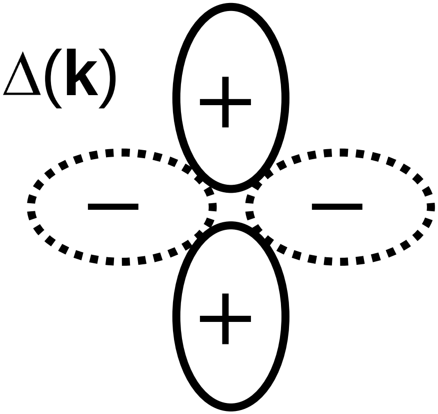
Theory of disordered D-wave superconductors

B.Spivak
University of Washington
with S. Kivelson, and P. Oreto
Stanford University

D-wave order parameter in pure superconductors

$\Delta(\mathbf{k})$



$$\Delta(\vec{r},\vec{r}\,') = \int \Delta(\vec{k}) e^{i\vec{k}(\vec{r}-\vec{r}\,')} d\vec{k}$$

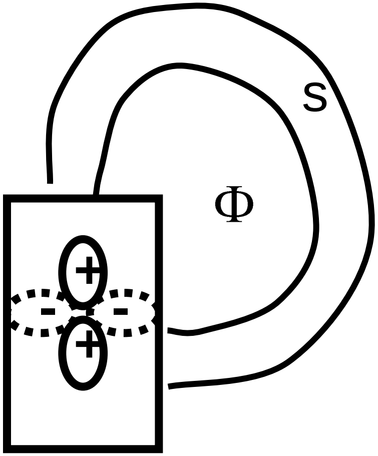
$$\Delta(\vec{r} = \vec{r}\,') = 0$$

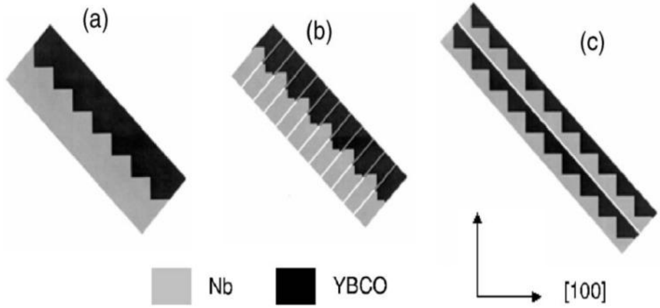
an assumption: e-e interaction in D-channel is attractive and in S-channel is repulsive

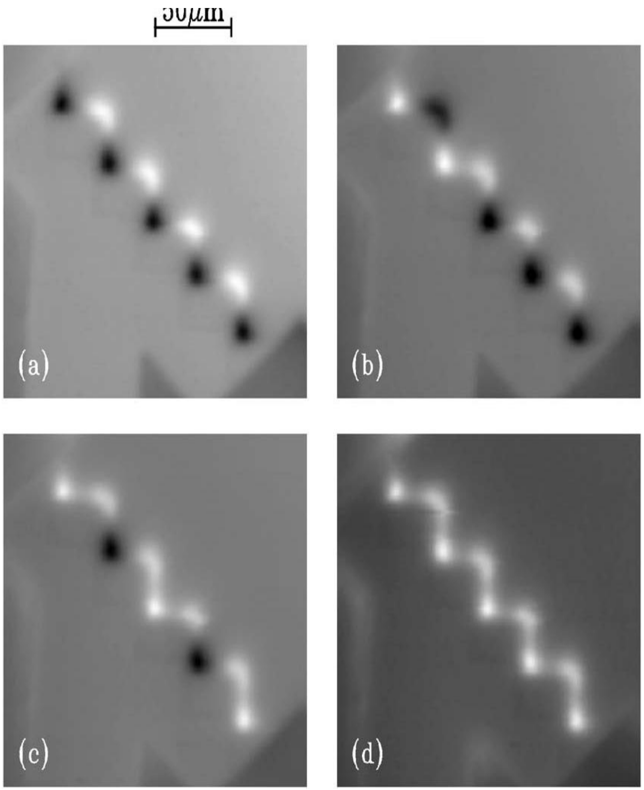
elastic electron scattering destroys D-wave superconductivity when the electron mean free path $l \sim \xi$ becomes of order of the superconducting coherence length.

