Giant Nernst effect due to fluctuating Cooper pairs in superconductors

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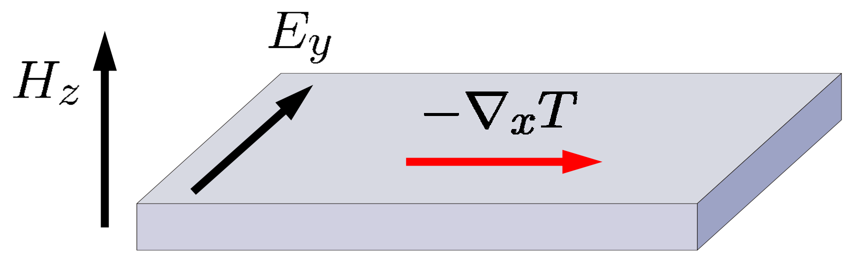
Nernst-Ettingshausen effect

A. V. Ettingshausen and W. Nernst (1886)

Walther Nernst
(1864–1941)

Nobel Prize
in Chemistry (1920)
for thermoelectricity



$$\begin{cases} j^e_\alpha = \sigma_{\alpha\beta} E_\beta - \beta_{\alpha\beta} \nabla_\beta T \\ j^Q_\alpha = \gamma_{\alpha\beta} E_\beta - \kappa_{\alpha\beta} \nabla_\beta T \end{cases}$$

Nernst coefficient:

$$\nu_N = \frac{E_y}{(-\nabla_x T) H_z} = \frac{1}{H} \frac{\beta_{xy}\sigma_{xx} - \beta_{xx}\sigma_{xy}}{\sigma_{xx}^2 + \sigma_{xy}^2}$$

Nernst effect in normal metals

E. H. Sondheimer (1948)

$$\nu_N = \frac{1}{H} \frac{\beta_{xy}\sigma_{xx} - \beta_{xx}\sigma_{xy}}{\sigma_{xx}^2 + \sigma_{xy}^2}$$

In metals, the thermoelectric tensor $\beta_{\alpha\beta}$ is due to the particle-hole asymmetry:

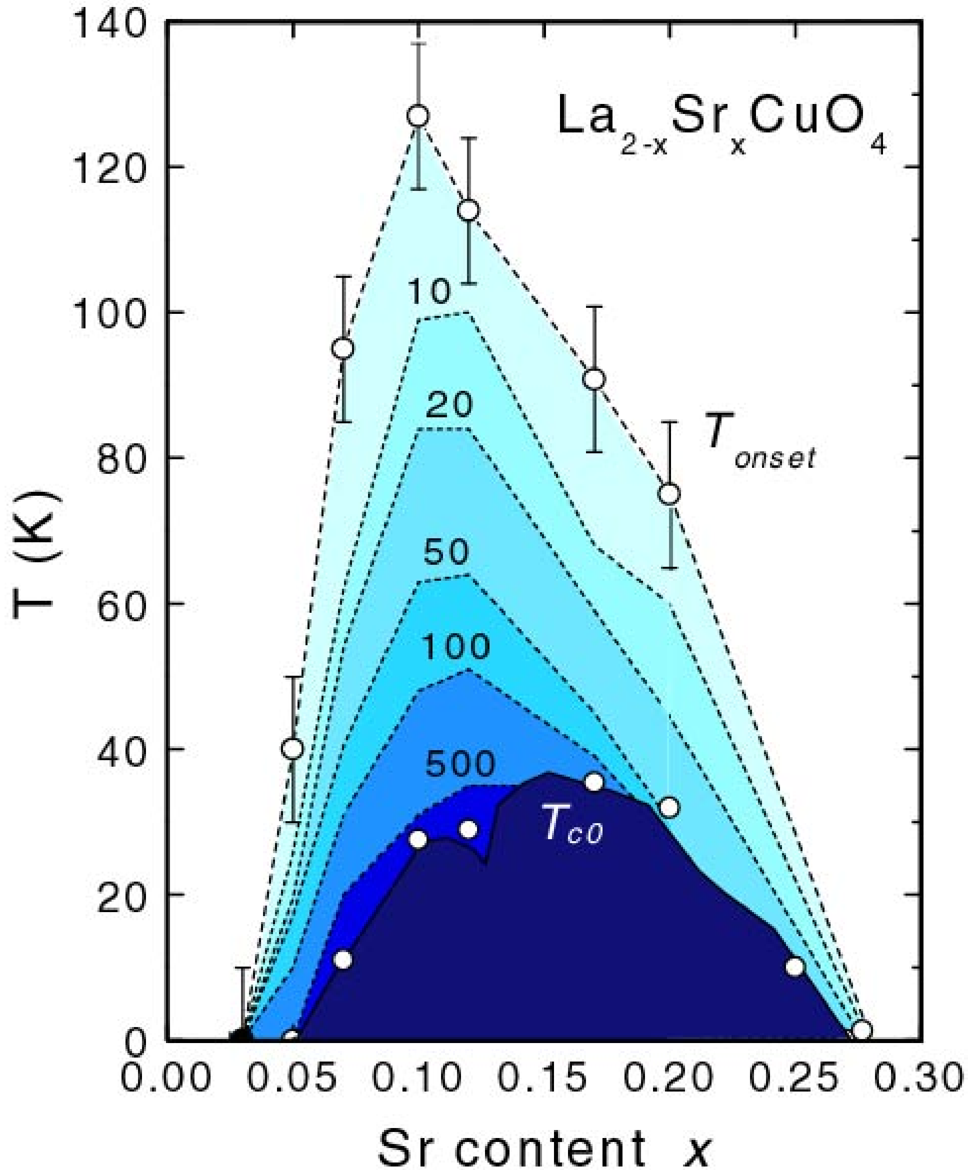
$$\beta_{\alpha\beta} = \frac{\pi^2}{3} \frac{cT}{e} \frac{\partial \sigma_{\alpha\beta}}{\partial \mu}$$

$$\nu_N \approx \frac{1}{H} \frac{\beta_{xy}\sigma_{xx} - \beta_{xx}\sigma_{xy}}{\sigma_{xx}^2} = \frac{\pi^2}{3} \frac{cT}{eH} \frac{\partial}{\partial \mu} \left(\frac{\sigma_{xy}}{\sigma_{xx}} \right) = \frac{\pi^2}{3} \frac{cT}{eH} \frac{\partial (\omega_c \tau)}{\partial \mu}$$

