

BFKL Pomeron and scattering amplitudes in $N = 4$ SUSY

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1 Gluon reggeization

Regge kinematics of scattering amplitudes

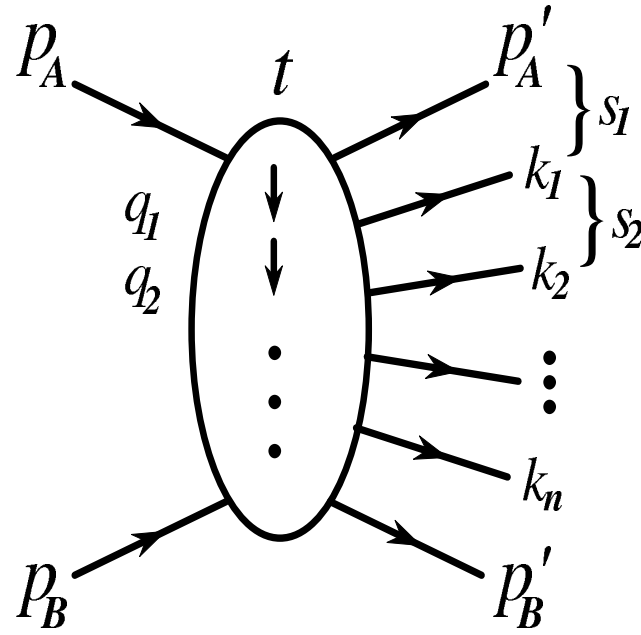
$$s = 4E^2 \gg -t = |q|^2 \approx E^2 \theta^2$$

Elastic amplitude in LLA of QCD

$$M_{AB}^{A'B'}(s, t)|_{LLA} = 2g T_{A'A}^c \delta_{\lambda_{A'}\lambda_A} \frac{s^{1+\omega(t)}}{t} g T_{B'B}^c \delta_{\lambda_{B'}\lambda_B}$$

Gluon Regge trajectory in LLA

$$\omega(-|q|^2) = -\frac{\alpha_s N_c}{4\pi^2} \int d^2k \frac{|q|^2}{|k|^2 |q-k|^2} \approx -\frac{\alpha_s N_c}{2\pi} \ln \frac{|q|^2}{\lambda^2}$$



Multi-Regge amplitudes (F.,K.,L. (1975))

$$M_{2 \rightarrow 1+n} \sim \frac{s_1^{\omega_1}}{|q_1|^2} g T_{c_2 c_1}^{d_1} C(q_2, q_1) \frac{s_2^{\omega_2}}{|q_2|^2} \dots C(q_n, q_{n-1}) \frac{s_n^{\omega_n}}{|q_n|^2},$$

$$C(q_2, q_1) = \frac{q_2 q_1^*}{q_2^* - q_1^*}, \quad \sigma_t = \sum_n \int d\Gamma_n |M_{2 \rightarrow 1+n}|^2$$

2 BFKL equation (1975)

Balitsky-Fadin-Kuraev-Lipatov equation

$$E \Psi(\vec{\rho}_1, \vec{\rho}_2) = H_{12} \Psi(\vec{\rho}_1, \vec{\rho}_2), \quad \sigma_t \sim s^\Delta, \quad \Delta = -\frac{\alpha_s N_c}{2\pi} E$$

BFKL Hamiltonian

$$H_{12} = \ln |p_1 p_2|^2 + \frac{1}{p_1 p_2^*} (\ln |\rho_{12}|^2) p_1 p_2^* \\ + \frac{1}{p_1^* p_2} (\ln |\rho_{12}|^2) p_1^* p_2 - 4\psi(1), \quad \rho_{12} = \rho_1 - \rho_2$$

Möbius invariance and conformal weights (L. (1986))

$$\rho_k \rightarrow \frac{a\rho_k + b}{c\rho_k + d},$$

$$m = \gamma + n/2, \quad \tilde{m} = \gamma - n/2, \quad \gamma = 1/2 + i\nu$$

3 BKP equation (1980)

Bartels-Kwiecinski-Praszalowicz equation

$$E \Psi(\vec{\rho}_1, \dots, \vec{\rho}_n) = H \Psi(\vec{\rho}_1, \dots, \vec{\rho}_n), \quad H = \sum_{k < l} \frac{\vec{T}_k \vec{T}_l}{-N_c} H_{kl}$$

Holomorphic separability at large N_c (L. (1988))

$$H = h + h^*, \quad h = \ln p_1 + \ln p_2 + \frac{1}{p_1} (\ln \rho_{12}) p_1 + \frac{1}{p_2} (\ln \rho_{12}) p_2 - 2\psi(1)$$

