Critical Quantum Spin Liquids in 2d

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Interest: Spin liquid phases of 2d Mott insulators

Three classes of 2d Spin liquids

a) **Topological**
b) **Critical**
c) **"Quantum spin metals"** - singular "Bose" surfaces

Results:

- Quantum spin metals have n-leg ladder descendents, a new class of quasi-1d spin liquid phases
- n-leg ladder descendents are accessible via controlled analysis
  (ED, DMRG, VMC, Bosonizaton of gauge theory)
- Existence of "Quantum spin metal" for XY AFM/ring-exchange on 2-leg ladder

Mott Insulators

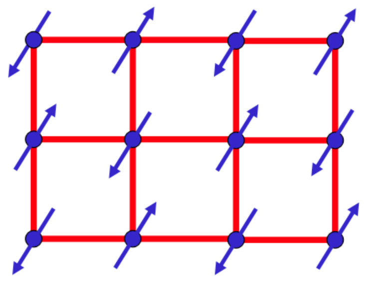
Insulator with an odd number of electrons/unit cell (eg. Hubbard with U>>t)

Spin physics: $$H_{spin} = J \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j$$

Symmetry Breaking ➡ Unit cell doubling ("Band Insulator")

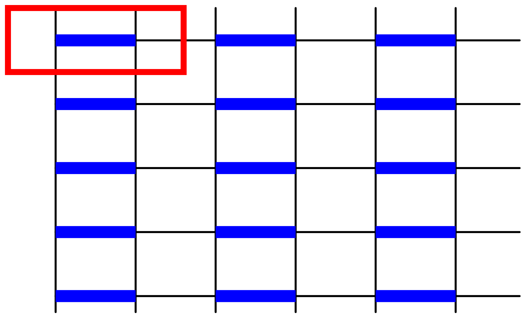
2 electrons/cell

- Magnetic Long Ranged Order



- Spin Peierls: Valence bond crystal

$— = |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$



2d Spin Liquids

Mott insulators with no broken symmetries

Theorem: 2d Mott insulator on an L by L torus has a gap that vanishes as ln(L)/L or faster (Matt Hastings, 2005)

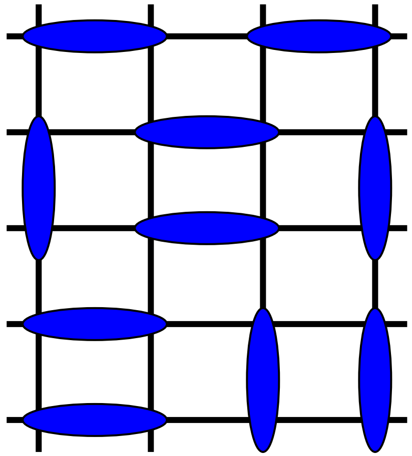
Implication: “Exotic” Quantum ground state guaranteed if no broken symmetries - a spin liquid

3 Classes of spin liquids

1) “Topological” Spin Liquids
2) Critical (“algebraic”) Spin Liquids
3) “Quantum Spin Metals”

Topological Spin Liquids

RVB state (Anderson)



- Ground state degeneracies on a torus
- Gapped excitations with fractional statistics; Abelian and non-Abelian

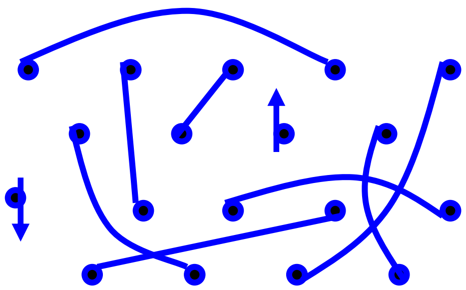
Hamiltonians Exhibiting Topological Spin Liquids

- Soluble lattice gauge theories (loop models) - contrived and not built with spin operators
- Triangular lattice quantum dimer model (not a spin model)
- Kagome $J_1=J_2=J_3$ spin model in easy-axis limit (not SU(2) invariant)

No physically reasonable spin model (yet) shown to have a Topological spin liquid ground state

