Anomalous localization and quantum Hall effect in disordered graphene

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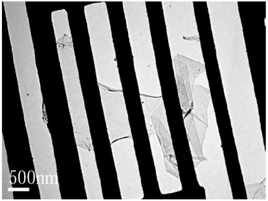
Outline

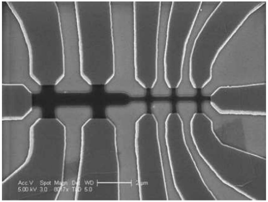
1. Introduction
   - Experimental facts
   - Model
2. Anomalous Quantum Hall effect
   - Odd quantization
   - Ordinary quantization
   - Absence of quantization
3. Absence of localization at $B = 0$
   - Unitary class
   - Symplectic class
4. Ballistic transport
   - Clean system
   - Disordered system
   - Single parameter scaling

Outline

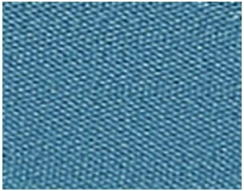
Graphene samples

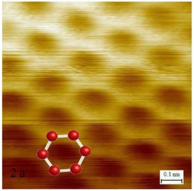


Suspended sample

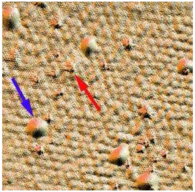


Hall bar



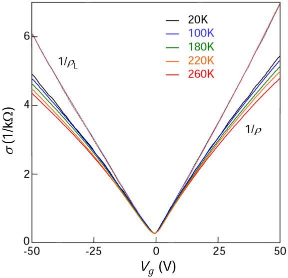


Micro-mechanical cleavage

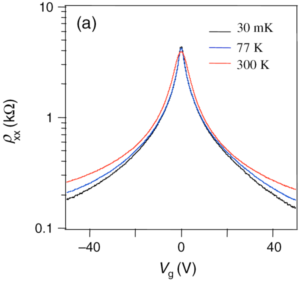


Epitaxial growth

Experiments on conductivity
Density dependence



Novoselov, Geim *et al.* '08

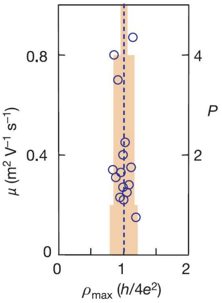


Zhang, Tan, Stormer, Kim '07

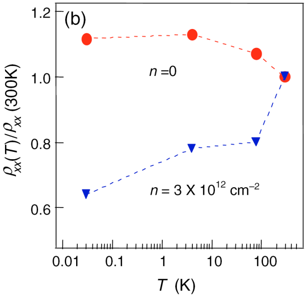
Conductivity is linear in density:

- long-range Coulomb impurities
- corrugations (ripples)

Experiments on conductivity
Minimal conductivity



Novoselov, Geim *et al.* '05

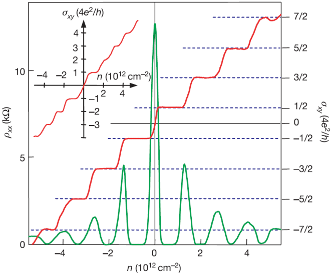


Zhang, Tan, Stormer, Kim '07

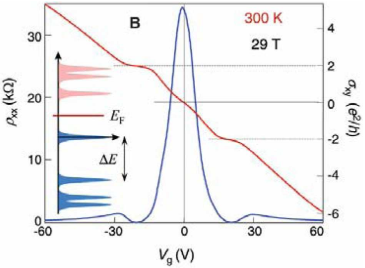
Minimal conductivity

- of order e^2/h
- temperature independent $\Longrightarrow$ no localization!

Experiments on QHE



Novoselov, Geim *et al.* '05

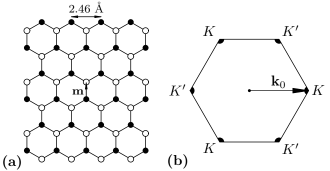


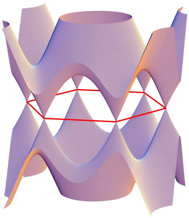
Novoselov, Geim, Stormer, Kim '07

Anomalous quantum Hall effect

- only odd plateaus: $\sigma_{xy} = (2n+1)2e^2/h$
- QHE transition at zero concentration
- visible up to room temperature!