Holomorphic factorization of wave functions

$$\Psi(\vec{\rho}_1, \vec{\rho}_2, \dots, \vec{\rho}_n) = \sum_{r,s} a_{r,s} \Psi_r(\rho_1, \dots, \rho_n) \Psi_s(\rho_1^*, \dots, \rho_n^*)$$

4 Integrability at $N_c \rightarrow \infty$ (1993)

Transfer and monodromy matrices (L. (1993))

$$T(u) = \text{tr } t(u), \quad t(u) = L_1 L_2 \dots L_n = \sum_{r=0}^n u^{n-r} q_r,$$

$$L_k = \begin{pmatrix} u + \rho_k p_k & p_k \\ -\rho_k^2 p_k & u - \rho_k p_k \end{pmatrix}$$

Yang-Baxter equation (L. (1993))

$$t_{r'_1}^{s_1}(u) t_{r'_2}^{s_2}(v) l_{r_1 r_2}^{r'_1 r'_2}(v-u) = l_{s'_1 s'_2}^{s_1 s_2}(v-u) t_{r_2}^{s'_2}(v) t_{r_1}^{s'_1}(u),$$

$$\hat{l} = u \hat{1} + i \hat{P}$$

Heisenberg spin model (L. (1994); F.,K. (1995))

5 Pomeron in $N = 4$ SUSY

BFKL kernel in two loops (F., L. (1998))

$$\omega = 4\hat{a} \chi(n, \gamma) + 4\hat{a}^2 \Delta(n, \gamma), \quad \hat{a} = g^2 N_c / (16\pi^2),$$

Hermitian separability in $N = 4$ SUSY (K.,L. (2000))

$$\Delta(n, \gamma) = \phi(M) + \phi(M^*) - \frac{\rho(M) + \rho(M^*)}{2\hat{a}/\omega}, \quad M = \gamma + \frac{|n|}{2},$$

$$\rho(M) = \beta'(M) + \frac{1}{2}\zeta(2), \quad \beta'(z) = \frac{1}{4} \left[\Psi'\left(\frac{z+1}{2}\right) - \Psi'\left(\frac{z}{2}\right) \right]$$

Maximal transcendentality (K.,L. (2002))

$$\phi(M) = 3\zeta(3) + \Psi''(M) - 2\Phi(M) + 2\beta'(M) \left(\Psi(1) - \Psi(M) \right),$$

$$\Phi(M) = \sum_{k=0}^{\infty} \frac{(-1)^k}{k+M} \left(\Psi'(k+1) - \frac{\Psi(k+1) - \Psi(1)}{k+M} \right)$$

6 Maximal transcendentality for γ

Three loop anomalous dimension (K.,L.,O.,V (2002-2004))

$$\gamma(j) = \hat{a}\gamma_1(j) + \hat{a}^2\gamma_2(j) + \hat{a}^3\gamma_3(j) + \dots, \quad \gamma_1(j+2) = -4S_1(j)$$