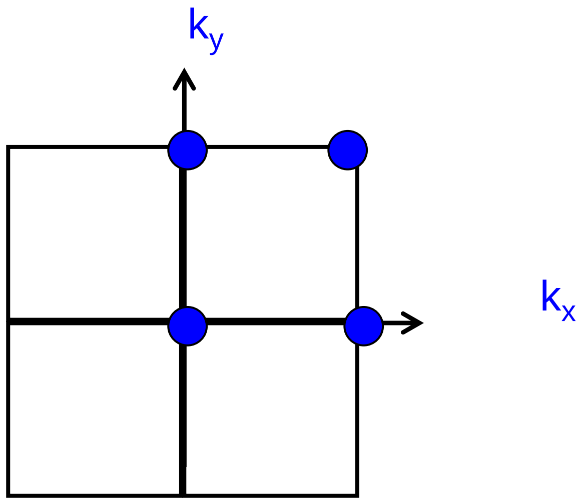
Critical Spin Liquids

- *Stable gapless phase,* no broken symmetries and no free particle description
- Power-law correlations at set of discrete momenta, **K_j , j=1,2,…N**



Valence bonds on all length scales

$$G(\mathbf{r}) \sim \sum_{j=1}^{N} \frac{\cos(\mathbf{K}_j \cdot \mathbf{r} + \phi_\alpha)}{|\mathbf{r}|^{\eta_j}}$$



Effective field theory is often "relativistic":

1) Compact QED3, staggered flux phase of fermionic spinons;
2) Non-compact QED3, Fermionized vortices

$$\mathcal{L}_{QED3} = \overline{\psi}_a \gamma^\mu (\partial_\mu - i\tilde{a}_\mu)\psi_a$$

No physically reasonable spin model shown to have a Critical spin liquid ground state

“Quantum spin metals”

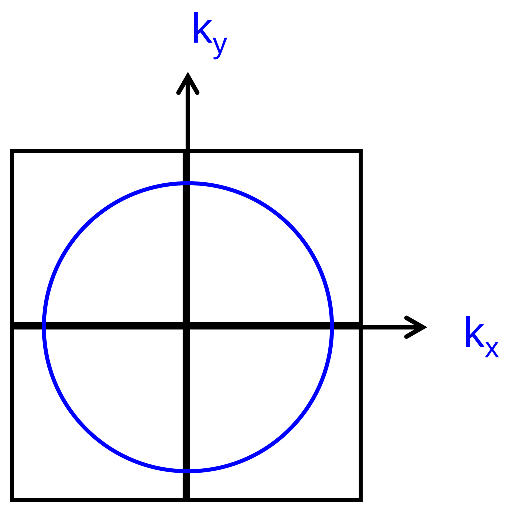
As in critical spin liquid: Gapless phase with no
broken symmetries or free particle description

Correlation functions singular along surfaces in momentum space

Effective field theory is not “relativistic”:

(eg. uniform RVB state with Fermi surface of
spinons coupled to a compact U(1) gauge field)

Stability of such phases? Field theory is uncontrolled



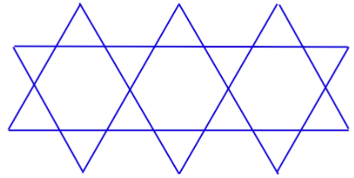
**No spin Hamiltonians known to have
a “Quantum spin metal” ground state**

Arguably the most complicated (entangled?) type of spin liquid

Experimental Candidates?

1) Topological Spin Liquids: no candidates

2) Critical Spin Liquids: Kagome lattice compounds



- Iron Jarosite, $KFe_3(OH)_6(SO_4)_2$: Fe^{3+} s=5/2 , $f = T_{cw}/T_N \sim 20$
- 2d "spinels" $SrCr_8Ga_4O_{19}$ Cr^{3+} s=3/2, f ~ 100
- Volborthite $Cu_3V_2O_7(OH)_2\ 2H_2O$ Cu^{2+} s=1/2 f ~ 75
- Herbertsmithite $ZnCu_3(OH)_6Cl_2$ Cu^{2+} s=1/2 , f > 600

