Entanglement and Noise in Quantum Conductors

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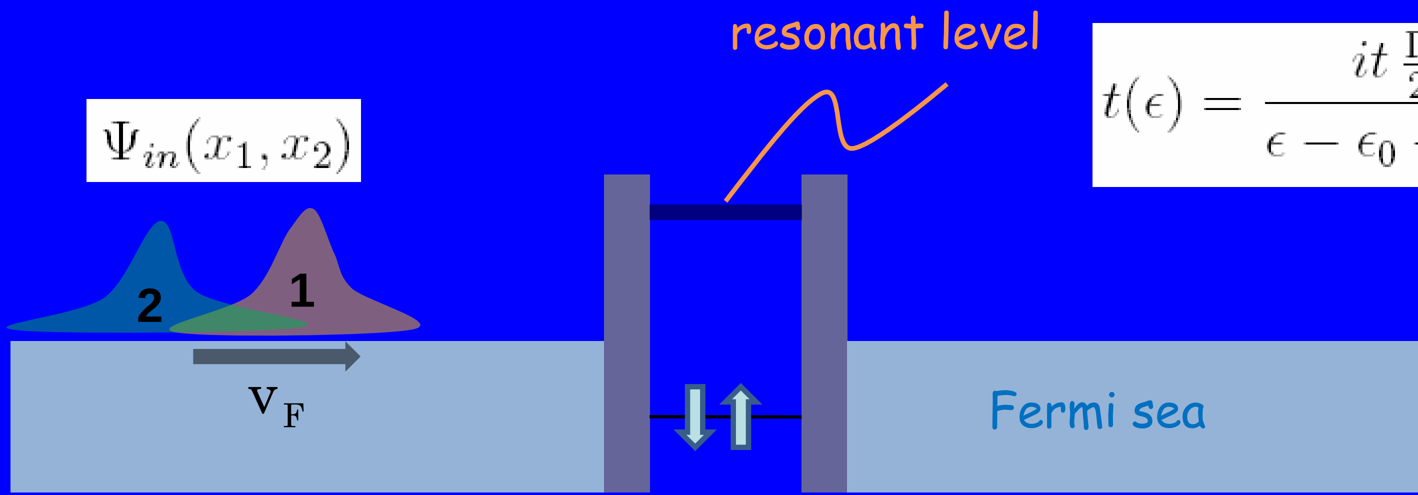
I will discuss producing and detecting of entangled states in quantum conductors. Generally, two particles become entangled due to interaction. In [1] we derive an exact formula for the scattering matrix for N particles interacting within a quantum dot. Characterizing the dot by its resonances, we find a compact form for the scattering matrix in a real-time representation. We make use of our results to study the transmission probabilities and interaction-induced orbital entanglement of two electrons incident on the dot in a spin-singlet state. For the identical particles there is a way to create an entangled state without direct interaction. In [2] we suggest an operating scheme for the deliberate generation of spin-entangled electron pairs in a normal-metal mesoscopic structure with fork geometry. Voltage pulses with associated Faraday flux equal to one flux unit Φ=hc/e drive individual singlet-pairs of electrons towards the beam splitter. The spin-entangled pair is created through a post-selection in the two branches of the fork. We analyze the appearance of entanglement in a Bell inequality test formulated in terms of the number of transmitted electrons with a given spin polarization.

1. Exact N-particle scattering matrix for electrons interacting on a quantum dot
A.V. Lebedev, G. B. Lesovik, G. Blatter, Phys. Rev. Lett. 100, 226805 (2008)
2. Generating spin-entangled electron pairs in normal conductorsusing voltage pulses .
A. V. Lebedev, G. B. Lesovik, G. BlatterPhys. Rev. B 72, 245314 (2005).