“corner SQUID” experiment which demonstrate d-wave symmetry of the order parameter in HTC

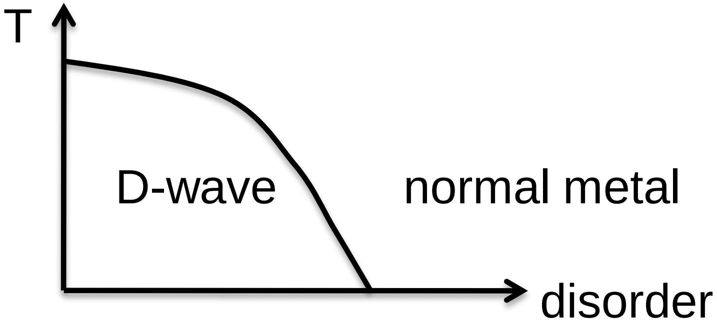
D. Geshkenbein, A.I. Larkin
J.R. Kirtley at all



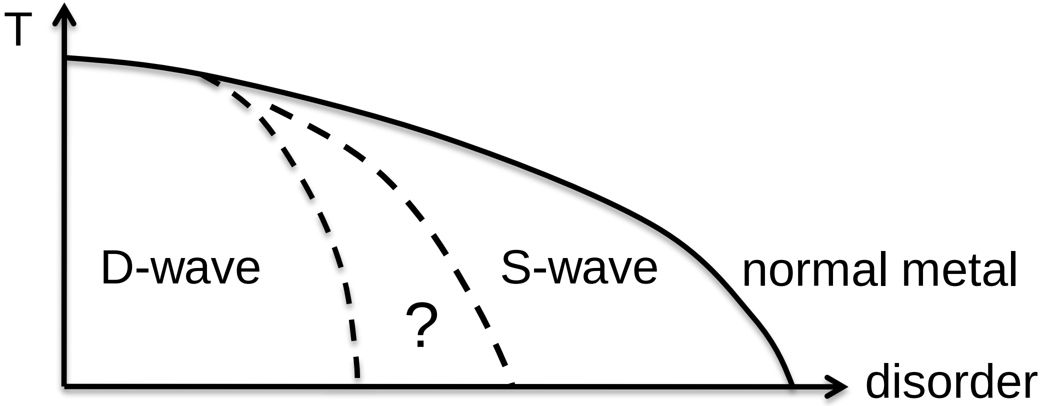




“Conventional” phase diagram



The phase diagram of disordered D-wave superconductors



near the point of the transition the order parameter has global S-wave symmetry regardless of its symmetry in pure state

before averaging over realizations of disordered potential the order parameter $\Delta(r,r')$ (and the anomalous Green function $F(r,r')$) do not have any symmetry.

Possible definitions of the global S-wave symmetry in bulk samples :

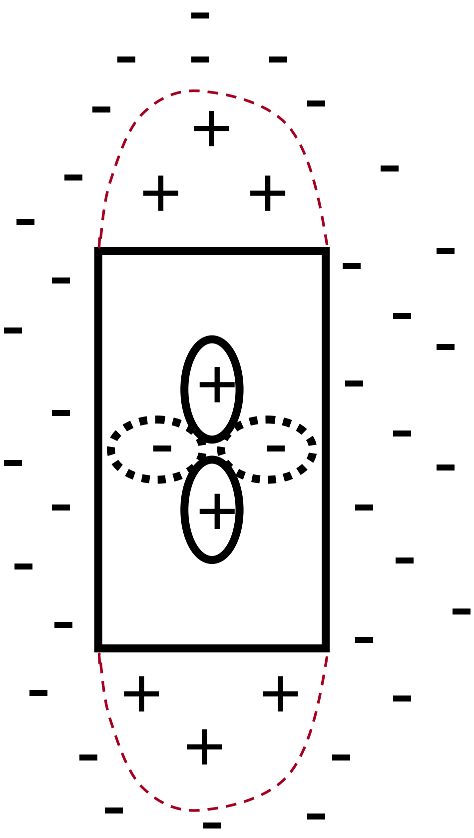
1. corner SQUID experiment shows global s-wave symmetry of the order parameter

2. the quantity $< F_s(r)> = <F(r=r')>$ is nonzero. (The brackets stand for averaging over realizations of random potential.)