Key elements? Frustration, low spin, low coordination number

3) "Quantum Spin Metals"

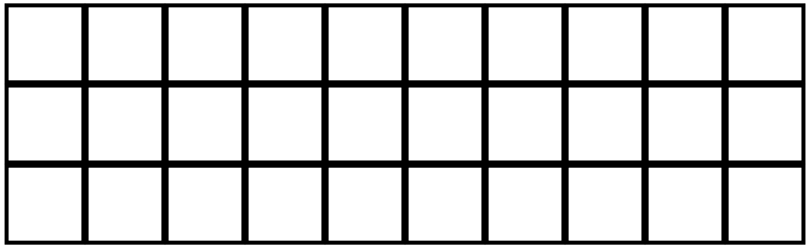
- Triangular lattice Organic: κ-$(ET)_2Cu_2(CN)_3$: s=1/2 ,f ~ 10^4

Key elements? Mott insulator with small charge gap, large ring exchange

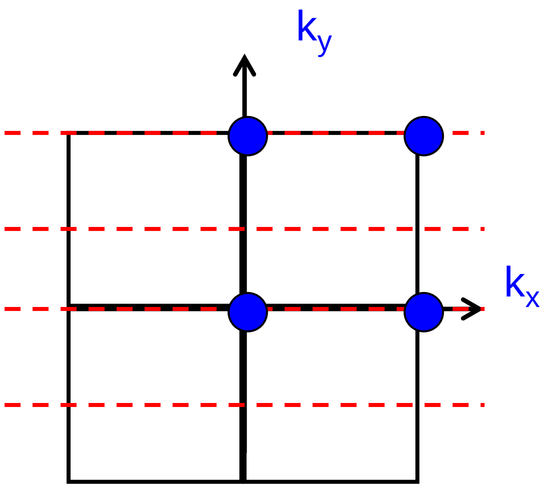
$$\mathcal{H} = J \sum_{<ij>} \mathbf{S}_i \cdot \mathbf{S}_j + K \sum_{rings} (\mathcal{P}_4 + \mathcal{P}_4^{-1}) \qquad J \sim K$$

“Quantum Spin Metals”: Ladders to the Rescue

n-leg Ladder:

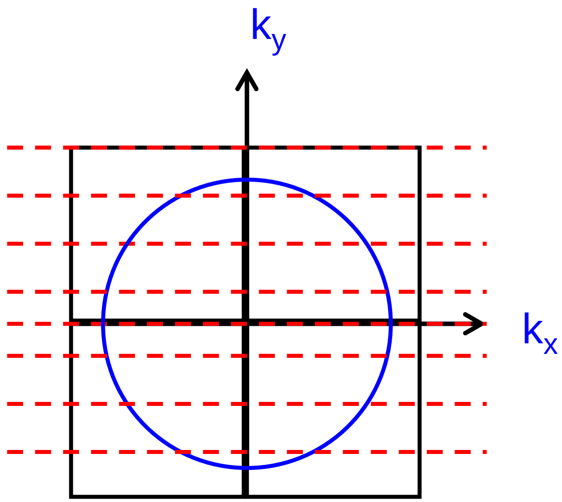


Neel or Critical Spin liquid



Few gapless 1d modes

“Quantum Spin Metal”



Fingerprint of 2d singular surface -
many gapless 1d modes, of order N

Expectation: New quasi-1d spin liquid phases on n-leg ladders, each a descendent of a 2d quantum spin metal

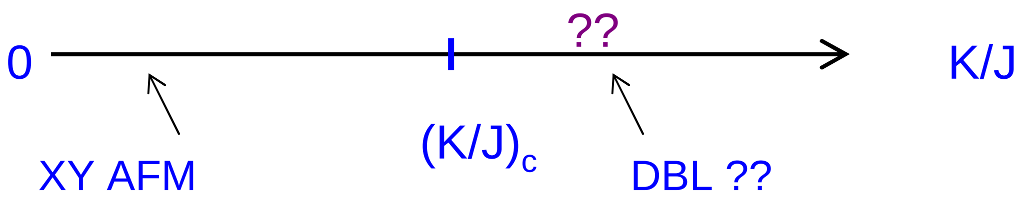
2d D-Wave Bose Liquid (DBL)
(a putative “quantum spin metal”)

