QUANTIZED BLACK HOLES, THEIR SPECTRUM AND RADIATION

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- Introduction: entropy, temperature, radiation of black holes
- Adiabatic invariance of area makes its quantization natural
- Holographic bound
- Count of states of area
- Quantization of area
- Radiation spectrum of quantized black holes
- Is radiation of quantized black holes observable?

Introduction

Wheeler's paradox. Entropy of black hole. Bekenstein.
Then temperature of black hole. Radiation! Gribov, Hawking.
Propagation of semiclassical wave packet from horizon vicinity.
Independently of its initial spectral content, at infinity it is
$|f(\omega)|^2 \sim \exp(-8\pi kM\omega)$. Boltzmann distribution with

$$T = (8\pi kM)^{-1}$$

Now, with $dE(= dM) = TdS = (8\pi kM)^{-1}dS$,
we obtain $S = 4\pi kM^2$. Since the horizon area is
$A = 4\pi r_g^2 = 16\pi k^2M^2$,

$$S = A/4l_p^2; \quad l_p^2 = k. \qquad \text{Hawking.}$$

Adiabatic invariance of area makes its quantization natural

The area of a black hole horizon behaves as an adiabatic invariant
Christodoulu, Ruffini, Bekenstein.
The minimum change of the horizon area under adiabatic process is, according to Bekenstein,

$(\Delta A)_{\min} = \xi l_p^2 \,.$

Quantization of horizon area! Bekenstein, later Mukhanov, Kogan (strings).
A simple-minded argument: $A = 8\pi\gamma l_p^2 N\,; \quad l_p^2 = \hbar k/c^3,$
N generalized quantum number (the same power as $\hbar$).
No sound arguments in favor of integer N, or equidistant spectrum!

$$A = 8\pi\gamma l_p^2 \sum_{jm} a(j)\nu_{jm}$$
$$S = A/4l_p^2 = 2\pi\gamma \sum_{jm} a(j)\nu_{jm} = 2\pi\gamma \sum_j a(j)\nu_j$$

Holographic bound

Bekenstein, 't Hooft, Susskind.
Entropy S of any spherical nonrotating body confined
inside a sphere of area A is bounded as follows:
$S \leq A/4l_p^2$,
with the equality attained only for a body that is a black hole.
To prove it, let this body collapse into a black hole. Then, we have
$S \leq S_{bh} = A_{bh}/4l_p^2 \leq A/4l_p^2$.
Quite unexpected,
but for common objects too mild to contradict the common experience.
Alternative formulation:
Among the spherical surfaces of a given area, it is
the surface of a black hole horizon that has the maximum entropy.

Number of quantum states

$S = \ln K\,,$
K is the total number of quantum states, which depends essentially on the assumptions related to the distinguishability of the sites.

1 Complete distinguishability

$K = \nu!\,, \quad \nu = \sum_{jm} \nu_{jm} = \sum_j \nu_j\,.$
$S = \ln \nu! = \nu \ln \nu$ reaches maximum for fixed A when all j correspond to the smallest possible $a(j) = a_{\rm min}$. Then $A \sim \nu$ and

$S \sim A \ln A.$

Wrong!

2 Complete indistinguishability

Number of states of given j is

$$K(j) = (\nu_j + g(j) - 1)!/\nu_j!(g(j) - 1)!\,,$$

$g(j)$ is the number of possible values of m for a given j.

$$S(j) = \ln K(j) = g(j)\ln j\,,$$

under natural assumption $\nu_j \gg g(j)$.

$$A(j) \sim a(j)\nu_j \gg g(j)\ln \nu_j\,.$$

Wrong!

3 (and quite popular)

Total number of states is $K = \sum_j g(j)^{\nu_j}$.
It corresponds to the assumptions:

same j,	different m	$\longrightarrow$	distinguishable,
different j,	any m	$\longrightarrow$	indistinguishable,
same j,	same m	$\longrightarrow$	indistinguishable.

Obviously, the first two of them, taken together, look unnatural!

except the case when only a single value j is allowed.

The reasonable assumption

Only sites with the same jm are indistinguishable. Then

$$S = \ln\left[\nu!\prod_{jm} 1/\left(\nu_{jm}!\right)\right].$$

But what is in common with

$$A \sim N = \sum_{jm} a(j)\nu_{jm}\,?$$

We need maximum S for given N ("microcanonical" entropy), or with logarithmic accuracy maximum of

$$\nu\ln\nu - \sum_{jm}\nu_{jm}\ln\nu_{jm} - \mu\sum_{jm} a(j)\nu_{jm}\,,$$

where μ is a Lagrangian multiplier.

Variation in ν_{jm} gives (with logarithmic accuracy)

1. $\ln \nu - \ln \nu_{jm} - \mu a(j) = 0,$ or

2. $\nu_{jm} = \nu\, \mathrm{e}^{-\mu a(j)}$. Sum 2. over jm and divide by ν :

$$\sum_{jm} \mathrm{e}^{-\mu a(j)} = \sum_j g(j)\, \mathrm{e}^{-\mu a(j)} = 1$$

(a secular equation for the Lagrange multiplier μ).