Sondheimer formula: $\nu_N = \frac{\pi^2}{3} \frac{T}{m} \frac{\partial \tau(\varepsilon)}{\partial \varepsilon} \Big|_{\varepsilon=\mu}$

Nernst effect in superconductors

Nernst effect in cuprates



Y. Wang et al. (2001)

$La_{2-x}Sr_xCuO_4$

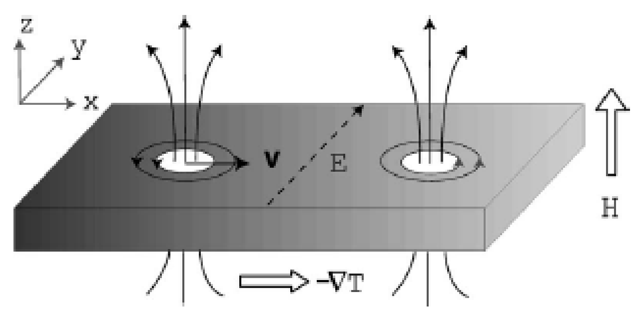
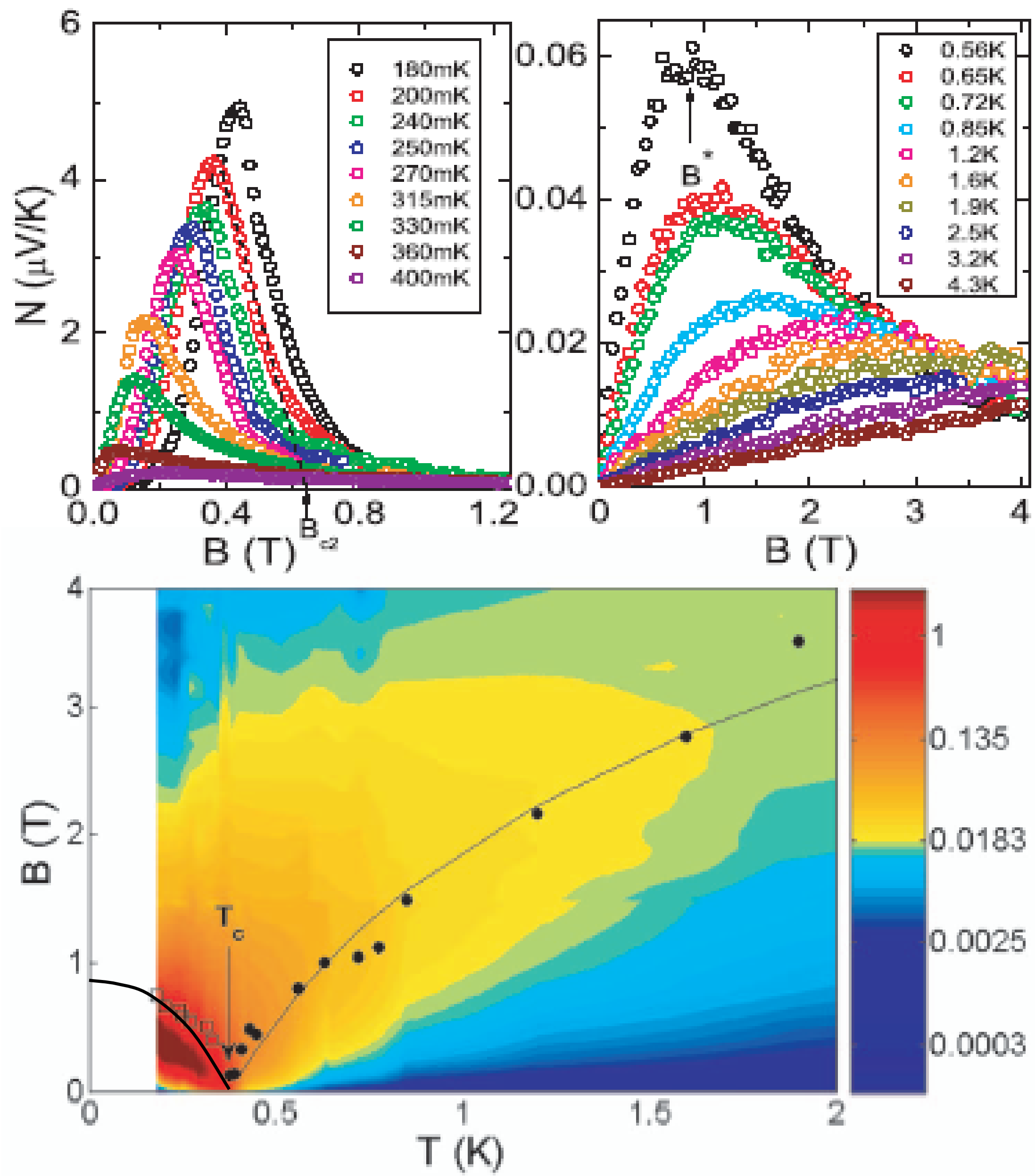


FIG. 1. The vortex-Nernst effect in a type-II superconductor. Concentric circles represent vortices.