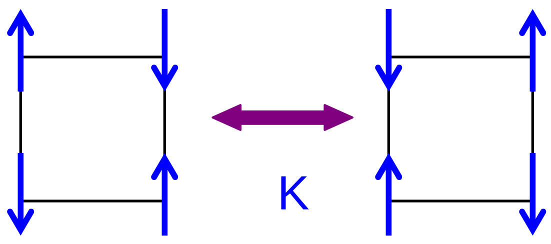
O. Motrunich/MPAF

Hamiltonian: s=1/2 square lattice with 4-site ring exchange,
Zeeman field in easy-plane limit (maybe unnecessary)

$$\mathcal{H}_{JK} = J \sum_{<ij>} (S_i^+ S_j^- + h.c.) + K \sum_{rings} (S_1^+ S_2^- S_3^+ S_4^- + h.c.) + h \sum_i S_i^z$$

Phase diagram: K/J and <S^z>



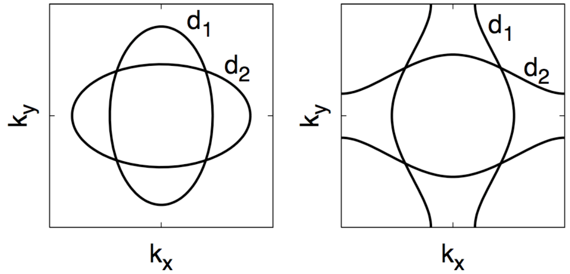


J-K Model has a sign problem - completely intractable

Theory of DBL phase

Slave Fermion mean field with anisotropic Fermi surfaces

$$S^+ = d_1^\dagger d_2^\dagger$$

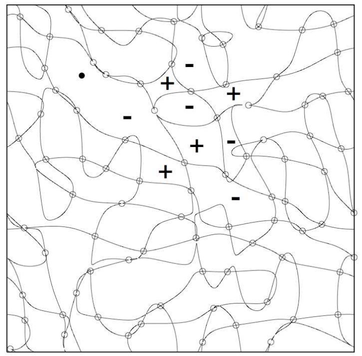


Gauge Theory: Fermions coupled to U(1) gauge field

Gutzwiller wavefunction: product of elongated determinants,

$$\Psi_{D_{xy}}(\mathbf{r}_1, ..., \mathbf{r}_N) = (det)_x \times (det)_y$$

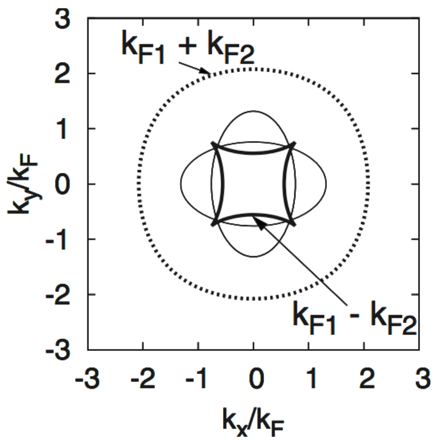
d_{xy} nodal structure



Spin-spin correlator singular on 2 “Bose” Surfaces

$$G_\pm(\mathbf{k}) = \int_\mathbf{r} e^{i\mathbf{k}\cdot\mathbf{r}} \langle S^+(\mathbf{r}) S^-(\mathbf{0}) \rangle$$

$$\mathbf{k}_{F_1}(\hat{\mathbf{r}}) \pm \mathbf{k}_{F_2}(\hat{\mathbf{r}})$$

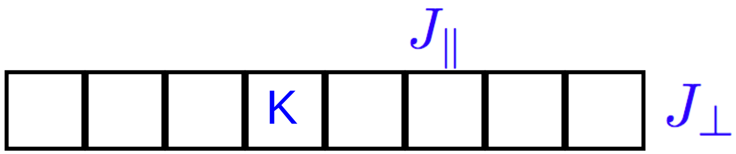


DBL "quantum spin metal" on the 2-Leg Ladder

Studied H_{JK} on 2-leg ladder

(E. Gull, D. Sheng, S. Trebst, O. Motrunich and MPAF)