$$\frac{\gamma_2(j+2)}{8} = 2S_1(S_2 + S_{-2}) - 2S_{-2,1} + S_3 + S_{-3}$$

$$\gamma_3(j+2)/32 = -12(S_{-3,1,1} + S_{-2,1,2} + S_{-2,2,1})$$

$$+6(S_{-4,1} + S_{-3,2} + S_{-2,3}) - 3S_{-5} - 2S_3S_{-2} - S_5$$

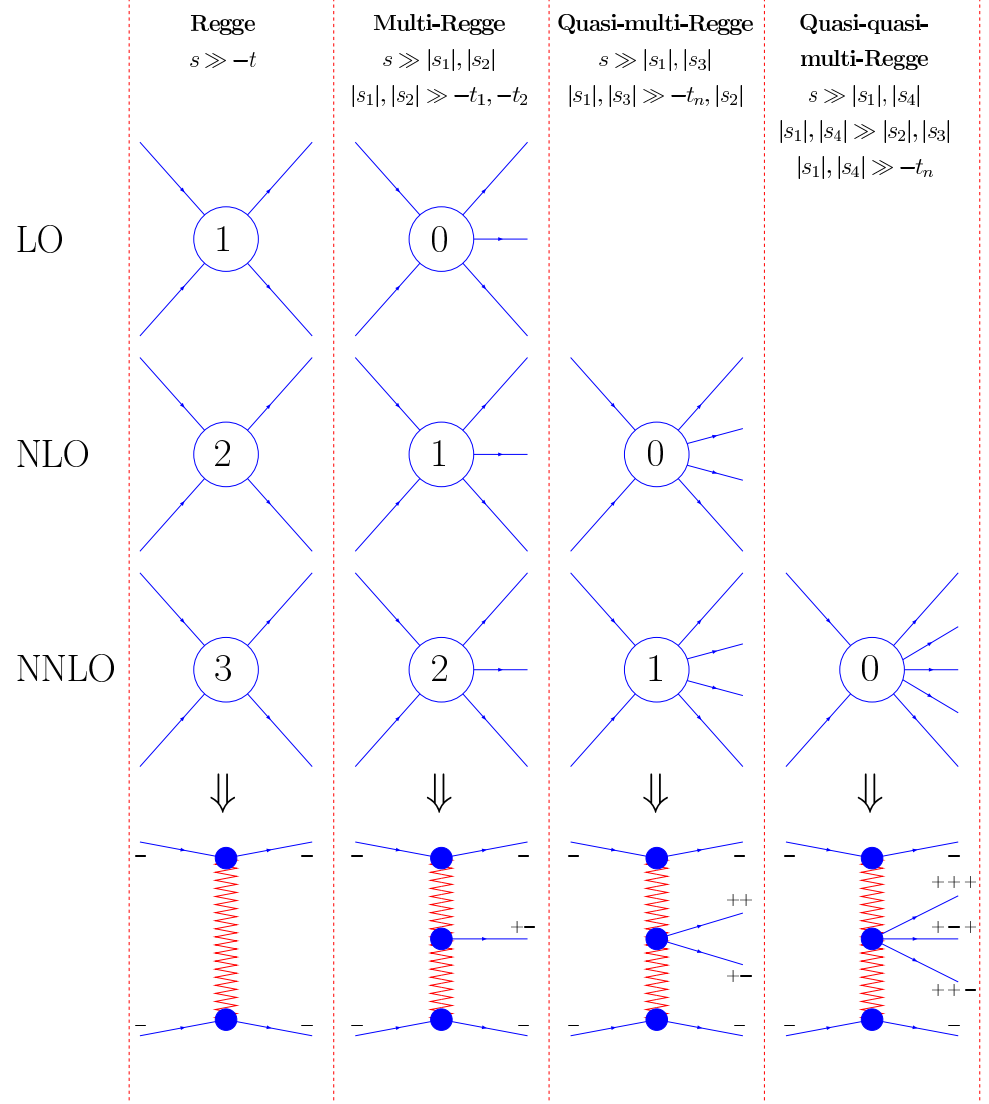
$$-2S_1^2(3S_{-3} + S_3 - 2S_{-2,1}) - S_2(S_{-3} + S_3 - 2S_{-2,1})$$

$$+24S_{-2,1,1,1} - S_1(8S_{-4} + S_{-2}^2 + 4S_2S_{-2} + 2S_2^2)$$

$$-S_1(3S_4 - 12S_{-3,1} - 10S_{-2,2} + 16S_{-2,1,1}),$$

$$S_{a,b,c,\dots}(j) = \sum_{m=1}^j \frac{1}{m^a} S_{b,c,\dots}(m), \quad S_{-a,b,\dots}(j) = \sum_{m=1}^j \frac{(-1)^m}{m^a} S_{b,\dots}(m),$$

7 Diagrams for the BFKL kernel H_0



8 Effective action approach

Gluon and Reggeized gluon fields

$$v_\mu(x) = -iT^a v_\mu^a(x), \quad A_\pm(x) = -iT^a A_\pm^a(x)$$

Local gauge transformations

$$\delta v_\mu(x) = \frac{1}{g} [D_\mu, \chi(x)], \quad \delta \psi(x) = -\chi(x) \psi(x), \quad \delta A_\pm(x) = 0$$

Effective action for reggeized gluons (L., 1995)

$$S = \int d^4x (L_0 + L_{ind}^{GR}), \quad L_0 = i\bar{\psi} \hat{D} \psi + \frac{1}{2} Tr G_{\mu\nu}^2$$

$$L_{ind}^{GR} = -\frac{1}{g} \partial_+ P \exp \left(-g \frac{1}{2} \int_{-\infty}^{x^+} v_+(x') d(x')^+ \right) \partial_\sigma^2 A_- + (+ \rightarrow -)$$