Nernst effect in conventional superconductors



A. Pourret et al., Nature Phys. (2006); Phys. Rev. B (2007)

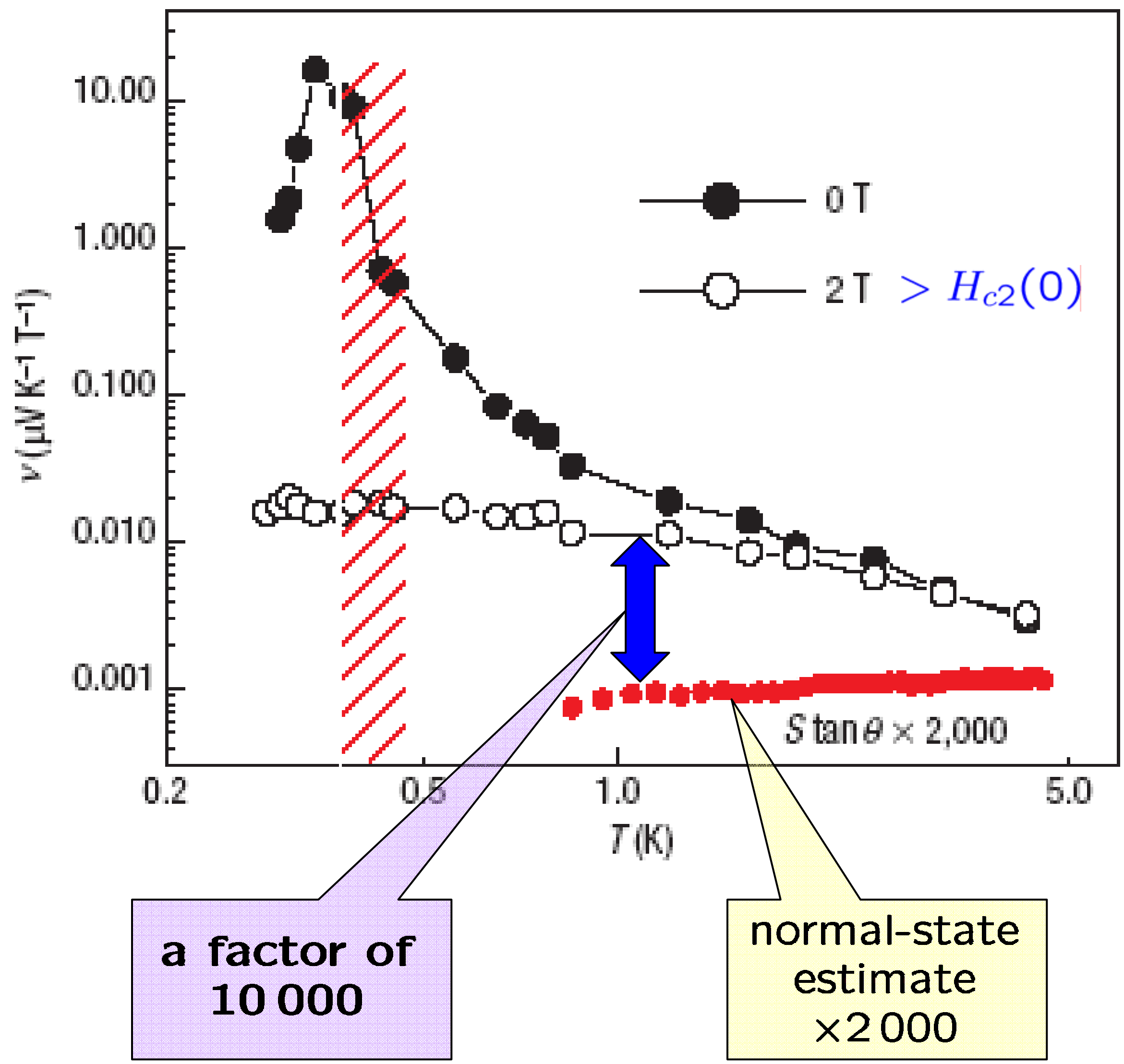
$Nb_{0.15}Si_{0.85}$ film

thickness
$d = 35$ nm

$T_c = 0.38$ K

$R_\square = 350$ Ohm

Nernst effect in conventional superconductors



A. Pourret et al.,
Nature Phys. (2006);
Phys. Rev. B (2007)

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Origin of the Nernst effect in NbSi?

A. Pourret et al., Nature Phys. (2006);

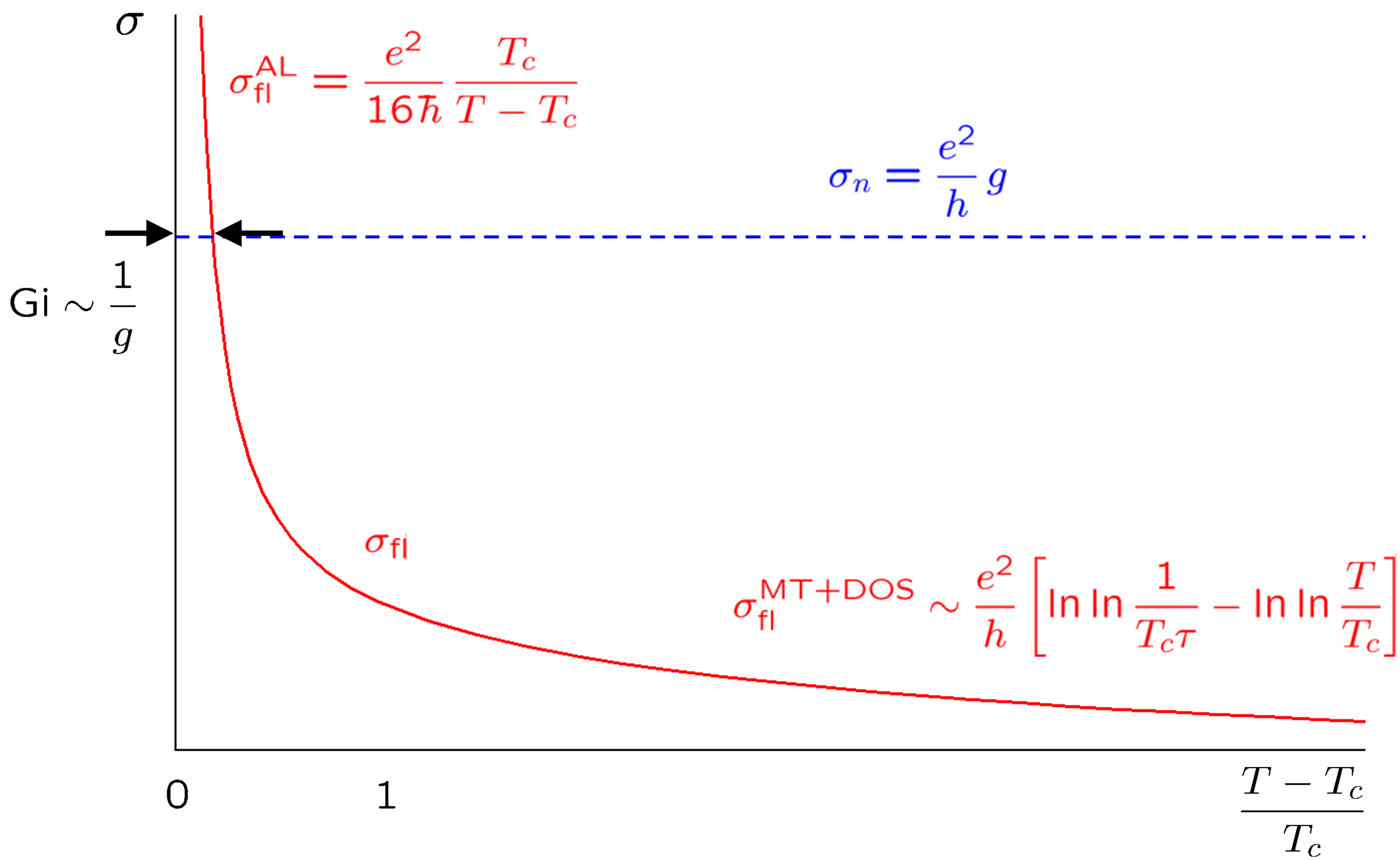
Could this signal be caused by phase fluctuations of the superconducting order parameter? This is also unlikely. In contrast to the underdoped cuprates, the carrier density in $Nb_{0.15}Si_{0.85}$ is comparable to any conventional metal. As the 'phase stiffness' of a superconductor is determined by its superfluid density[19], there is no reason to speculate on the presence of preformed Cooper pairs without phase coherence in a wide temperature window above T_c as has been the case in the pseudogap state of the cuprates. In contrast to granular superconductors[20], decreasing the thickness leads to a shift of the sharp superconducting transition and does not reveal a temperature scale other than the mean-field BCS (Bardeen–Cooper–Schrieffer) critical temperature. The variation of T_c with thickness has been attributed to the enhancement of the Coulomb interactions with the increase in the sheet resistance, R_{square} (ref. 21).