- Exact Diagonalization (2 x 18)
- Variational Monte Carlo
- DMRG (2 x 50)
- quasi-1d gauge theory via bosonization

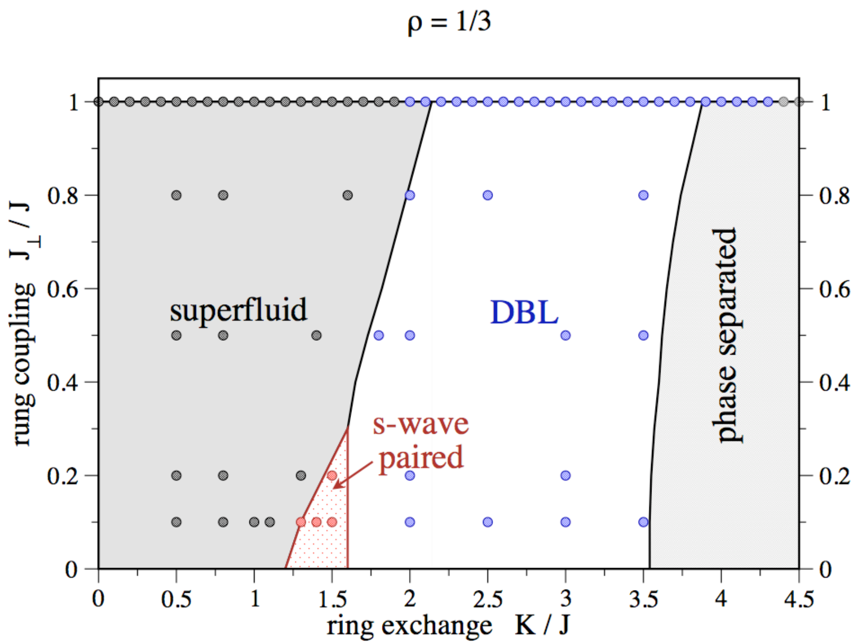


Phase Diagram

Two Phases:

XY AFM (Superfluid)