Multiply 1. by ν_{jm} and sum over jm :

$$S_{\max} = \mu N = \mu\, A/(8\pi\gamma l_p^2). \quad \gamma = \mu/2\pi.$$

$$A = 8\pi\,\gamma\, l_p^2\, \nu \sum_j \mathrm{e}^{-\mu a(j)}\, g(j)\, a(j).$$

Example

In Loop Quantum Gravity (LQG) $a(j) = \sqrt{j(j+1)}; \quad g(j) = 2j + 1$.

The secular equation is $\sum_{j=1/2}^{\infty} (2j+1)\, \mathrm{e}^{-\mu\sqrt{j(j+1)}} = 1$,

with the solution $\mu = 1.722$,

and Barbero – Immirzi parameter of LQG $\gamma = \mu/(2\pi) = 0.274$.

Khriplovich, Korkin (2001); Ghosh, Mitra (2004); Corichi et al (2006).

Radiation spectrum of quantized black holes

In virtue of the same Bekenstein limit
$(\Delta A)_{\min} = \xi l_p^2\,,$
for the change of area under adiabatic process,
there are no "combinatorial" frequencies,
i.e., the radiation occurs when a site of given j disappears:

$$\Delta N = a(j), \quad \omega_j = \mu T a(j).$$

Discrete spectrum! $\omega_{\min} = \mu\, T\, a(j_{\min})$.

Even finite number of lines: since $\nu_{jm} \geq 1$,
$a(j_{\max}) = \ln \nu/\mu$. $\omega_{\max} = T \ln(A/l_p^2)$.
However, anyway, exponential decrease (due to which the last conclusion is possibly beyond the accuracy).

Natural assumption: $\Gamma_j \sim \nu_j = \nu g(j)\mathrm{e}^{-\omega_j/T}$. Then
$I_j \sim \omega_j \nu_j = \nu \omega_j g(j)\mathrm{e}^{-\omega_j/T}$
Exponential Wien profile for $\omega_j \gg T$ (almost free)!

As distinct from the common thermal radiation, here the typical wave length is roughly on the same order of magnitude as the radius of the black body. Therefore, due to the centrifugal barrier, this radiation is strongly dominated by the lowest possible partial wave, $J_{\min} = 1$ for photon.

Still, according to calculations by Page, the intensity of γ radiation is about the same as that given by the naïve Planck formula.

For electrons, positrons and neutrinos $J_{\min} = 1/2$, and the radiation intensity is higher.

Using the Page profile as the envelope for the discrete spectrum, one can demonstrate that

the ratio of the total natural line width to the line separation is $\lesssim \mathbf{0.03}$.

Spectrum is really discrete!

Is Radiation of Quantized Black Holes Observable?

Dark matter of density $\rho(r)$ in Solar system (interacting with the usual one only gravitationally) results in additional perihelion rotation

$$\frac{\delta\phi}{2\pi} = -\frac{2\pi\rho(r)r^3}{M}.$$

We assume that $\rho(r)$ is spherically-symmetric and centered on the Sun, and that the orbit eccentricity is small; M is mass of the Sun.

Analysis of observational data for secular perihelion precession of Earth and Mars results in the upper limit (Khriplovich, Pitjeva):

$$\rho(r) < 3 \times 10^{-19}\ \mathrm{g/cm^3}.$$

Estimates for the expected signal from PBHs (Khriplovich, Produit) were performed under the (optimistic) assumption that their density is $\boldsymbol{\rho \simeq 10^{-19}}$ g/cm^3.

PBH with initial mass $\boldsymbol{m \lesssim m_0 = 5 \times 10^{14}}$ g cannot survive till our time due to their radiation. For masses larger than $\mathbf{10^{17}}$ g, the signal gets hopelessly small. The estimates for reasonable masses are as follows:

$\boldsymbol{m}$, g	$\boldsymbol{n}$, cm^{-3}	$\bar{\boldsymbol{r}}$, cm	$\boldsymbol{T}$, MeV	$\boldsymbol{N}$, ph s^{-1}	$\boldsymbol{\nu}$, ph cm^{-2} s^{-1}
$\mathbf{5 \times 10^{14}}$	$\mathbf{2 \times 10^{-34}}$	$\mathbf{1.7 \times 10^{11}}$	20	$\mathbf{6 \times 10^{19}}$	$\mathbf{1.6 \times 10^{-4}}$
$\mathbf{2 \times 10^{15}}$	$\mathbf{5 \times 10^{-35}}$	$\mathbf{2.7 \times 10^{11}}$	5	$\mathbf{1.5 \times 10^{19}}$	$\mathbf{1.6 \times 10^{-5}}$
$\mathbf{10^{16}}$	$\mathbf{10^{-35}}$	$\mathbf{0.5 \times 10^{12}}$	1	$\mathbf{3 \times 10^{18}}$	$\mathbf{10^{-6}}$
$\mathbf{10^{17}}$	$\mathbf{10^{-36}}$	$\mathbf{10^{12}}$	0.1	$\mathbf{3 \times 10^{17}}$	$\mathbf{2 \times 10^{-8}}$

Table 1: Predictions for radiation of primordial black holes in Solar system

The typical signature of radiating PBH would be 2 — 3 relatively strong lines in the spectrum, for instance, with energies about 1, 2, 3 MeV, for PBH with mass $\mathbf{10^{16}}$ g.

It can be demonstrated that the observational data do not preclude the searches for quantized PBHs in the Solar system.

Thank you!