Clean graphene model





Tight-binding approximation

- two sublattices: A, B
- two valleys: K, K′
- linear dispersion: $\varepsilon = v_0|\mathbf{p}|$
- massless Dirac Hamiltonian:
  K: $H = v_0\boldsymbol{\sigma}\mathbf{p}$ K′: $H = -v_0\boldsymbol{\sigma}^T\mathbf{p}$
  $\boldsymbol{\sigma} = \{\sigma_x, \sigma_y\}$
- velocity: $v_0 \approx 10^8$ cm/s
- band width: $\Delta \sim 1$ eV

Disorder model

- valleys decouple for long-range disorder
- Dirac equation with disorder:

$$-iv_0\,\boldsymbol{\sigma}\nabla\Psi + V(x,y)\Psi = \epsilon\Psi$$

- two-component wave function $\Psi = \{\phi_A, \phi_B\}$
- $V = \sum_\mu \sigma_\mu V_\mu$ random field (with structure in sublattices)

Types of disorder

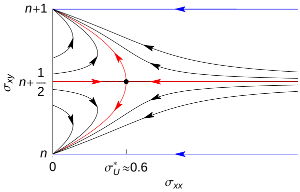
- $\sigma_0 = \hat{1}$: random potential (charged impurities)
- σ_x, σ_y: random vector potential (ripples)
- σ_z: random mass

  [Ludwig *et al.* '94; Nersesyan, Tsvelik, Wenger '94]

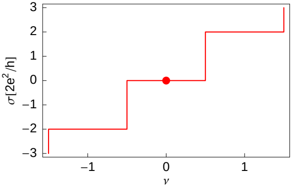
Outline

Decoupled valleys: paradox?

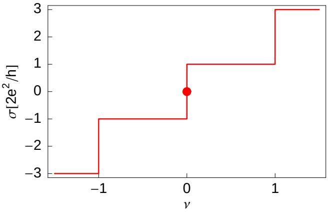
Conventional field theory [Pruisken '84, Khmelnitskii '84]



2 valleys × 2 spin $\Longrightarrow$



Experiment



- Why odd plateaus?
- What is the RG flow?
- When may this happen?
- What are other options?

Single valley conductivities

$$\sigma_{xx} = -\frac{1}{2}\,\mathrm{Tr}\left[j_x(G^R - G^A)j_x(G^R - G^A)\right] \qquad \text{(bulk)}$$

$$\sigma_{xy}^{I} = \frac{1}{2}\,\mathrm{Tr}\left[j_x(G^R - G^A)j_y(G^R + G^A)\right] \qquad \text{(bulk)}$$

$$\sigma_{xy}^{II} = \frac{ie}{2}\,\mathrm{Tr}\left[(xj_y - yj_x)(G^R - G^A)\right] \qquad \text{(edge)}$$

Boundary conditions important!

Single valley $\Longrightarrow$ infinite mass boundary condition

$H = v_0 \boldsymbol{\sigma}\mathbf{p} + m\sigma_z,$ $m \to \infty$ at the edge

Hall conductivity: $$2\sigma_{xy} = \underbrace{\left(\sigma_{xy} + \frac{1}{2}\right)}_{\text{valley K}} + \underbrace{\left(\sigma_{xy} - \frac{1}{2}\right)}_{\text{valley K}'}$$

appears in σ-model

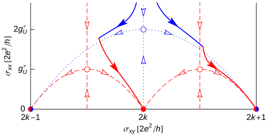
Effective field theory: σ-model

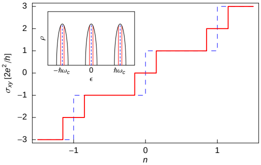
Single valley (unitary σ-model with topological term $\theta = 2\pi\sigma_{xy} + \pi$):