On the other hand, there is no reason to doubt the presence of amplitude fluctuations of the superconducting order parameter

Fluctuation effects in conventional superconductors

Fluctuation corrections to conductivity (2D)

Aslamazov–Larkin + Maki–Thompson + Density-of-states



Fluctuation corrections to thermoelectricity

Close to T_c:

$$\beta_n \propto \begin{pmatrix} \frac{T}{E_F} & \frac{T}{E_F}\omega_c\tau \\ -\frac{T}{E_F}\omega_c\tau & \frac{T}{E_F} \end{pmatrix}$$

$$\beta_{\text{fluct}} \propto \begin{pmatrix} \frac{T}{E_F}\ln\frac{T_c}{T-T_c} & \frac{T_c}{T-T_c}\frac{H}{H_{c2}(0)} \\ -\frac{T_c}{T-T_c}\frac{H}{H_{c2}(0)} & \frac{T}{E_F}\ln\frac{T_c}{T-T_c} \end{pmatrix}$$

$$\nu_N = \frac{\beta_{xy}}{H\sigma_{xx}}$$

Role of magnetization

microscopic heat current

transport heat current

magnetization heat current

N. R. Cooper,
B. I. Halperin,
I. M. Ruzin
(1997)

$$\mathbf{j}^Q = \mathbf{j}^Q_{\mathsf{tr}} + c\mathbf{M} \times \mathbf{E}$$

$$\beta^{\alpha\beta} = \tilde{\beta}^{\alpha\beta} + \epsilon^{\alpha\beta\gamma}\frac{cM^\gamma}{T}$$

transport experiment

Kubo response

fluctuation magnetization

$$j^{Q\alpha} = T\tilde{\beta}^{\alpha\beta}E^\beta$$

Vicinity of T_c: GL approach

I. Ussishkin, S. L. Sondhi, D. A. Huse (2002)

$$\beta^{xy} = \beta_0 \frac{\pi e D H}{48c(T - T_c)} \sim \beta_0 \frac{\xi^2(T)}{l_H^2}$$