3. the system has s-wave global symmetry if $P_+ - P_- > (<)0$. P_+ and P_- are volume fractions where $F(r=r')= F_s(r)$ has positive or negative sign, respectively.

d-wave superconducting puddle embedded into disordered normal metal.

outside the puddle s-wave component of the order parameter is generated. Only this component survives on distances larger than elastic mean free path l



+ and – indicate signs of the s-components of the anomalous Green function $F_s(\mathbf{r},\mathbf{r})$

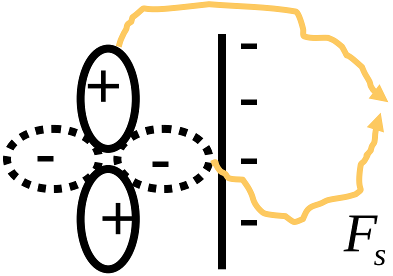
in diffusive metal s-component of the anomalous Green function $F_s(r)=F(r,r)$ is described by the Usadel equation

$$D_{tr}\frac{d^2\theta(\varepsilon,\vec r)}{d^2\vec r}+i\varepsilon\sin\theta(\varepsilon,\vec r)=0;\quad F_s(\vec r,\varepsilon)=-i\sin\theta(\varepsilon,\vec r)$$

$$F_s(\vec r)=\int F_s(\vec r,\varepsilon)\,d\varepsilon$$

D_{tr} *is the electron diffusion coefficient in the normal metal*

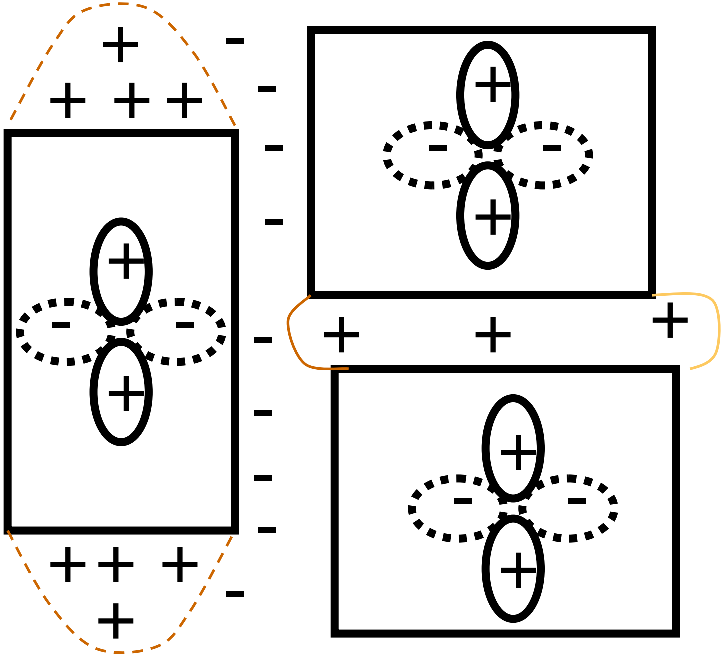
boundary conditions at D-N boundary are after Nazarov at all



$$F_s(\vec r,\varepsilon)\propto\left(\frac{1}{r^{D-2}}\right)\exp\left[-\frac{r}{L_\varepsilon}\right];\qquad L_\varepsilon=\sqrt{\frac{D_{tr}}{\varepsilon}}$$