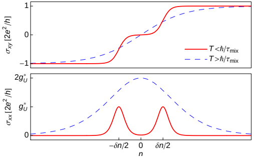
$$S[Q] = \frac{1}{4}\,\mathrm{Str}\left[-\frac{\sigma_{xx}}{2}(\nabla Q)^2 + \left(\sigma_{xy} + \frac{1}{2}\right) Q\nabla_x Q\nabla_y Q\right]$$

Weakly mixed valleys:

$$S[Q_K, Q_{K'}] = S[Q_K] + S[Q_{K'}] + \frac{\rho}{\tau_{\mathrm{mix}}}\,\mathrm{Str}\; Q_K Q_{K'}$$







Even plateau width $\sim (\tau/\tau_{\mathrm{mix}})^{0.45}$, visible at $T < T_{\mathrm{mix}} \sim \hbar/\tau_{\mathrm{mix}}$

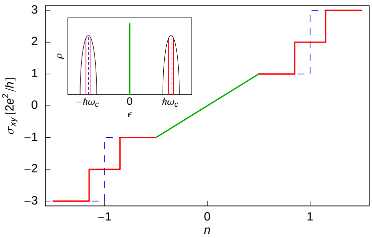
Estimate for Coulomb scatterers: even plateaus 5%, $T_{\mathrm{mix}} \sim 100$ mK,

Chiral disorder: "Classical" quantum Hall effect

- Ripples ⇔ Abelian random vector potential
- Dislocations ⇔ non-Abelian random vector potential

Atiyah–Singer theorem: Zero Landau level remains degenerate

$\Longrightarrow$ no localization — Aharonov, Casher '79



- Ripples: odd plateaus
- Ripples + Dislocations: all non-zero plateaus

Outline

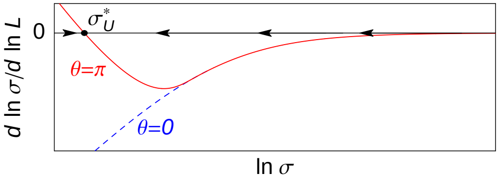
Unitary class

Generic single-valley disorder (e.g. charged impurities + ripples), $B = 0$

$\Longrightarrow$ effective time-reversal symmetry broken

Unitary sigma model with $\sigma_{xy} = 0$: anomalous θ-term with $\theta = \pi$

$$S[Q] = \frac{1}{8} \operatorname{Str} \left[-\sigma_{xx} (\nabla Q)^2 + Q \nabla_x Q \nabla_y Q \right]$$



no localization, QHE criticality instead!

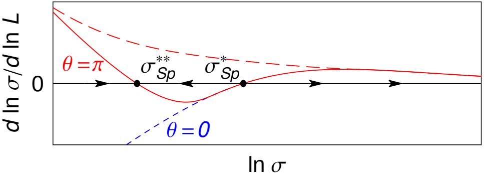
Minimal conductivity: $\sigma = 4\sigma_U^* \approx (2.0 \div 2.4) e^2/h$

Symplectic class

Random potential (e.g. charged impurities)
$\Longrightarrow$ effective time-reversal symmetry preserved

Symplectic sigma model: anomalous θ-term with $\theta = \pi$!

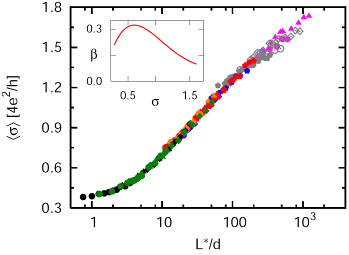
$$S[Q] = \frac{\sigma_{xx}}{16}\,\mathrm{Str}(\nabla Q)^2 + i\theta N[Q] \qquad N[Q] = 0, 1$$



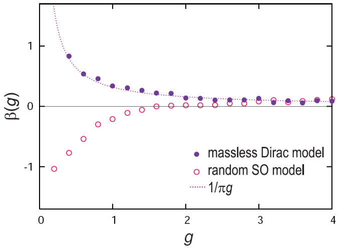
no localization! criticality?

- Minimal conductivity: $\sigma = 4\sigma^{**}_{Sp} \sim e^2/h$, **or**
- Absolute antilocalization: $\sigma \to \infty$

Scaling of conductance: numerical results



Bardarson, Tworzydło, Brower, Beenakker '07



Nomura, Koshino, Ryu '07

- Absence of localization confirmed
- Absolute antilocalization scenario
- From ballistics to diffusion: single parameter scaling???