1. General concepts in describing noise and entanglement
2. Sources of entanglement
3. Entanglement due to Coulomb interaction
4. Entanglement of identical noninteracting particles



$$t(\epsilon) = \frac{it\,\frac{\Gamma}{2}}{\epsilon - \epsilon_0 + i\frac{\Gamma}{2}}$$

incoming two-particle state

$$\Psi_{in}(x_1, x_2) = \frac{\psi_1(x_1)\psi_2(x_2) - \psi_2(x_1)\psi_1(x_2)}{\sqrt{1 - |O|^2}}$$

overlap

$$O = \int \psi_1(x)\psi_2^*(x)\,dx$$

$$|\Psi_{out}\rangle = \hat{S}\,|\Psi_{in}\rangle$$

What is effect of interaction?
(transport, entanglement)

Model:

First quantized form:

$$\hat{H} = \frac{\hat{p}_1^2}{2m} + U(x_1) + \frac{\hat{p}_2^2}{2m} + U(x_2) + \frac{\hat{Q}^2}{2C}$$

$U(x)$ - dot potential

$\hat{Q}$ - charge in the dot

Second quantized form:

$$\hat{H} = \sum_{k\sigma} \xi_k \left(\hat{a}^\dagger_{k\sigma}\hat{a}_{k\sigma} + \hat{b}^\dagger_{k\sigma}\hat{b}_{k\sigma} \right) + \sum_{n\sigma} \epsilon_n \, \hat{c}^\dagger_{n\sigma}\hat{c}_{n\sigma}$$

$$+ \sum_{nk\sigma} \Delta_n^{(l)} (\hat{c}^\dagger_{n\sigma}\hat{a}_{k\sigma} + \hat{a}^\dagger_{k\sigma}\hat{c}_{n\sigma}) + \Delta_n^{(r)} (\hat{c}^\dagger_{n\sigma}\hat{b}_{k\sigma} + \hat{b}^\dagger_{k\sigma}\hat{c}_{n\sigma})$$

$$+ \sum_{nm} U_{nm} \, \hat{N}_n \hat{N}_m$$

tunneling

quadratic interaction

$$\hat{N}_n = \hat{c}^\dagger_{n\uparrow}\hat{c}_{n\uparrow} + \hat{c}^\dagger_{n\downarrow}\hat{c}_{n\downarrow}$$

- level occupancy

Two-particle propagator

$$\Psi_{out}(x_1, x_2, t) = \int dx_1' dx_2' \, K^{(2)}(x_1, x_2, t; x_1', x_2', t') \, \Psi_{in}(x_1', x_2', t')$$

$$K^{(2)}(\vec{x}, t; \vec{x}', t') = \int Dx_1(t) Dx_2(t) \, \exp\Big(\frac{i}{\hbar} \int_{t'}^{t} L(\vec{x}, \dot{\vec{x}}, \tau) d\tau\Big)$$

System Lagrangian:

$$L(\vec{x}, \dot{\vec{x}}, t) = \sum_{i=1,2} \left(\frac{m\dot{x}_i^2}{2} - U(x_i) \right) - \frac{U_c}{4} (\chi(x_1) + \chi(x_2))^2$$

$\chi(x)$ **- characteristic function of the dot**

$U_c = \frac{2e^2}{C}$ **- Coulomb energy**

Calculation scheme:

- Decouple electrons by applying a Hubbard-Stratanovich transformation

- Integrating out the electrons degrees of freedom

- Integrating out the Hubbard-Stratanovich field

Decoupling interaction term $\propto U_c\,(\chi(x_1)+\chi(x_2))^2$

A Hubbard-Stratanovich transformation:

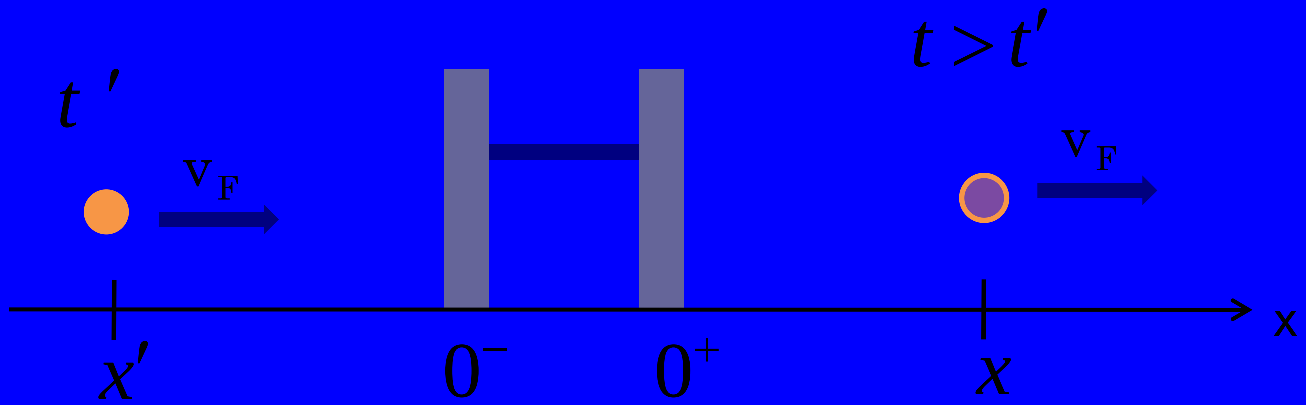
$$K^{(2)}(1,2)=\int Dz(t)\,\exp\Big(\frac{iU_c}{\hbar}\int z^2(t)dt\Big)\,K^{(1)}_{z(t)}(1)\,K^{(1)}_{z(t)}(2)$$

$z(t)$ - auxiliary real field $\quad \langle z(t_1)z(t_2)\rangle=\frac{i\hbar}{2U_c}\,\delta(t_1-t_2)$

$K^{(1)}_{z(t)}(x,t;x',t')$ - one particle propagator in the presence of alternating field:

$$K^{(1)}_{z(t)}(x,t;x',t')=\int Dx(t)\,\exp\Bigg(\frac{i}{\hbar}\int_{t'}^{t}dt\,\Big(\frac{m\dot{x}^2}{2}-U(x)-U_c z(t)\chi(x)\Big)\Bigg)$$

Relation between scattering matrix and propagator for ballistic leads with linear spectrum:



$$K^{(1)}_{z(t)}(x_\alpha, t; x'_\beta, t') = \frac{1}{v} S^{(1)}_{\alpha\beta, z(t)}\left(t - \frac{|x|}{v}, t' + \frac{|x'|}{v}\right)$$

$S^{(1)}_{\alpha\beta, z(t)}(\tau, s)$ one-particle scattering matrix in the presence of alternating field

$$\hat{H} = \frac{\hat{p}^2}{2m} + U(x) \quad \Longrightarrow \quad S^{(1)}_{\alpha\beta}(\tau)$$

$$\hat{H} = \frac{\hat{p}^2}{2m} + U(x) + U_c z(t)\chi(x) \quad \overset{?}{\Longrightarrow} \quad S^{(1)}_{\alpha\beta, z(t)}(\tau, s)$$

One-particle non-stationary scattering problem

One-particle transition amplitude:

$$t^{(1)}(\epsilon) = \frac{i\frac{\Gamma}{2}\,\tilde{t}}{\epsilon - \epsilon_0 + i\frac{\Gamma}{2}}$$

Fourier transform:

$$t^{(1)}(\tau) = \tilde{t}\,\frac{\eta}{2}\,\theta(\tau)\,\exp\Big(-\frac{i\epsilon_0\tau}{\hbar} - \frac{\eta}{2}\tau\Big)$$

$$\eta = \Gamma/\hbar$$

phase factor

One- particle transition amplitude for non-stationary situation:

$$t^{(1)}_{z(t)}(\tau, s) = \tilde{t}\,\frac{\eta}{2}\,\theta(\tau - s)\,\exp\Big(-\frac{i}{\hbar}(\epsilon_0 - i\frac{\Gamma}{2})(\tau - s)\Big)\,\exp\Big(-\frac{iU_c}{\hbar}\int_s^\tau z(t)dt\Big)$$

Scattering matrix in non-stationary case

$$S^{(1)}_{\alpha\beta,z(t)}(\tau,s) = S^{(1)}_{\alpha\beta}(\tau-s)\ \exp\Big(-\frac{iU_c}{\hbar}\int_s^{\tau} z(t)dt\Big)$$

$$S^{(1)}_{\alpha\beta}(\tau) = \int \frac{d\epsilon}{2\pi\hbar}\, S^{(1)}_{\alpha\beta}(\epsilon)\ \exp\Big(-\frac{i\epsilon\tau}{\hbar}\Big)$$

ballistic travel time

Validity of the phase approximation:

$$\int Dz(t)\ \exp\Big(\frac{iU_c}{\hbar}\int_t^{t+\delta t} z^2(t)dt\Big) \longrightarrow z^2 \sim \frac{2\pi\hbar}{U_c\delta t} \qquad \delta t \sim \frac{2\pi\hbar}{\Delta}$$

$$U_c z(t) \sim \sqrt{U_c\Delta} \sim U_c\sqrt{\epsilon\,\frac{v}{c}\,\alpha^{-1}}$$