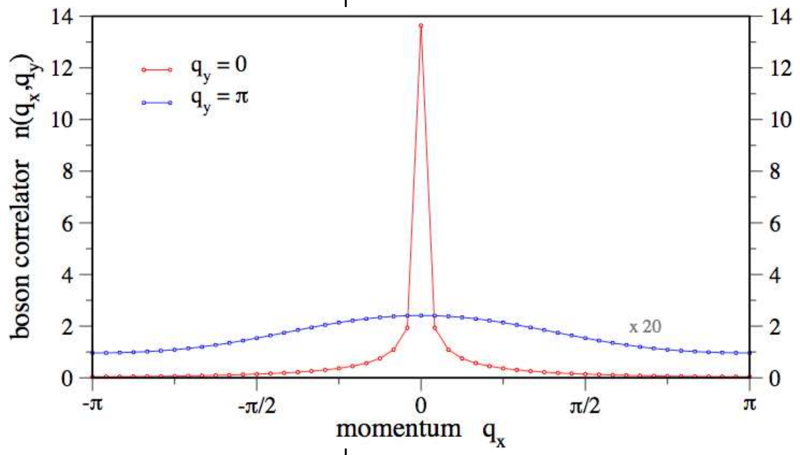
D-Wave Bose Liquid - DBL



Spin-correlators for 2-leg XY AFM and DBL Phases

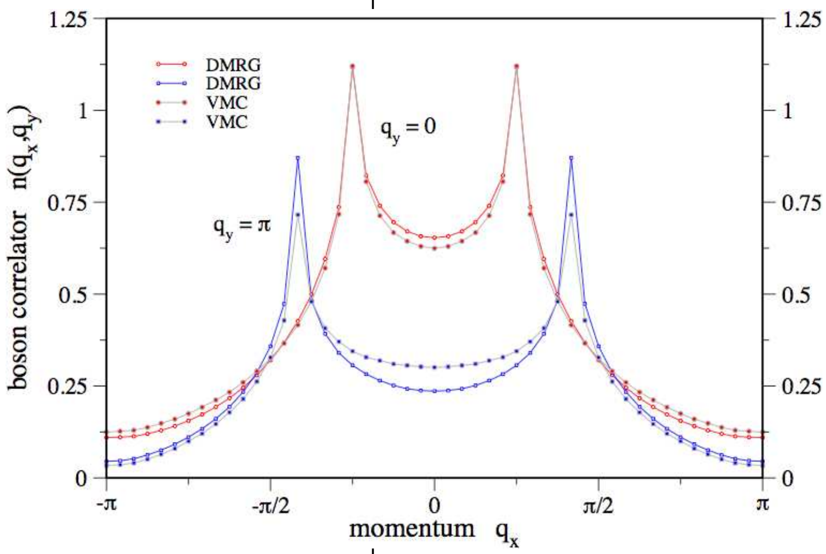
$$G_{\pm}(k_x, k_y) = \langle S^{+}_{\mathbf{k}} S^{-}_{\mathbf{k}} \rangle \qquad k_y = 0, \pi$$

DMRG 2 x 48



XY AFM
"1d Bose superfluid"

DMRG/VMC 2 x 48



DBL "quantum spin metal"

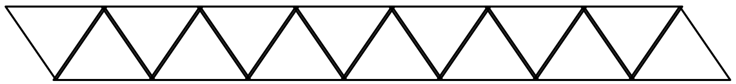
Triangular AFM with cyclic 4-site ring exchange

$$\mathcal{H}_\Delta = J \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j + K \sum_{\langle 1234 \rangle} [\mathcal{P}_{1234} + \mathcal{P}_{1234}^{-1}]$$

Motivation:

- 2d Wigner crystal of electrons
- Triangular lattice Organic: κ-$(ET)_2Cu_2(CN)_3$ (Kanoda et. al.)
  a "weak" Mott insulator near Mott transition
  s=1/2 triangular lattice AFM with J = 250 K, no magnetic ordering down to 50mK

Approach triangular lattice via quasi-1d strips



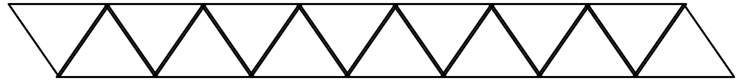
Underway: Analysis of J-K model on zigzag strip
via ED, DMRG, VMC, Bosonization of Gauge theory (D. Sheng, O. Motrunich, MPAF)
Preliminary data: "Quantum spin metal" present for K/J > 0.2

Summary & Outlook

- Three types of 2d spin liquids: Topological, critical and "quantum spin metal"
- "Quantum spin metals" have singular "Bose" surfaces and ladder descendents
- Established existence of the 2-leg ladder descendent of the DBL for 2d XY J-K ring Hamiltonian

Future generalizations (DMRG, VMC, gauge theory):

- n-leg ladders approaching 2d DBL
- Triangular strips: Heisenberg/ring and Hubbard models
  Quasi-1d descendents of "spinon fermi surface" phase?
- "D-Wave Metal" on the n-leg ladder?