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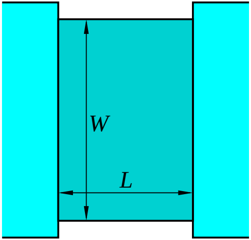
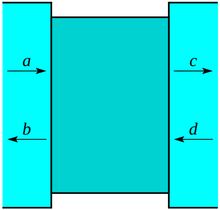
Ballistic setup



- rectangular sample with dimensions $L \times W$
- large aspect ratio: $W \gg L$
  $\Longrightarrow$ boundary conditions (edge modes) irrelevant
- ballistic regime: $L \ll l$
  $\Longrightarrow$ treat disorder perturbatively
- ideal contacts
- perfect metallic leads (highly doped regions of graphene)

Transfer matrix technique



Scattering matrix vs. Transfer matrix

$$\begin{pmatrix} c \\ b \end{pmatrix} = \mathcal{S} \begin{pmatrix} a \\ d \end{pmatrix} = \begin{pmatrix} t & r' \\ r & t' \end{pmatrix} \begin{pmatrix} a \\ d \end{pmatrix}$$

$$\begin{pmatrix} c \\ d \end{pmatrix} = \mathcal{T} \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} t^{+\,-1} & -r' t'^{-1} \\ -t'^{-1} r & t'^{-1} \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix}$$

Transport properties

- determined by transmission eigenvalues T_n of $t^+ t$
- e.g. conductance G and Fano factor F

$$G = \frac{4e^2}{h} \mathrm{Tr}(t^+ t) \qquad F = 1 - \frac{\mathrm{Tr}(t^+ t)^2}{\mathrm{Tr}(t^+ t)}$$

- Clean limit: $T_{p_y}(x) = \left[1 + \frac{p_y^2}{p_y^2 - \epsilon^2} \sinh^2 \left(\sqrt{p_y^2 - \epsilon^2}\, x \right) \right]^{-1}$

[Tworzydło *et al.* '06; Titov '07]

Clean graphene: transmission distribution

Measure in channel space

$$P(T)dT = 2W\frac{dp_y}{2\pi} \quad \Longrightarrow \quad P(T) = \frac{W}{\pi}\left|\frac{dp_y}{dT}\right|$$

Low energies: $\epsilon L \ll 1$

Expansion in small energy

$$P(T) = \frac{W}{2\pi L}\,\frac{1}{T\sqrt{1-T}}\left[1 + (\epsilon L)^2\left(\frac{\sqrt{1-T}}{\operatorname{arcosh}^3\frac{1}{\sqrt{T}}} - \frac{1+T}{2\operatorname{arcosh}^2\frac{1}{\sqrt{T}}}\right)\right]$$

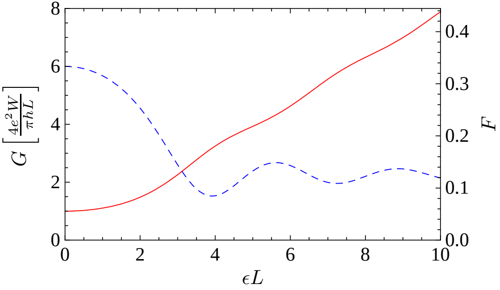
High energies: $\epsilon L \gg 1$

$T(p_y)$ is a rapidly oscillating function

After averaging over oscillations

$$P(T) = \frac{W|\epsilon|}{\pi^2}\,\frac{K(\sqrt{T}) - E(\sqrt{T})}{T\sqrt{1-T}}$$

Conductance and Fano factor



Limit	Conductance	Fano factor
$\epsilon L \ll 1$	$\frac{4e^2}{\pi h}\frac{W}{L}\left[1+0.101\,(\epsilon L)^2\right]$	$\frac{1}{3}\left[1-0.05\,(\epsilon L)^2\right]$
$\epsilon L \gg 1$	$\frac{e^2}{h}W\lvert\epsilon\rvert\left[1+\frac{\sin(2\epsilon L-\frac{\pi}{4})}{2\sqrt{\pi}(\epsilon L)^{3/2}}\right]$	$\frac{1}{8}\left[1+\frac{9\sin(2\epsilon L-\frac{\pi}{4})}{2\sqrt{\pi}(\epsilon L)^{3/2}}\right]$