Two-particle scattering matrix

$$\mathbf{S}^{(2)}(\tau_1, \tau_2; s_1, s_2) = \mathbf{S}^{(1)}(\tau_1 - s_1) \otimes \mathbf{S}^{(1)}(\tau_2 - s_2)\ \exp\Big(-\frac{iU_c}{2\hbar}\,\tau_{12}\Big)$$

Time spent by two particles together in the quantum dot:

interaction phase

$$\tau_{12} = \frac{1}{2}\Big(|\tau_1 - s_2| + |\tau_2 - s_1| - |\tau_1 - \tau_2| - |s_1 - s_2|\Big)$$

N-particle scattering matrix

$$\mathbf{S}^{(N)}(\vec{\tau}; \vec{s}) = \prod_{n=1}^{N} \otimes \mathbf{S}^{(1)}(\tau_n - s_n) \prod_{j>k}^{N} \exp\Big(-\frac{iU_c}{2\hbar}\,\tau_{jk}\Big)$$

Two-particle scattering matrix in energy representation

$$\mathbf{S}^{(2)}(\epsilon_1', \epsilon_2'; \epsilon_1, \epsilon_2) = (2\pi)^2\delta(\epsilon_1 - \epsilon_1')\delta(\epsilon_2 - \epsilon_2')\,\mathbf{S}^{(1)}(\epsilon_1) \otimes \mathbf{S}^{(2)}(\epsilon_2)$$
$$+(2\pi)\delta(\epsilon_1 + \epsilon_2 - \epsilon_1' - \epsilon_2')\,\mathbf{S}^{(2)}_{int}(\epsilon_1', \epsilon_2'; \epsilon_1, \epsilon_2)$$

Inelastic scattering contribution:

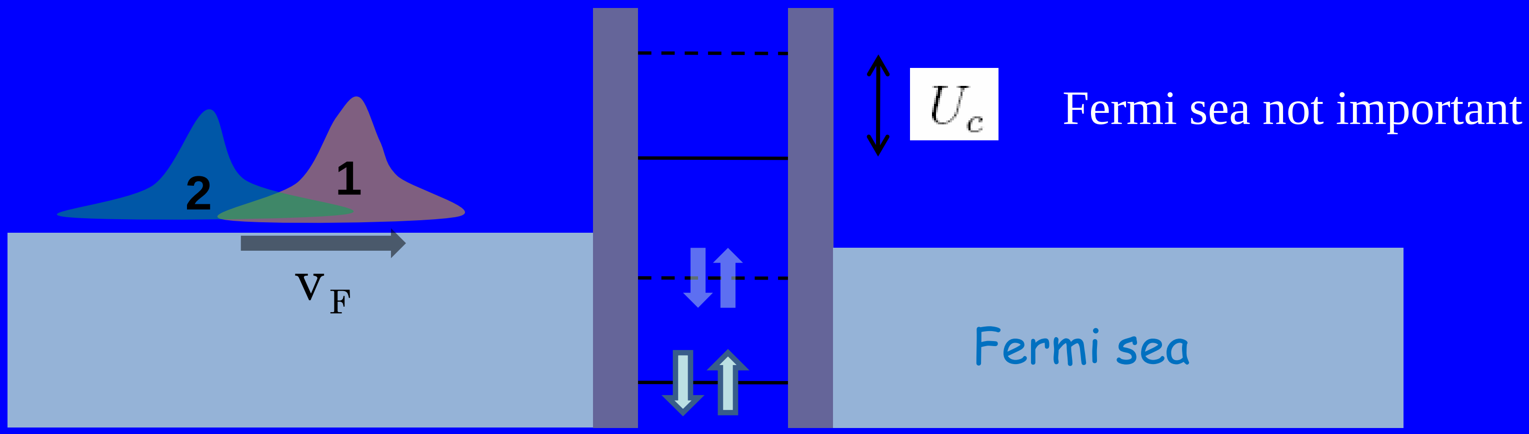
$$\mathbf{S}^{(2)}_{int} = \frac{iU_c/2}{\epsilon_1 + \epsilon_2 - 2\epsilon_0 - U_c + i\Gamma}\,\frac{i\frac{\Gamma}{2}}{\epsilon_1 - \epsilon_0 + i\frac{\Gamma}{2}}\,\frac{i\frac{\Gamma}{2}}{\epsilon_2 - \epsilon_0 + i\frac{\Gamma}{2}}$$
$$\times \left(\frac{1}{\epsilon_1' - \epsilon_0 + i\frac{\Gamma}{2}} + \frac{1}{\epsilon_2' - \epsilon_0 + i\frac{\Gamma}{2}}\right) \mathbf{t} \otimes \mathbf{t}$$