$$F_s(r)\propto\frac{1}{r^D}$$

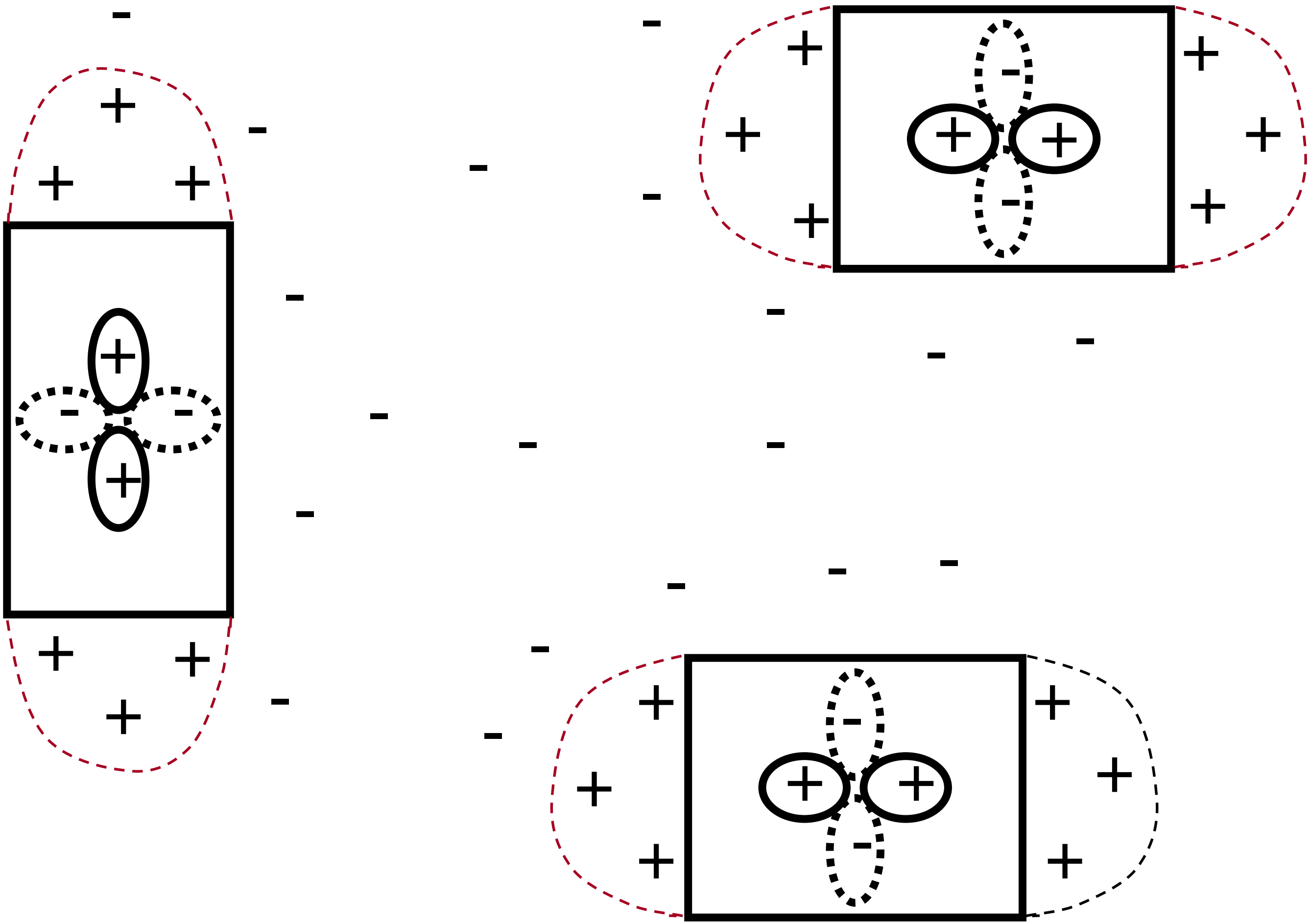
if puddle concentration is big the order parameter has global d-wave symmetry, while the s-component has random sample specific sign