thermoelectric conductance quantum

$$\beta_0 = \frac{k_B e}{\pi \hbar} = 6.68 \, \frac{\text{nA}}{\text{K}}$$

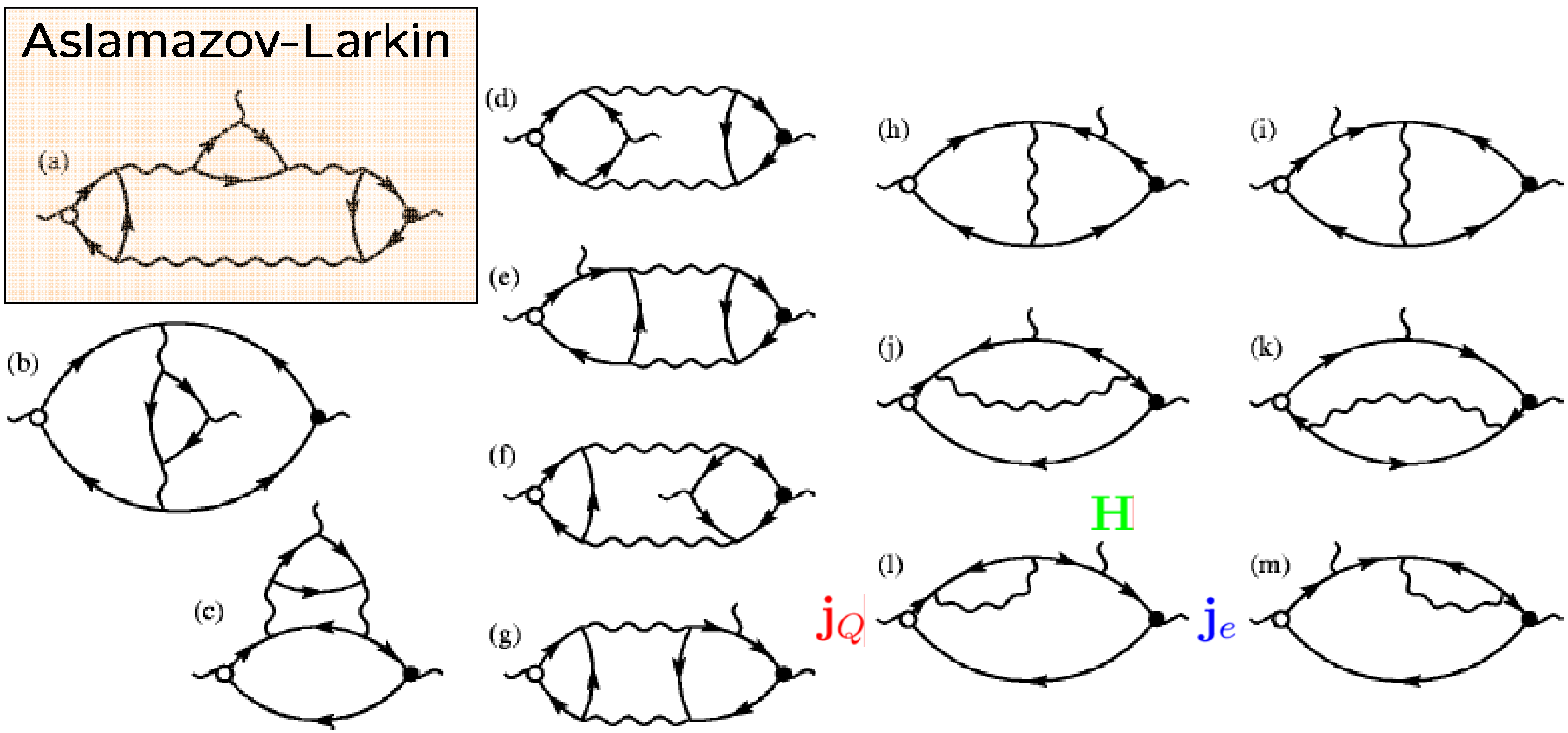
$$\beta^{xy} \propto 3 - 2$$

Kubo (3)

magnetization (2)

Vicinity of T_c: microscopic approach

I. Ussishkin, Phys. Rev. B (2003)

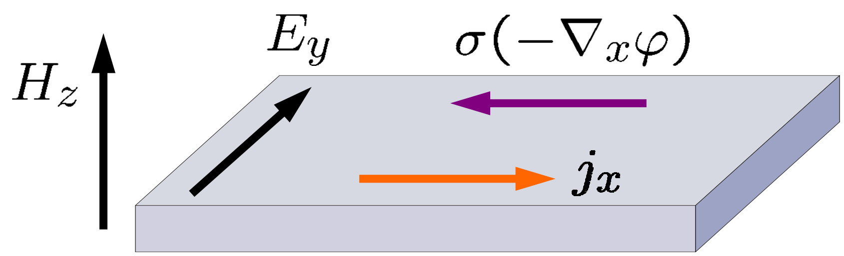


(cooperons are not shown)

Qualitative explanation of the Nernst effect

Nernst effect & chemical potential

- Drift velocity: $\bar{v}_x = cE_y/H_z$
- Drift current: $j_x = ne\bar{v}_x$
- Compensated by $\nabla_x\varphi = j_x/\sigma$
- Electroneutrality: $\nabla_x\mu = -e\nabla_x\varphi$
- Temperature gradient: $\nabla_x\mu = (d\mu/dT)\nabla_x T$



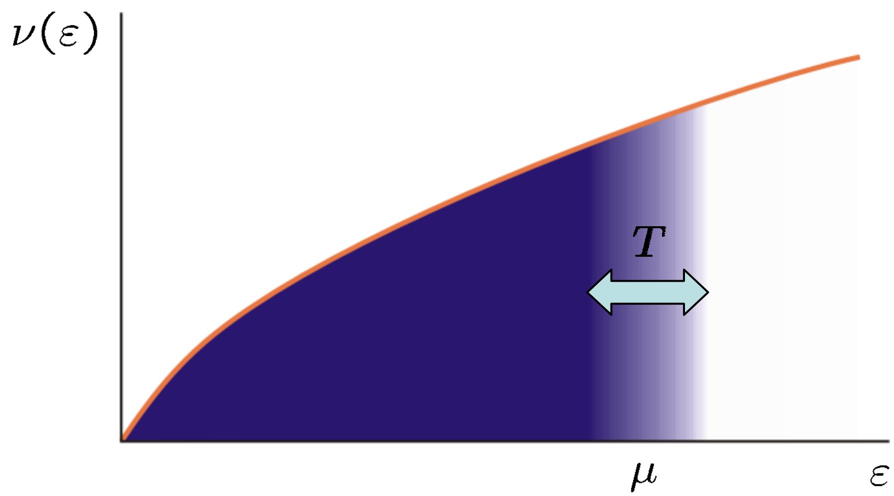
$$\nu_N \equiv \frac{E_y}{(\nabla_x T)H_z} = -\frac{\sigma}{ne^2c}\frac{d\mu(T)}{dT}$$

works for any types of carriers

Nernst effect: degenerate Fermi gas

$$\nu_N = -\frac{\sigma}{ne^2c}\frac{d\mu(T)}{dT}$$

Chemical potential of a degenerate Fermi gas: $\mu(T) = \mu_0 - \frac{\pi^2}{6}T^2\frac{d\ln\nu(\mu)}{d\mu}$



$$\nu_N = -\frac{\pi^2}{3}\frac{T}{m}\frac{\partial\tau(\mu)}{\partial\mu} \sim \frac{\tau}{m}\frac{T}{\mu}$$

origin of smallness

Nernst effect: fluctuating Cooper pairs near T_c

$$\nu_N = -\frac{\sigma}{ne^2c}\frac{d\mu(T)}{dT}$$

In the GL region close to T_c:

- chemical potential of Cooper pairs: $\mu(T) = T_c - T$
- density of Cooper pairs: $n = \langle|\Psi(\mathbf{r})|^2\rangle = \frac{mT_c}{\pi}\ln\frac{T_c}{T-T_c}$
- paraconductivity: $\sigma^{\mathrm{AL}} = (e^2/16)\,T_c/(T-T_c)$

↓

$$\nu_N \sim \frac{1}{mc(T-T_c)}$$

(Ussishkin result with an unknown coefficient;
magnetization not taken into account)