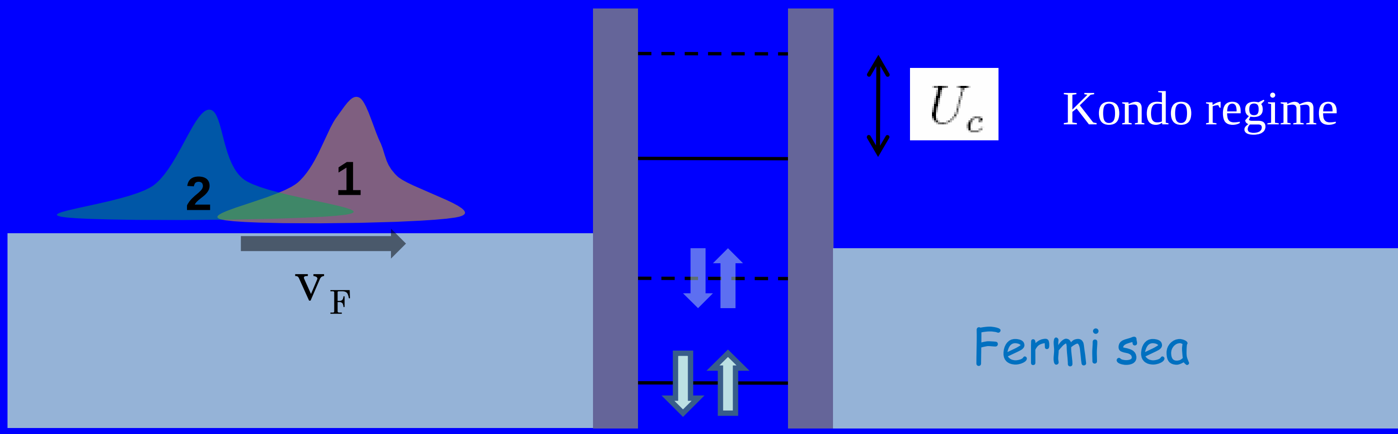
First-order perturbation calculation of Goorden and Buttiker, PRL (2007)

$$\mathbf{S}_{int} = -\frac{\pi i}{2} U_c \frac{\mathbf{S}^{(1)}(\epsilon_1) - \mathbf{S}^{(1)}(\epsilon_1')}{2\pi i(\epsilon_1 - \epsilon_1')} \frac{\mathbf{S}^{(1)}(\epsilon_2) - \mathbf{S}^{(1)}(\epsilon_2')}{2\pi i(\epsilon_2 - \epsilon_2')}$$