9 Maximal helicity violation

BDS amplitudes in $N = 4$ SUSY at $N_c \gg 1$ (2005)

$$A^{a_1, \dots, a_n} = \sum_{\{i_1, \dots, i_n\}} \text{Tr} T^{a_{i_1}} T^{a_{i_2}} \dots T^{a_{i_n}} f(p_{i_1}, p_{i_2}, \dots, p_{i_n}), \quad f = f_B M_n$$

Invariant amplitudes

$$\ln M_n = \sum_{l=1}^{\infty} a^l \left(f^{(l)}(\epsilon) \left(-\frac{1}{2\epsilon^2} \sum_{i=1}^n \left(\frac{\mu^2}{-s_{i,i+1}} \right)^\epsilon + F_n^{(1)}(0) \right) + C^{(l)} \right),$$

$$a = \frac{\alpha N_c}{2\pi} (4\pi e^{-\gamma})^\epsilon, \quad C^{(1)} = 0, \quad C^{(2)} = -\zeta_2^2/2, \quad f^{(l)}(\epsilon) = \sum_{k=0}^2 \epsilon^k f_k^{(1)}$$

Cusp anomalous dimension

$$f_0^{(l)} = \frac{1}{4} \gamma_K^{(l)}, \quad f_1 = \beta(f) = -a\zeta_3/2 + a^2(2\zeta_5 + 5\zeta_2\zeta_3/3) + \dots$$

10 Elastic BDS amplitude

Its Regge asymptotics at $s/t \rightarrow \infty$

$$M_{2 \rightarrow 2} = \Gamma(t) \left(\frac{-s}{\mu^2} \right)^{\omega(t)} \Gamma(t)$$

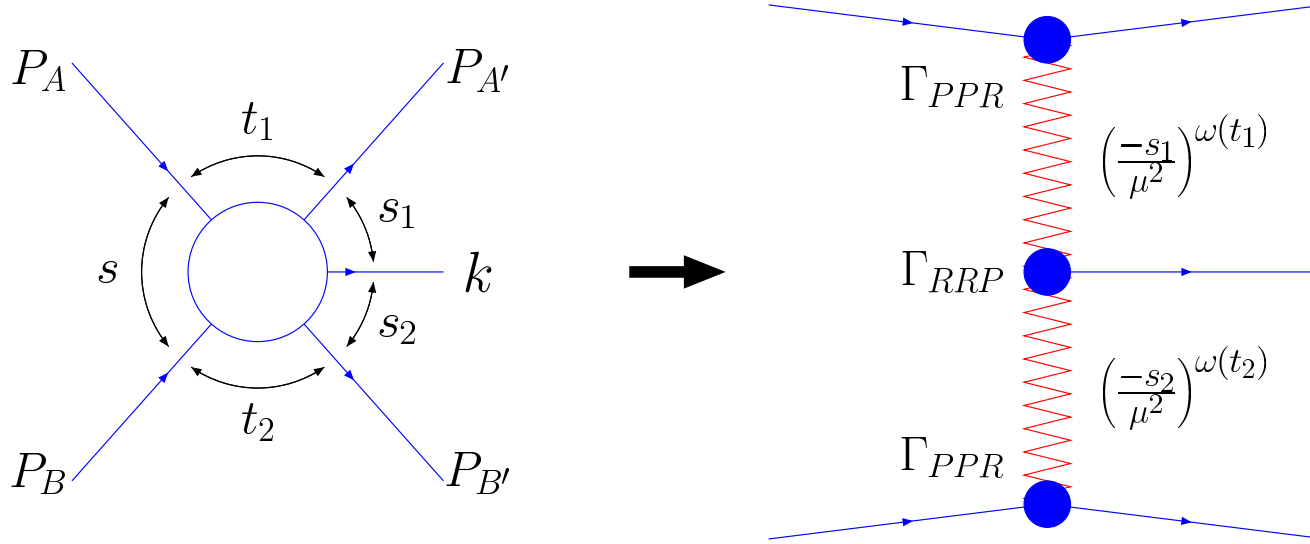
Reggeized gluon trajectory

$$\omega(t) = -\frac{\gamma_K(a)}{4} \ln \frac{-t}{\mu^2} + \int_0^a \frac{da'}{a'} \left(\frac{\gamma_K(a')}{4\epsilon} + \beta(a') \right)$$