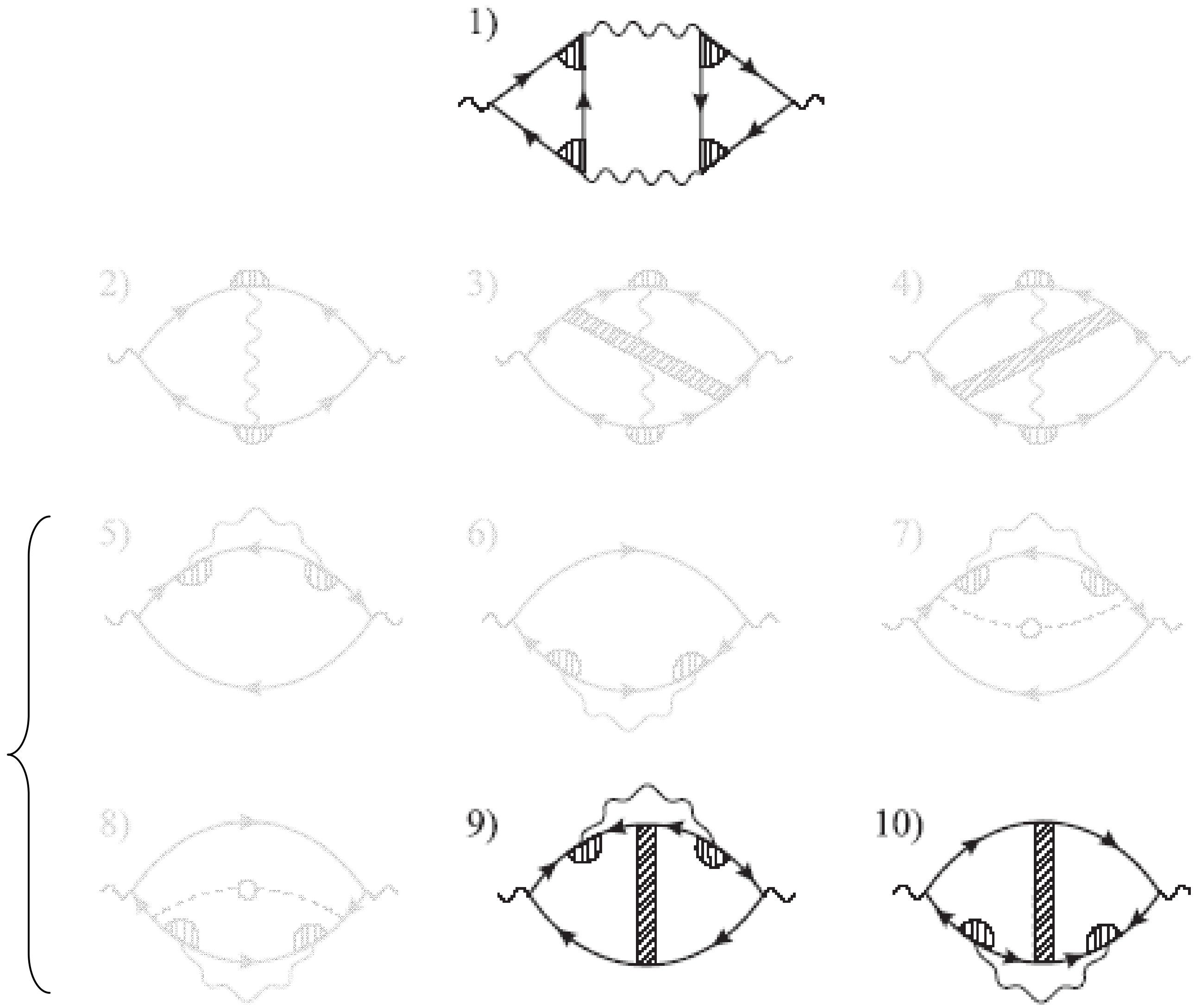
Complete microscopic theory of the fluctuation Nernst effect

Hierarchy of one-loop diagrams

Aslamazov-Larkin

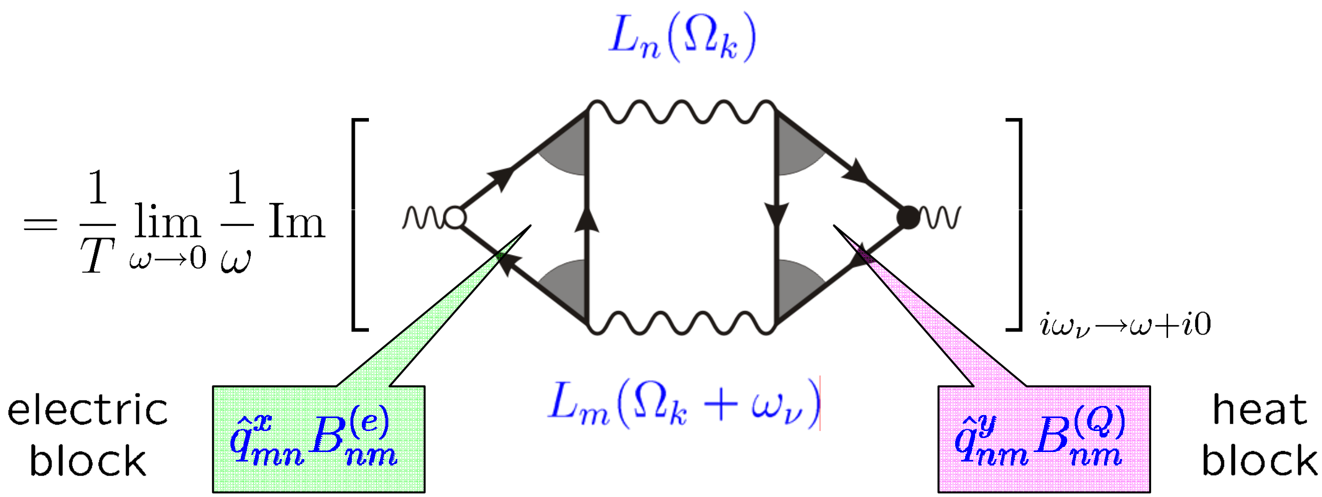
Maki-Thompson

Density-of-states



Aslamazov-Larkin diagram

$$\tilde{\beta}^{xy}_{\mathrm{AL}} = \frac{1}{T} \lim_{\omega \to 0} \frac{1}{\omega} \,\mathrm{Im} \Big[\cdots \Big]_{i\omega_\nu \to \omega + i0}$$



- Fluctuation propagator:

$$L_n(\Omega) = -\nu^{-1} \left[\ln \frac{T}{T_c} + \psi\left(\frac{1}{2} + \frac{|\Omega| + 4eDH(n+1/2)}{4\pi T} \right) - \psi\left(\frac{1}{2}\right) \right]$$

- Momentum operator in the Landau basis:

$$\hat{q}^{x,y}_{mn} = \sqrt{eH/c} \begin{pmatrix} i \\ 1 \end{pmatrix} \left(\sqrt{m}\, \delta_{m,n+1} \mp \sqrt{n}\, \delta_{n,m+1} \right)$$