$$\frac{U_c}{\Gamma} \ll 1$$

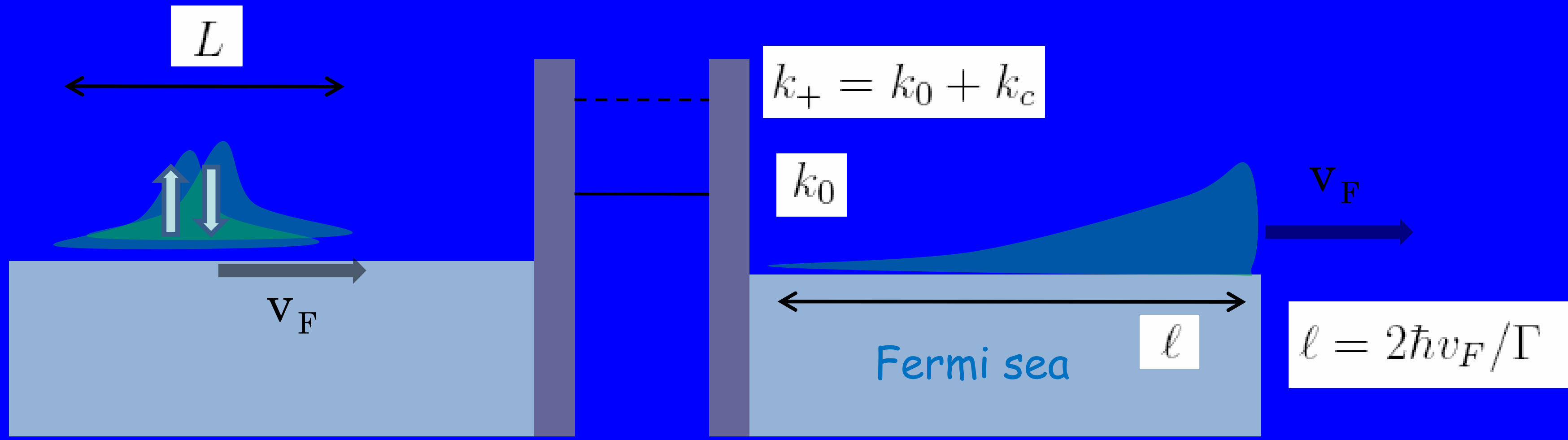
Including Fermi sea:





Scattering of the singlet state

$$k_c = U_c/(2\hbar v_F)$$



Incoming state:

$$\Psi_{in} = \phi(x_1)\phi(x_2)$$

Scattered state (both e transmitted component):

$$\Psi_{RR}(\xi_1,\xi_2) = \frac{2}{\ell} e^{ik_+ \max \xi} e^{ik_0 \min \xi} e^{(\xi_1+\xi_2)/\ell}$$