Ballistic transport experiment

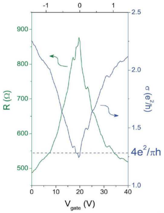
Danneau *et al.* '07

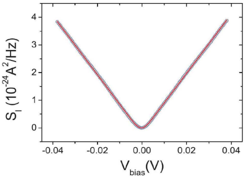
Setup

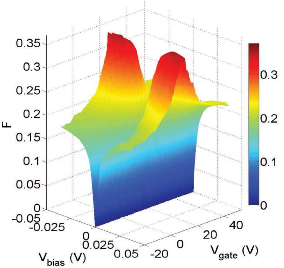
- Rectangular sample
- Temperature $4.2 \div 30$ K
- Large aspect ratio $W/L = 24$
- Ballistic limit $L \sim 200$ nm

Observations

- Conductance
$$G(\epsilon = 0) \approx \frac{4e^2}{\pi h}\frac{W}{L}$$
- Fano factor $F(\epsilon = 0) \approx 1/3$
- Conductance grows with ϵ
- Fano factor decreases with ϵ







Lowest-order disorder correction

Transfer matrix evolution

$$\mathcal{T}(x) = \mathcal{T}_0(x) - i \int_0^x dx' \mathcal{T}_0(x - x') \sigma_z \hat{V}(x') \mathcal{T}(x')$$

Gaussian white-noise disorder

$V(x,y) = \sum_\mu \sigma_\mu V_\mu(x,y)$ $\quad \langle V_\mu(x,y) \rangle = 0$ $\quad \langle V_\mu^2(x,y) \rangle = 2\pi \alpha_\mu$

$$\alpha = \alpha_0 + \alpha_x - \alpha_y - \alpha_z$$

Lowest order perturbative correction

Low energy $\epsilon L \ll 1$:

$$P(T) \mapsto (1 + \alpha)\, P(T)$$

The functional dependence is not changed!

Higher order corrections

Second-order correction logarithmically diverges!

Example: zero energy, random potential α_0

Conductance: $G = \frac{4e^2}{\pi h}\frac{W}{L}\Big(1 + \underbrace{\alpha_0 + 2\alpha_0^2 \log(L/a) + \ldots}_{\alpha_0(L)}\Big)$

Divergence is cut by the sample size L and lattice constant a

How to proceed?

Include logarithmic terms into renormalized parameter $\alpha_0(L)$

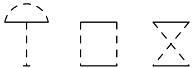
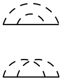
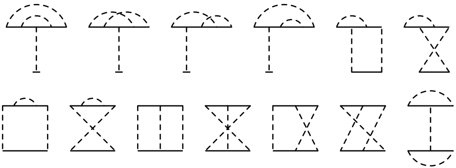
$\Longrightarrow$ Renormalization Group

[Dotsenko, Dotsenko '83, Ludwig *et al.* '94; Nersesyan *et al.* '94, Aleiner, Efetov '06]

Renormalization group

2D action for Dirac fermions in random potential

$$S[\psi] = \int d^2x \left[\bar{\psi} \boldsymbol{\sigma} \nabla \psi + i\epsilon \bar{\psi}\psi + \pi\alpha_0 (\bar{\psi}\psi)^2 \right]$$

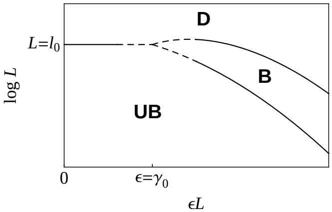
	Energy	Disorder
1-loop		
2-loop		

$$\frac{d\epsilon}{d\log\Lambda} = \epsilon\left(\alpha_0 + \alpha_0^2/2\right) \qquad \frac{d\alpha_0}{d\log\Lambda} = 2\alpha_0^2 + 2\alpha_0^3$$