- Electric & heat blocks:

$$\cdots$$

Electric and heat vertices

- Electric vertex:

$$B^{(e)}_{nm}(\Omega_k,\omega_\nu) = e\nu D\left[\frac{\psi_m(\omega_\nu+|\Omega_k|)-\psi_n(|\Omega_k|)}{\omega_\nu+\alpha_m-\alpha_n}+\frac{\psi_n(\omega_\nu+|\Omega_{k+\nu}|)-\psi_m(|\Omega_{k+\nu}|)}{\omega_\nu-\alpha_m+\alpha_n}\right]$$

- Heat vertex:

$$B^{(Q)}_{nm}(\Omega_k,\omega_\nu) = \frac{-i\nu D}{2}\left[\frac{(\Omega_k-\alpha_m)\psi_m(|\Omega_k|+\omega_\nu)-(\Omega_{k+\nu}-\alpha_n)\psi_n(|\Omega_k|)}{\omega_\nu+\alpha_m-\alpha_n}\right.$$

$$\left.+\frac{(\Omega_{k+\nu}+\alpha_n)\psi_n(|\Omega_{k+\nu}|+\omega_\nu)-(\Omega_k+\alpha_m)\psi_m(|\Omega_{k+\nu}|)}{\omega_\nu+\alpha_n-\alpha_m}\right]$$

$$\alpha_n \equiv 4eDH(n+1/2), \qquad \psi_n(\Omega)\equiv\psi\left(\frac{1}{2}+\frac{|\Omega|+\alpha_n}{4\pi T}\right)$$

Vicinity of T_c

$$\beta^{xy} = \tilde{\beta}^{xy}_{\text{AL}} + cM_z/T$$

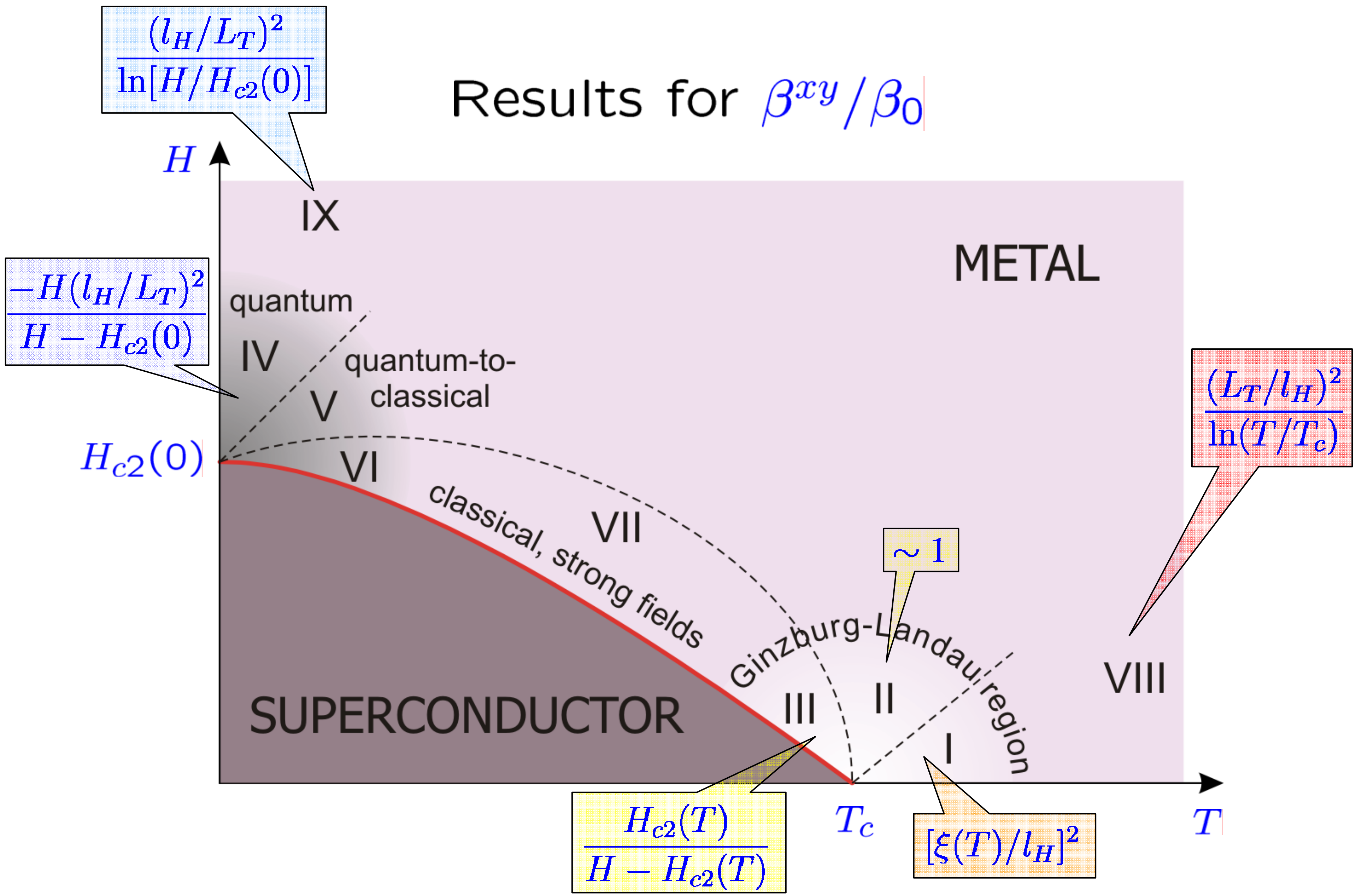
$$\beta^{xy} = \beta_0 \frac{\pi e D H}{12c(T - T_c)} = 4\beta^{xy}_{\text{USH}}$$

$$\beta^{xy} \propto 2 \times 3_{\text{AL}} - 2_{\text{M}} = 4$$

$$\beta^{xy}_{\text{USH}} \propto 3_{\text{AL}} - 2_{\text{M}} = 1$$

due to complicated analytic structure of the the heat vertex $B^{(Q)}_{mn}(\Omega_k, \omega_\nu)$