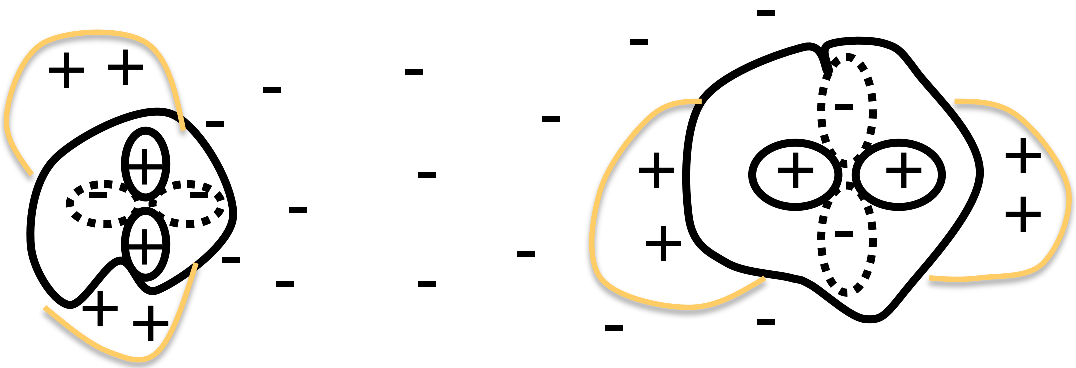
Effective mean field energy

$$E = \sum_{ij} j_{ij}^{(d)} e^{i(\varphi_i - \varphi_j)} + c.c.$$

$J_{ij}^{(d)}$ is the Joshepson coupling energy between D-wave components if the concentration of superconducting puddles is small the order parameter has s-wave global symmetry, while the d-wave component has random sample specific sign



more realistic picture superconducting puddles embedded into a metal



effective energy of the system is equivalent to Mattic model in the spin glasses theory :

$$E = -\sum_{ij} j_{ij}^{(s)} \eta_j \eta_i e^{i(\varphi_i - \varphi_j)} + c.c.; \quad \eta_i = \pm 1 \; are \; random$$

$$in \; the \; ground \; state \;\; e^{i\varphi_i} = \eta_i$$

$J_{ij}^{(s)}$ is the Joshepson coupling energy between S-wave components

an effective energy at intermediate concentration of superconducting puddles

$$E = \sum_{ij} \left[J_{ij}^{(s)} \eta_j \eta_i + J_{ij}^{(d)} \right] e^{i\left(\varphi_i - \varphi_j\right)} + c.c.$$

$$\eta_i = \pm 1 \; are \;\; random$$

is there a superconducting glass phase when $J^{(s)} \sim J^{(d)}$?

near the point of disordered quantum phase transition the system can be visualized as superconducting puddles connected by Joshepson couplings. Characteristic interpuddle distance is much bigger than their characteristic size.

a criterion of the transition:

$$X_{ij} = \chi_i \chi_j J_{ij} J_{ji} \approx 1$$

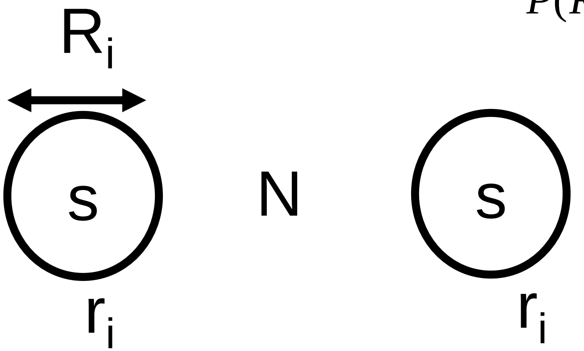
χ_i is the susceptibility of a puddle
J_{ij} is the Joshepson coupling between puddles

χ_ι and J_{ij} are random quantities

a model: superconducting puddles of random radiuses R_i are embedded into a repulsive normal metal .

this model demonstrates a superconductor-metal transition

$$P(R_i) = \frac{N}{\sqrt{2\pi}\sigma_R R} \exp\left[- \frac{\left(R_i - \overline{R}\right)^2}{\sigma_R^2 \overline{R}^2} \right]$$



$R_c \sim \xi_0$ is the critical radius

In the framework of superconductor-insulator transition this model has been considered by Caldeira, A. Legget, B. Haperin, S. Kivelson, A. Luther, S. Chakrovarty, S. Girvin, M. Fisher, S. Sachdev, N. Read, V. Ambegaokar, G. Schoen, U. Escern, D. Fisher, P. Lee, ………………

to find a critical puddle concentration N_c we use a procedure similar to that introduced by N. Mott in the theory of variable range hopping conductivity

1. in a space of R_i we introduce an interval of a width of order σ_R which is is centered at R_{opt}