Solution to RG equations

$$\alpha_0(\Lambda) = \frac{1}{2\log(l_0/\Lambda)} \qquad \epsilon(\Lambda) = \frac{\epsilon}{\sqrt{2\alpha_0 \log(l_0/\Lambda)}}$$

$$l_0 = a\sqrt{\alpha_0}\, e^{1/2\alpha_0}$$
$$\gamma_0 = \Delta e^{-1/2\alpha_0}$$



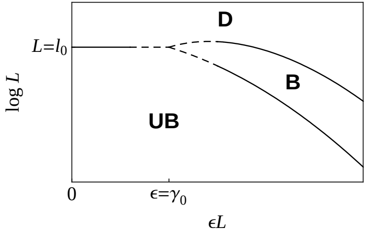
RG stops when

- $\Lambda \sim L \Longrightarrow$ ultra-ballistic $[\epsilon(\Lambda)L \ll 1]$
- $\epsilon(\Lambda)\Lambda \sim 1 \Longrightarrow$ ballistic $[\epsilon(\Lambda)L \gg 1]$
- $\alpha_0(\Lambda) \sim 1 \Longrightarrow$ diffusive

Crossover between regimes

- UB–D: $L \sim l_0 \Rightarrow$ zero-energy mean free path
- UB–B: $L \sim \lambda_F(\epsilon) = \sqrt{2\alpha_0 \log(\epsilon/\gamma_0)}/\epsilon \Rightarrow$ Fermi wave length
- B–D: $L \sim l(\epsilon) = [2\alpha_0 \log(\epsilon/\gamma_0)]^{3/2}/\epsilon \Rightarrow \epsilon \neq 0$ mean free path

Results for conductance and noise



Regime	Conductance	Fano factor
UB	$\frac{4e^2}{\pi h}\frac{W}{L}\left[1+\frac{\alpha_0+0.101(\epsilon L)^2}{2\alpha_0\log(l_0/L)}\right]$	$\frac{1}{3}\left[1+\frac{0.05(\epsilon L)^2}{2\alpha_0\log(l_0/L)}\right]$
B	$\frac{e^2}{h}\frac{W\epsilon}{2\alpha_0\log(l_0/L)}$	$\frac{1}{8}$
D	$\frac{8e^2}{\pi h}\log\frac{\epsilon}{\gamma_0}$	$\frac{1}{3}$

Single parameter scaling

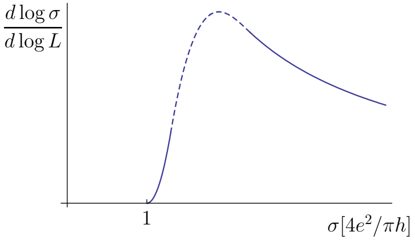
Assume

- Zero energy
- Gaussian white-noise random potential

Transmission distribution is universal ! ! !

$$P(T) = \frac{W}{2\pi L} \frac{\sigma}{T\sqrt{1-T}} \quad \text{with} \quad \sigma = \begin{cases} 1 + \alpha_0(L), & \text{ultra-ballistics} \\ G\,\pi h/4e^2, & \text{diffusion} \end{cases}$$

[Diffusive limit: Dorokhov '83]



Unified scaling

$$\frac{d\log\sigma}{d\log L} = \begin{cases} 2(\sigma-1)^2/\sigma, & \text{ballistic} \\ 1/\sigma, & \text{diffusive} \end{cases}$$

Conclusions

Results

1. Anomalous QHE
   - Decoupled valleys $\Longrightarrow$ odd quantum Hall effect
   - Mixed valleys $\Longrightarrow$ even plateaus appear
   - Chiral disorder (ripples) $\Longrightarrow$ classical Hall effect at the lowest LL
2. Absence of localization at $B = 0$
   - Decoupled valleys $\Longrightarrow$ no localization
   - Charged impurities + ripples $\Longrightarrow$ quantum Hall critical state
3. Ballistic transport
   - Transmission distribution including disorder
   - Two-loop RG for random potential
   - Single parameter scaling at the Dirac point

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