Reggeon residues

$$\begin{aligned} \ln \Gamma(t) = & \ln \frac{-t}{\mu^2} \int_0^a \frac{da'}{a'} \left(\frac{\gamma_K(a')}{8\epsilon} + \frac{\beta(a')}{2} \right) + \frac{C(a)}{2} + \frac{\gamma_K(a)}{2} \zeta_2 \\ & - \int_0^a \frac{da'}{a'} \ln \frac{a}{a'} \left(\frac{\gamma_K(a')}{4\epsilon^2} + \frac{\beta(a')}{\epsilon} + \delta(a') \right) \end{aligned}$$

11 One particle production



$$\ln \Gamma_{\kappa=s_1 s_2/s} = -\frac{1}{2} \left(\omega(t_1) + \omega(t_2) - \int_0^a \frac{da'}{a'} \left(\frac{\gamma_K(a')}{4\epsilon} + \beta(a') \right) \right) \ln \frac{-\kappa}{\mu^2} -$$

$$\frac{\gamma_K(a)}{16} \left(\ln^2 \frac{-\kappa}{\mu^2} - \ln^2 \frac{-t_1}{-t_2} - \zeta_2 \right) - \frac{1}{2} \int_0^a \frac{da'}{a'} \ln \frac{a}{a'} \left(\frac{\gamma_K(a')}{4\epsilon^2} + \frac{\beta(a')}{\epsilon} + \delta(a') \right)$$

12 Steinman relations

Overlapping channels

$$(s_1, s_2) (2 \rightarrow 3); (s_1, s_2), (s_2, s_3), (s_{012}, s_2), (s_{123}, s) (2 \rightarrow 4)$$

Dispersion representation for $M_{2 \rightarrow 3}$ in the Regge ansatz

$$M_{2 \rightarrow 3} = c_1(-s)^{j(t_2)}(-s_1)^{j(t_1)-j(t_2)} + c_2(-s)^{j(t_1)}(-s_2)^{j(t_2)-j(t_1)}$$

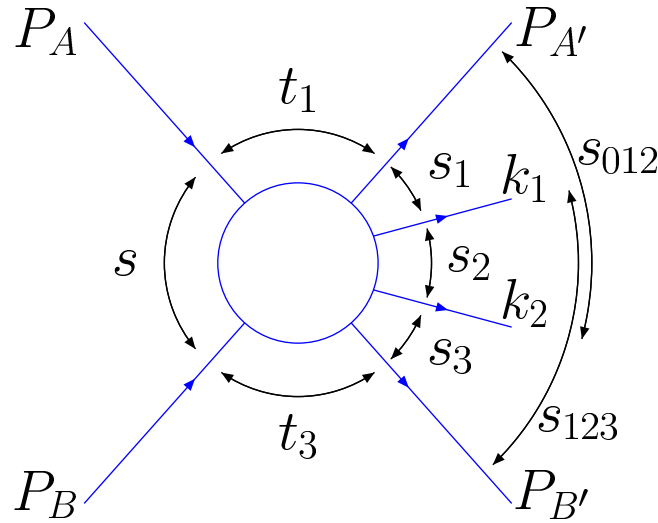
Violation of the dispersion representation for $M_{2 \rightarrow 4}$

$$\begin{aligned} M_{2 \rightarrow 4} \neq & d_1(-s)^{j_3}(-s_{012})^{j_2-j_3}(-s_1)^{j_1-j_2} + d_2(-s)^{j_1}(-s_{123})^{j_2-j_1}(-s_3)^{j_3-j_2} \\ & + d_3(-s)^{j_3}(-s_{012})^{j_1-j_3}(-s_2)^{j_2-j_1} + d_4(-s)^{j_1}(-s_{123})^{j_3-j_1}(-s_2)^{j_2-j_3} \\ & + d_5(-s)^{j_2}(-s_1)^{j_1-j_2}(-s_3)^{j_3-j_2}, \quad j_r = j(t_r) \end{aligned}$$

Important relations

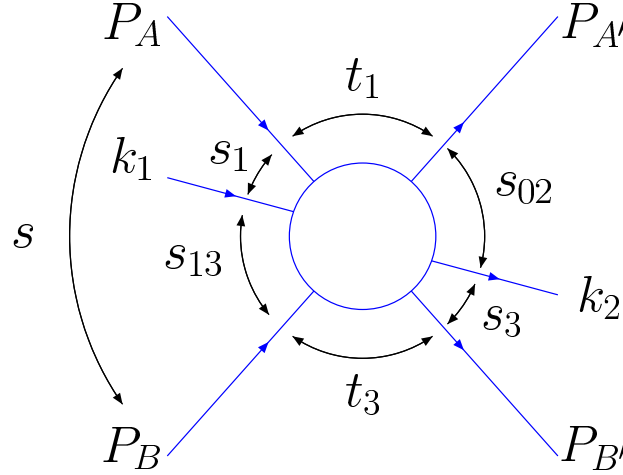
$$\Phi \equiv \frac{(-s)(-s_2)}{(-s_{012})(-s_{123})}, \quad Li_2(1-\Phi)_{\Phi \rightarrow \exp(2\pi i)} = \ln(1-\Phi) \approx \ln \frac{(\vec{k}_1 + \vec{k}_2)^2}{s_2}$$