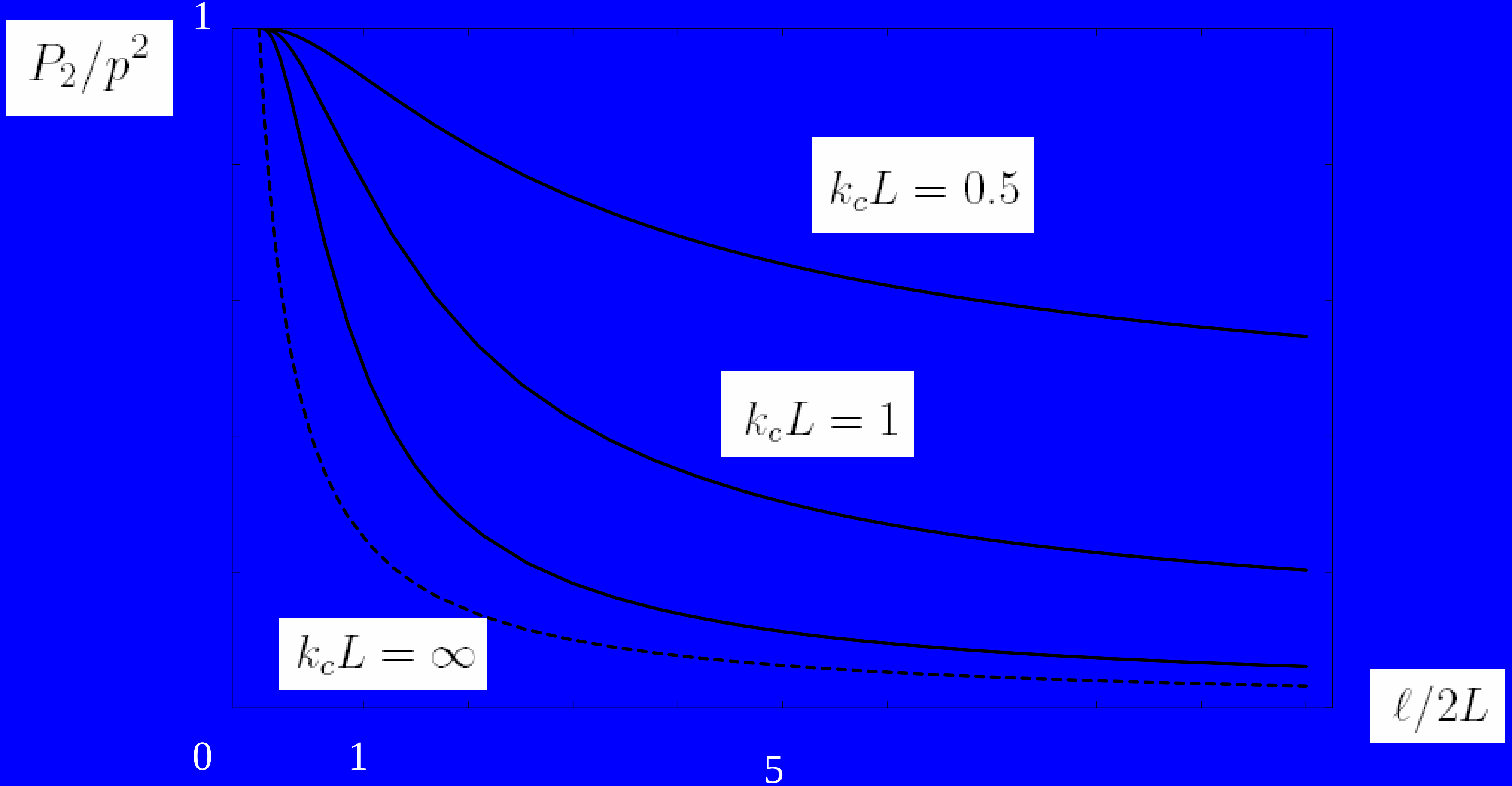
retarded variable

$$\xi_{1,2} = x_{1,2} - v_F t$$

Scattering of the singlet state.

$$\phi(x) \propto \theta(a-x)e^{ik_0(x-a)}e^{(x-a)/2L}$$

$$P_2 = p^2 f(k_c L, \frac{\ell}{2L})$$



Orbital entanglement of the transmitted state.

$$\Psi_{\mathrm{RR}} \propto e^{i(k_0+k_c/2)(\xi_1+\xi_2)} \exp\Big(i\frac{k_c}{2}|\xi_1-\xi_2|\Big) e^{(\xi_1+\xi_2)/\ell}$$

EPR –like two-particle state!

Degree of entanglement.

