" = dim. of a SYM operator:

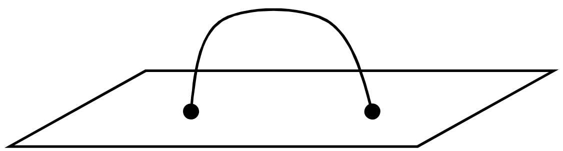
time translation generator

$$\Delta = \int_0^{2\pi} d\sigma \, \frac{\delta S}{\delta[\partial_\tau t(\sigma,\tau)]} = \sqrt{\lambda}\,\kappa$$

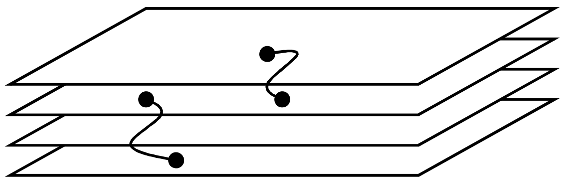
AdS/CFT: motivation

- Effective action of the IIB string theory with N_c D-branes contains both the N=4 SYM and the $AdS_5 \times S^5$ supergravity at low energies.

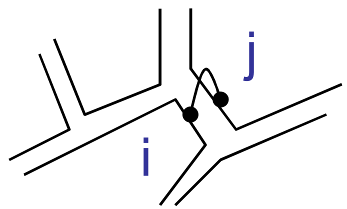
3-brane with open string



stack of Nc 3-branes



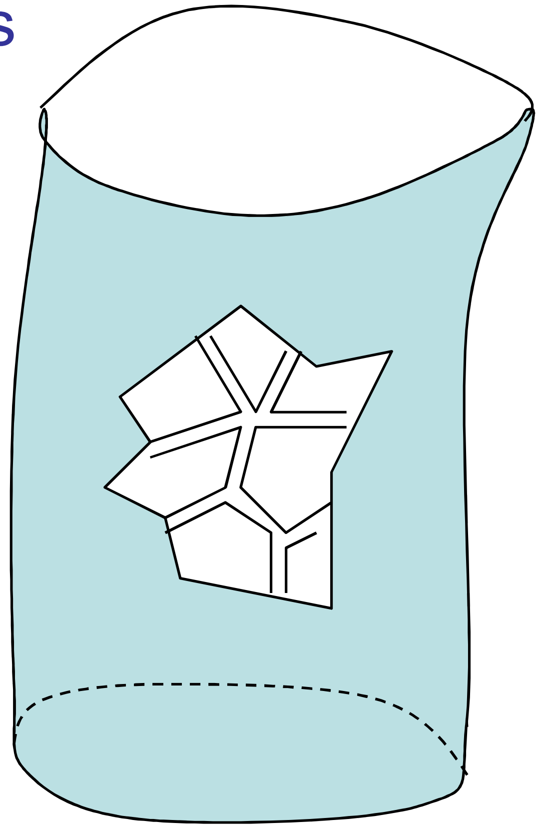
String eff. action contains N=4 SYM with $SU(N_c)$ gauge group and coupling $\lambda = N_c g_{YM}^2 = \frac{1}{\alpha'^2}$



Feynman Graphs in 4d

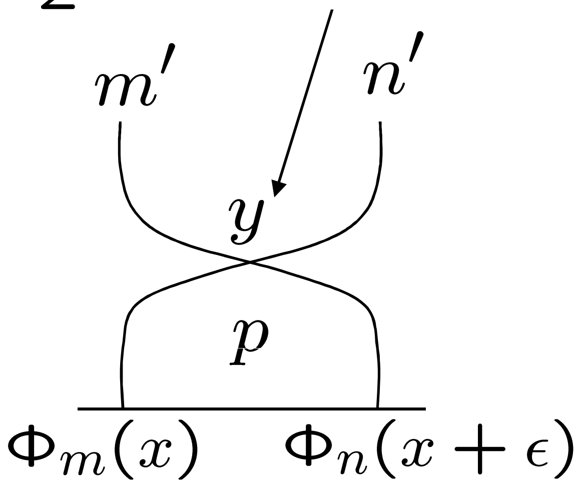
Closed strings from planar graphs of SYM

- SYM graphs ➡ closed strings
- 4D ➡ 10D, due to extra fields
- Condensate of closed strings creates a nontrivial supergravity background $AdS_5 \times S^5$



Diagrams.....

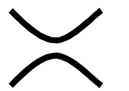
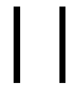
$$\frac{1}{2}\mathrm{Tr}\,[\Phi_m,\Phi_n]^2 = \mathrm{Tr}\,(\Phi_m\Phi_n)^2 - \mathrm{Tr}\,\Phi_m^2\Phi_n^2$$



$$\frac{1}{2}\int \frac{d^4p\, e^{i(p\cdot\epsilon)}}{(2\pi)^4\,p^4} = \frac{1}{8\pi^2}\log\Lambda$$

$$\Lambda = 1/|\epsilon|$$

Index structures: $\delta_{mn}\delta_{m'n'} - 2\delta_{mn'}\delta_{nm'} + C\delta_{mm'}\delta_{nn'}$

- **Permutation (first term contributes):** $\mathcal{P} = \delta_{mn'}\delta_{nm'}$ 
- **Trace operator (last term):** $\mathcal{K} = \delta_{mn}\delta_{m'n'}$ 
- **Unity (last and all other graphs):** $\mathcal{I} = \delta_{mm'}\delta_{nn'}$ 

AdS time

- Radial coordinate *z* and Lorentzian space-time of AdS x_μ recovered from

$$X_{-1} + X_{10} = R/z,\ (X_0, X_7, X_8, X_9) = R\frac{x_\mu}{z},$$
$$\text{giving } ds^2 = R^2\ \frac{dx^2+dz^2}{z^2}.$$

- AdS time: $t = \log z$ - Isometry on AdS; related to SYM RG scale $\log \Lambda.$

- Intro (N=4 SYM, SCFT, dimensions, D – spin chain, integrability) 4
- AdS/CFT and MT model (def, sym., operator ➔ string state) 2
- Integrability of MT ➔ algebraic curve 3
- « Quantization » of the curve ➔ ABA   2
- S-matrix and dressing factor   3
- Problem: wrapping interactions and finite size effects ➔ TBA   2
- Conclusions 1

Dilatation operator in SYM

- Dilatation operator in perturbation theory: $\lambda = N g_{YM}^2$

Point-splitting and renormalization:

$$\mathcal{O}(x/\Lambda) = \Lambda^{\hat{D}} \mathcal{O}(x) = \Lambda^{\hat{D}^{(0)}} \left(1 + \lambda \log \Lambda \hat{D}^{(2)} + \ldots\right)$$

$$\hat{D} = \hat{D}^{(0)} + \lambda \hat{D}^{(2)} + \lambda^2 \hat{D}^{(4)} + \ldots$$

Conf. dimensions are eigenvalues of ``Hamiltonian" $\hat{D}$

$$\Delta(\lambda) = \Delta^{(0)} + \lambda \Delta^{(2)} + \lambda^2 \Delta^{(4)} + \ldots$$

Plan

- Basic facts on AdS/CFT: duality of N=4 Super-Yang-Mills gauge theory and superstring on AdS_5xS^5 background
- String sigma model and finite gap eqs.
- Quantization of superstring
- Superconformal symmetry of SYM
- SYM dilatation operator as integrable spin chain
- S-matrix and bootstrap: asymptotic Bethe ansatz
- Finite size operators and wrapping.