13 Regge factorization violation



$$\begin{aligned}
 M_{2 \rightarrow 4} |_{s_2 > 0; s_1, s_3 < 0} &= \exp \left[\frac{\gamma_K(a)}{4} i\pi \left(\ln \frac{t_1 t_2}{(\vec{k}_1 + \vec{k}_2)^2 \mu^2} - \frac{1}{\epsilon} \right) \right] \\
 &\times \Gamma(t_1) \left(\frac{-s_1}{\mu^2} \right)^{\omega(t_1)} \Gamma(t_2, t_1) \left(\frac{-s_2}{\mu^2} \right)^{\omega(t_2)} \Gamma(t_3, t_2) \left(\frac{-s_3}{\mu^2} \right)^{\omega(t_3)} \Gamma(t_3)
 \end{aligned}$$

14 Factorization violation in scattering



$$\begin{aligned}
 M_{3 \rightarrow 3} |t'_2 > 0; s_1, s_3 < 0 = & \exp \left[\frac{\gamma_K(a)}{4} i\pi \ln \frac{(\vec{q}_1 + \vec{q}_3 - \vec{q}_2)^2 q_2^2}{(\vec{q}_1 - \vec{q}_2)^2 (\vec{q}_2 - \vec{q}_3)^2} \right] \\
 & \times \Gamma(t_1) \left(\frac{-s_1}{\mu^2} \right)^{\omega(t_1)} \Gamma(t_2, t_1) \left(\frac{-t'_2}{\mu^2} \right)^{\omega(t_2)} \Gamma(t_3, t_2) \left(\frac{-s_3}{\mu^2} \right)^{\omega(t_3)} \Gamma(t_3)
 \end{aligned}$$