$$E = -\mathrm{Tr}\{\hat\rho \ln \hat\rho\}$$

$$\rho(x,x') = \int dy\, \Psi(x,y)\Psi^*(x',y)$$

$$E > 1-\mu(\hat\rho), \quad \mu(\hat\rho) = \mathrm{Tr}\{\hat\rho^2\}$$

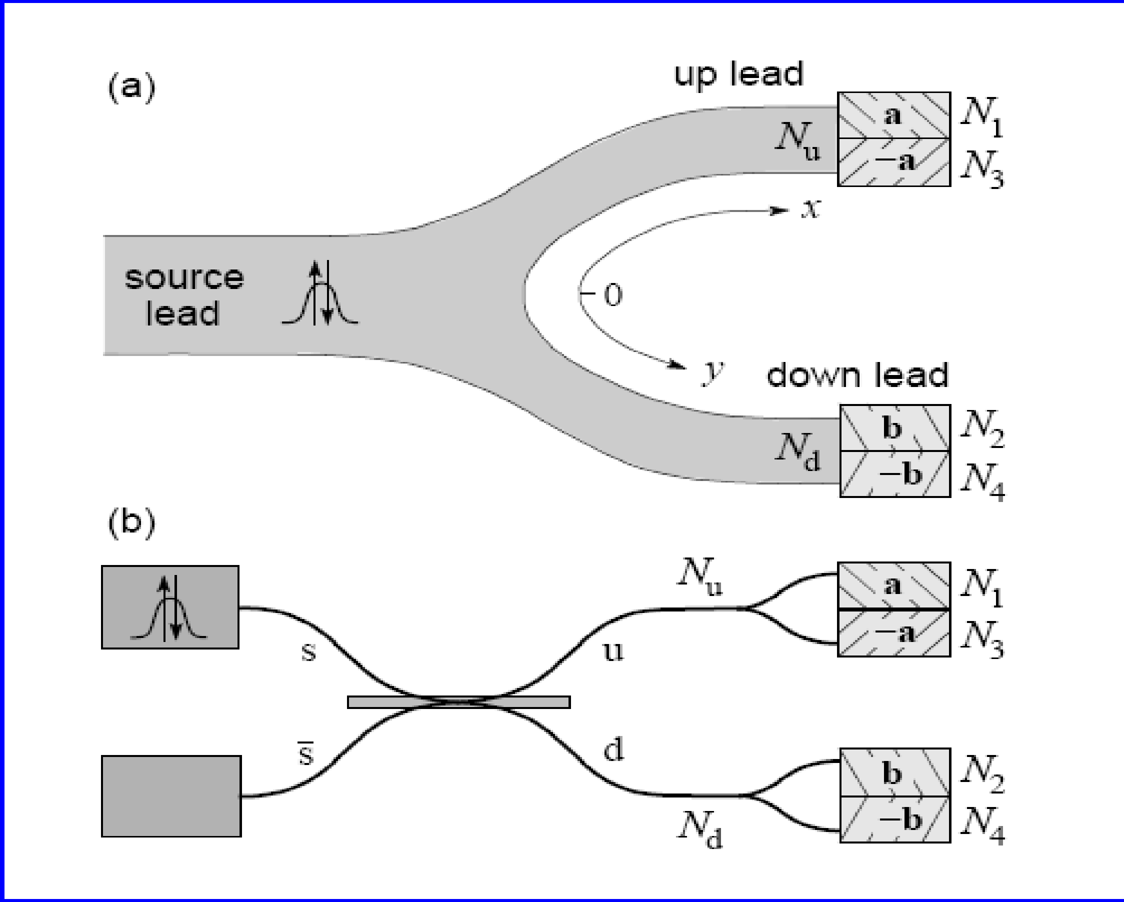
-purity of the state

$$\mu(\hat\rho) = \frac{1}{3} + \frac{2}{3}\frac{1}{1+(U_c/4\Gamma)^2}.$$

- The scattering matrix of the interacting quantum dot has been found
- Scattering probabilities are calculated
- Orbital entanglement of scattered state is demonstrated

Generating spin-entangled electron pairs in normal conductors using voltage pulses

A.V. Lebedev, G.B. Lesovik and G. Blatter 2005



Generic entanglement generation…S. Bose, D. Home 2001

BELL INEQUALITY WITH NUMBER CORRELATORS

$$|xy - x\bar{y} + \bar{x}y + \bar{x}\bar{y}| \leq 2|XY|$$

$x = N_1 - N_3$ $y = N_2 - N_4$, $X = N_1 + N_3$, and $Y = N_2 + N_4$

$$\hat{N}_i(t_{\rm ac}) = \int_0^{t_{\rm ac}} dt' \, \hat{I}_i(t').$$

$$\begin{aligned} K_{ij}(\mathbf{a}, \mathbf{b}) &= \langle \hat{N}_i(t_{\rm ac}) \hat{N}_j(t_{\rm ac}) \rangle \\ &= \int_0^{t_{\rm ac}} dt_1 dt_2 \, \langle \hat{I}_i(t_1) \hat{I}_j(t_2) \rangle \end{aligned}$$

$$E_{\mathrm{BI}} = \left| \frac{1}{2n-1} \right| \leq \frac{1}{\sqrt{2}},$$

FCS for transmission of the finite number of electrons, wave packet description

- **Th.Martin, R.Landauer, PRB 1992**

Individual electrons

- **Excitation by Lorentzian voltage pulses suggested by Ivanov, Lee, Levitov 1995**
- **Experiment - e.g. «An On-Demand Single Electron Source» Feve et al (2007)**

Two electrons

- **Exchange effects Hassler, Lesovik, Blatter 2007**