Asymptotic regimes in the phase diagram



Asymptotic regimes: analytic results

	Regimes	β^{xy}/β_0
I	$\tilde{h} \ll \epsilon \ll 1$	$\tilde{h}/3\epsilon$
II	$\epsilon \ll \tilde{h} \ll 1$	$1-(\ln 2)/2$
III	$\epsilon + \tilde{h} \ll h \ll 1$	$\tilde{h}/(\epsilon+\tilde{h})$
IV	$t \ll \eta \ll 1$	$-2\gamma t/9\eta$
V	$t^2/\ln(1/t) \ll \eta \ll t \ll 1$	$\ln(t/\eta)$
VI	$\eta \ll t^2/\ln(1/t) \ll 1$	$8\gamma^2 t^2/3\eta$
VII	$\eta \to 0$	$\frac{1}{\eta}\left[1+\frac{h}{4\gamma t}\frac{\psi''(1/2+h/4\gamma t)}{\psi'(1/2+h/4\gamma t)}\right]$
VIII	$(1,h) \ll t$	$\frac{eDH}{6\pi cT\ln(T/T_c)}$
IX	$(1,t) \ll h$	$\frac{\pi cT}{12eDH\ln[H/H_{c2}(0)]}$

$$\epsilon = \ln\frac{T}{T_c}, \quad t = \frac{T}{T_c}, \quad h = \frac{H}{H_{c2}(0)}, \quad \tilde{h} = \frac{\pi^2\gamma H}{8H_{c2}(0)}, \quad \eta = \frac{H_{c2}(T)}{H - H_{c2}(T)}, \quad \gamma = 1.78\ldots$$

Away from T_c

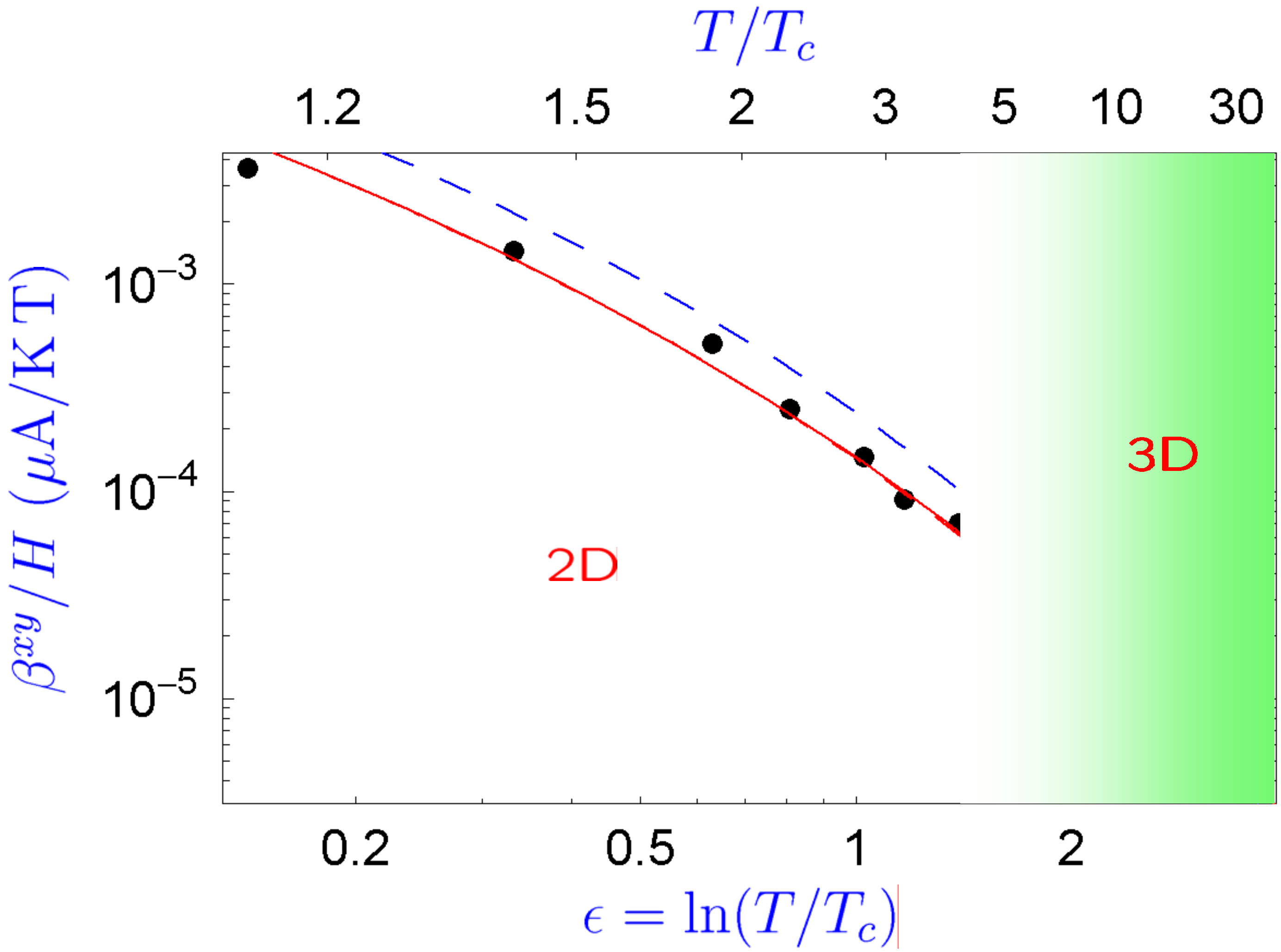
$$\beta^{xy} = \beta_0 \frac{eDH}{6\pi c T \ln(T/T_c)} \sim \beta_0 \frac{L_T^2}{l_H^2} \frac{1}{\ln(T/T_c)}$$

Relative value of the effect:

$$\frac{\beta^{xy}}{\beta_n^{xy}} = \frac{l}{d}\left(\frac{1}{T\tau}\right)^2 = \frac{l}{d}\left(\frac{L_T}{l}\right)^4$$

$\sim 10^3 \ldots 10^5$ for NbSi

Comparison with experiment



$Nb_{0.15}Si_{0.85}$ film

A. Pourret et al., Nature Phys. (2006)

thickness $d = 12.5$ nm

$T_c = 0.165$ K

$R_{\square} = 1250$ Ohm

$D = 0.145\ \mathrm{cm^2/s}$ (blue line)

$D = 0.087\ \mathrm{cm^2/s}$ (red line)

Conclusions

- Qualitative explanation of the Nernts effect in terms of $\mu(T)$
- Complete microscopic theory of the Nernst effect at arbitrary T and H above the transition line
- Crucial importance of the magnetization term
- Giant Nernst signal due to fluctuating Cooper pairs well above the transition line
- Relevance to cuprates