15 Regge cuts in j_2 -plane

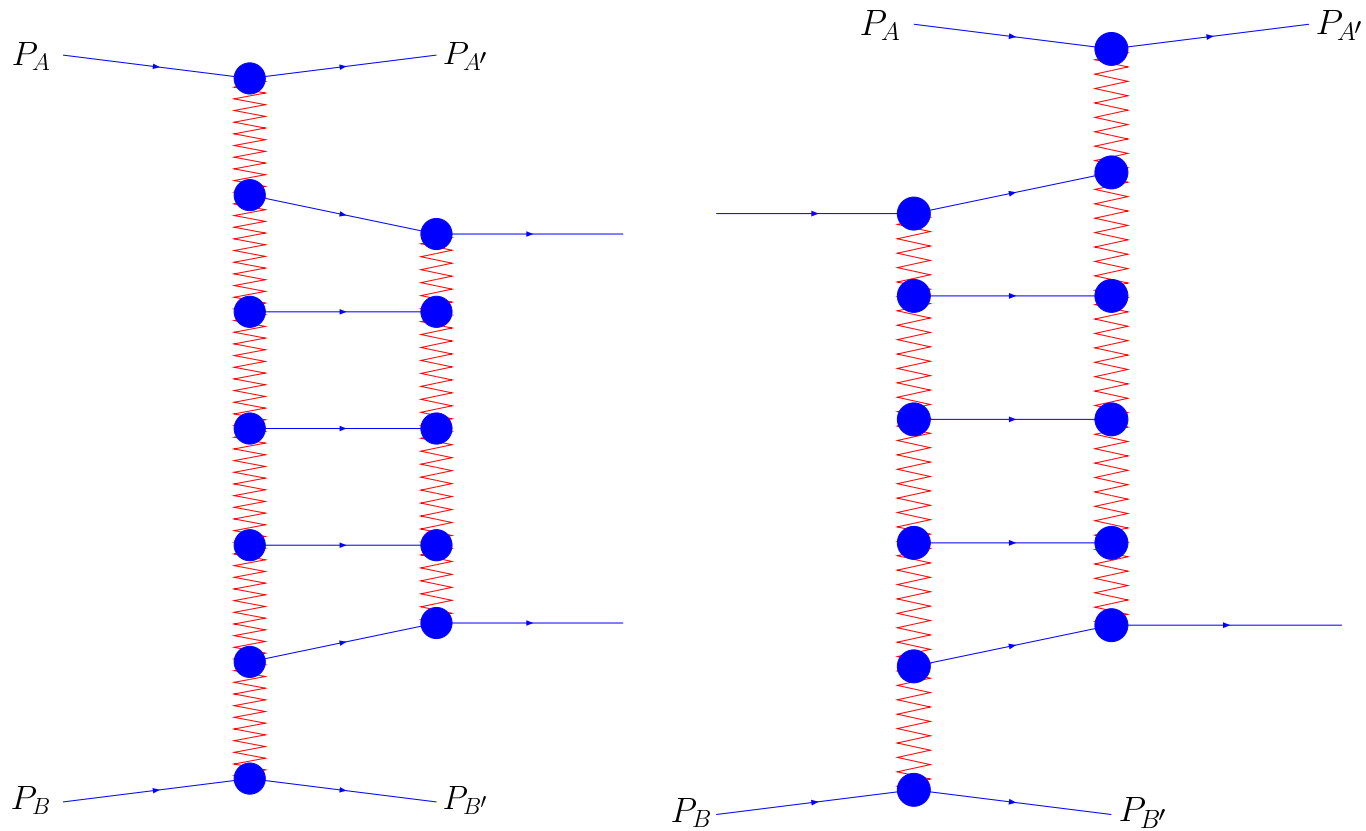


Figure 2: BFKL ladders in $M_{2 \rightarrow 4}$ and $M_{3 \rightarrow 3}$

16 BFKL equation in octet channels

Factorization of infrared divergencies in LLA

$$\lim_{\epsilon \rightarrow 0} M_{2 \rightarrow 4}^{LLA} = f_{2 \rightarrow 4}^{LLA} \lim_{\epsilon \rightarrow 0} M_{2 \rightarrow 4}^{BDS},$$

Renormalization of the intercept in the s_2 -channel

$$\Delta_2 = -a \left(E + \ln \frac{t_2}{\mu^2} - \frac{1}{\epsilon} \right)$$

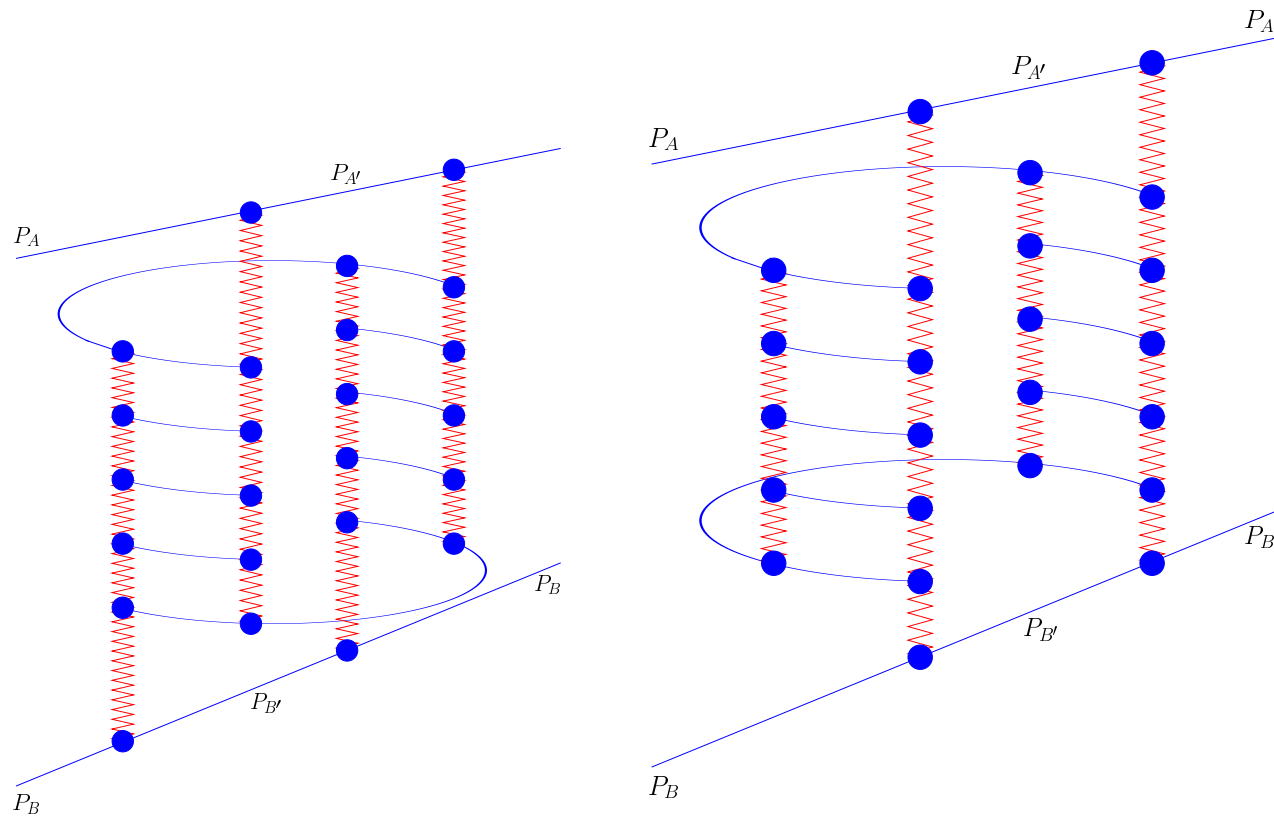
BFKL hamiltonian for the partial wave f_{j_2}