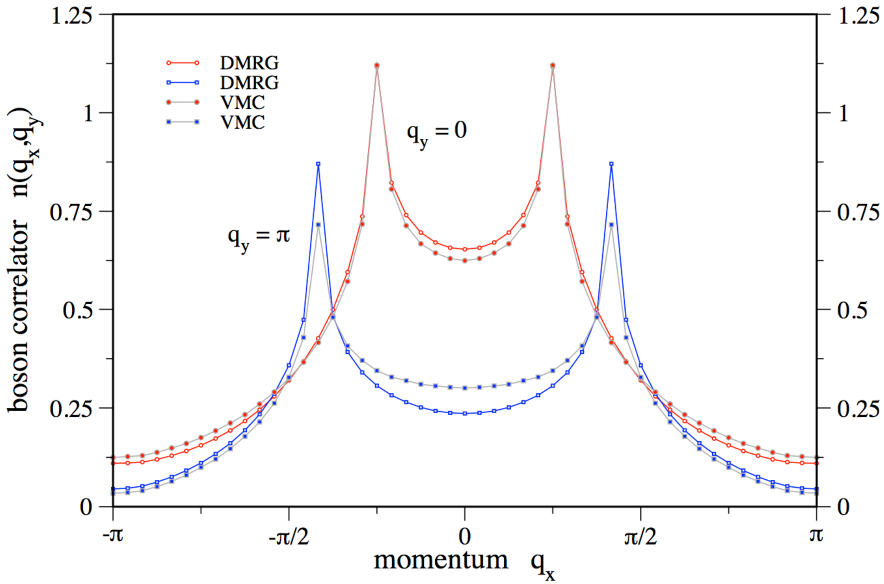


Ampere's Law

Gauge mean field theory predicts singularities at:

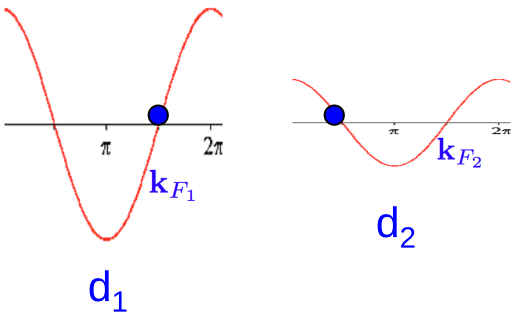
$$\mathbf{k}_{F_1} \pm \mathbf{k}_{F_2}$$

Both DMRG and $\det_1 \times \det_2$ Wavefunction show singular cusps *only* at: $\mathbf{k}_{F_1} - \mathbf{k}_{F_2}$



Why? Gauge theory encodes Ampere's Law - Parallel currents attract

d_1 and d_2 Fermions have opposite gauge charge, so right moving d_1 attracts left moving d_2 to form boson at momentum: $\mathbf{k}_{F_1} - \mathbf{k}_{F_2}$



D-Wave Metal

Itinerant non-Fermi liquid phase of 2d electrons

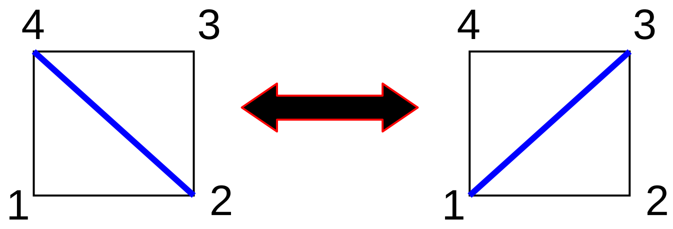
$$c_\alpha^\dagger(\mathbf{r}) = b^\dagger(\mathbf{r}) f_\alpha^\dagger(\mathbf{r}) = d_x^\dagger(\mathbf{r}) d_y^\dagger(\mathbf{r}) f_\alpha^\dagger(\mathbf{r})$$

Wavefunction:

$$\Psi_{\mathrm{electron}}(\uparrow,\downarrow) = [(\det)_x \times (\det)_y](\uparrow,\downarrow) \ \det(\uparrow)\ \det(\downarrow)$$

t-K Ring Hamiltonian (from strong coupling gauge theory)
(no double occupancy constraint)

$$\mathcal{H}_t = -t \sum_{\langle ij \rangle} c_{i\alpha}^\dagger c_{j\alpha} + h.c.$$



$$\mathcal{H}_K = K \sum_{plaquettes} [\mathcal{S}_{13}^\dagger \mathcal{S}_{24} + h.c.]$$

$$\mathcal{S}_{ij}^\dagger = \frac{1}{\sqrt{2}}[c_{i\uparrow}^\dagger c_{j\downarrow}^\dagger - c_{i\downarrow}^\dagger c_{j\uparrow}^\dagger]$$

Electron singlet pair
“rotation” term

t >> K Fermi liquid
t ~ K D-metal (?)