$$H = \ln \frac{|p_1 p_2|^2}{|p_1 + p_2|^2} + \frac{1}{2} p_1 p_2^* \ln |\rho_{12}|^2 \frac{1}{p_1 p_2^*} + \frac{1}{2} p_1^* p_2 \ln |\rho_{12}|^2 \frac{1}{p_1^* p_2} + 2\gamma$$

Eigenfunctions and eigenvalues

$$\Psi_{n,\nu} = \left(\frac{p_1}{p_2} \right)^{i\nu+n/2} \left(\frac{p_1^*}{p_2^*} \right)^{i\nu-n/2}, \quad E_{n,\nu} = 2\text{Re} \psi(i\nu + \frac{|n|}{2}) - 2\psi(1)$$

17 Corrections to the total cross-section



18 Multi-gluon states in octet channels

Equation for n-gluon composite states

$$\frac{1}{2}(h + h^*)\Psi = E\Psi, \quad h = \ln(Z_1^2 \partial_1) - 2\psi(1) + \ln \partial_{n-1} + \sum_{k=1}^{n-2} h_{k,k+1}$$

Pair hamiltonian of the spin chain

$$h_{1,2} = \ln(Z_{12}^2 \partial_1) + \ln(Z_{12}^2 \partial_2) - 2 \ln Z_{12} - 2\psi(1)$$

Monodromy matrix

$$t(u) = L_1(u)L_2(u)\dots L_{n-1}(u) = \begin{pmatrix} A(u) & B(u) \\ C(u) & D(u) \end{pmatrix}$$

Integrals of motion for the open spin chain

$$[D(u), h] = 0$$

19 Three-gluon composite state

Integrals of motion for three gluons

$$\hat{\lambda} = -i(Z_1 p_1 + Z_2 p_2), \quad \hat{q} = -Z_1 Z_{12} p_1 p_2$$

Two independent eigenfunctions

$$\Psi_{\lambda q}^{\pm} = Z_2^{\lambda} \sum_{\mu - \lambda_{\pm} = 0}^{\infty} \frac{\Gamma(\mu) \Gamma(\mu - \lambda) (Z_1/Z_2)^{\mu}}{\Gamma(\mu - \lambda_+ + 1) \Gamma(\mu - \lambda_- + 1)}, \quad \lambda_{\pm} = \frac{\lambda}{2} \pm \sqrt{\frac{\lambda^2}{4} + q}$$

Holomorphic factorization of the wave function

$$\Psi = \sum_{r=\pm} C_r \Psi_{\lambda q}^r(Z_1, Z_2) \Psi_{\tilde{\lambda} \tilde{q}}^r(Z_1^*, Z_2^*)$$

Quantization of λ and q from the single-valuedness for Ψ

$$\lim_{Z_1/Z_2 \rightarrow 1} \Psi, \quad \lim_{Z_1/Z_2 \rightarrow \infty} \Psi$$

20 Discussion

1. Pomeron is a composite state of reggeized gluons.
2. BFKL dynamics is integrable in LLA.
3. Remarkable properties of the kernel in $N = 4$ SUSY.
4. Next-to-next-to-leading corrections.
5. Effective action for Reggeized gluon interactions.
6. BDS amplitudes in the multi-Regge kinematics.
7. Breakdown of the Regge factorization.
8. BFKL cuts in the planar amplitudes M_n for $n > 5$.
9. Integrability of equations for multi-gluon states.