2. $$X_{opt} = \left(X_{opt} J_{opt} \right)_{R=R_{opt}}$$

3. let us find a maximum of X_{opt} as a function of $R_{opt.}$

4. one can find the value of $N=N_c$ from a requirement

$$X_{opt}^{\max} \approx 1$$

inter puddle Joshepson energy has a long range character

$$J_{ij}^{(s)} \approx \frac{C}{|\vec{r}_i - \vec{r}_j|^D} \frac{1}{(1+\lambda_N \ln^2 |\vec{r}_i - \vec{r}_j|)} \qquad T = 0$$

$$a) \quad R - R_c << R_c; \qquad |\Delta_i| = \Delta_0 (R_i - R_c)/R_c,$$

$$C = \nu V_j V_i (\Delta_i \Delta_j^* J_{ij} + c.c.);$$

$$b) \quad R - R_c > R_c; \; |\Delta_i = |\Delta_0| e^{i\varphi_i},$$

$$C \approx G_{eff} \frac{D_{tr}}{R^2} V_i \cos \varphi_{ij}$$

$$J_{ij}^{(s)}(T) = J_{ij}^{(s)}(0) \exp(-\frac{|\vec{r}_i - \vec{r}_j|}{L_T}); \qquad L_T = (D_{tr}/T)^{1/2}$$

D is the dimensionality of space, D_{tr} is the diffusion coefficient,
V~R^D is the volume of the grain,
G_{eff} is the “effective” conductance of the grain
λ_N is the repulsive interaction constant in s-channel

susceptibility of an individual puddle depends on value of (R_i-R_c)

$$\chi_i = \int \langle \Delta_i^*(0)\,\Delta_i(t)\rangle\, dt$$

1) $|R-R_c| \ll R_c$

$$\mathit{Ginzburg-Landau\ action}$$

$$S_i = \alpha_i \int d\tau \left[\frac{(R_c - R_i)}{R_c} |\Delta_i|^2 + \frac{|\Delta_i|^4}{4\Delta_0^2} \right] + \beta_i \int d\tau\, d\tau' \, \frac{|\Delta_i(\tau) - \Delta_i(\tau')|^2}{(\tau - \tau')^2}$$

$$\alpha = \nu V, \quad \beta = \nu V / \Delta_0$$

a) $R-R_c<0$,

$$\chi_i(\omega) \sim \frac{1}{\beta|\omega| + \alpha(R_i - R_c)/R_i}; \qquad \chi_i \sim \frac{R_c}{\alpha_i (R_c - R_i)}$$

b) R-R_c>0,

$$|\Delta_i| = \Delta_0 (R - R_c)/R > 0; \qquad \Delta = |\Delta| e^{i\varphi}$$

Contribution of the phase fluctuations

$$S = A \int \frac{\sin^2 \left(\varphi(t) - \varphi(t')\right)}{(t - t')^2} dt\, dt'; \qquad A \propto \Gamma_i \frac{(R_i - R_c)}{R_c} \approx V_i \nu \Delta_0 \frac{(R_i - R_c)}{R_c}$$

$$\chi_i \propto \Delta_0 \exp\left(\Gamma_i \frac{(R_i - R_c)}{R_c} \right);$$

$$(\textit{fluctuations of } |\Delta| \textit{ gave a contrubution to } \chi \textit{ of the same order})$$

susceptibility increases exponentially with (R_i-R_c) !

2. R~Rc

quantum fluctuations of the order parameter are governed by the quantum fluctuation of EMF

$$S = G_{eff} \int \frac{\sin^2\left(\varphi(t) - \varphi(t')\right)}{(t - t')^2} dt\, dt'$$

$$\chi = e^{G_{fff}} \qquad 3D \qquad \text{Kosterliz}$$

$$\chi = e^{\sqrt{G_{2D}}} \qquad 2D, \qquad \text{Fegelman, Larkin, Skvortsov}$$

G_{eff} and G_{2D} are conductances of a cube of normal metal of size R, and 2D normal film respectively

susceptibility is an exponential function of G_{eff}

at T=0 the distance between optimal puddles is exponentially larger than they size !

$$N_{opt} \propto \exp[-\Gamma^2 \sigma_R^2] \qquad\qquad T = 0$$

this is a generic picture of any quantum phase transition in a metal with disorder.

more realistic model :

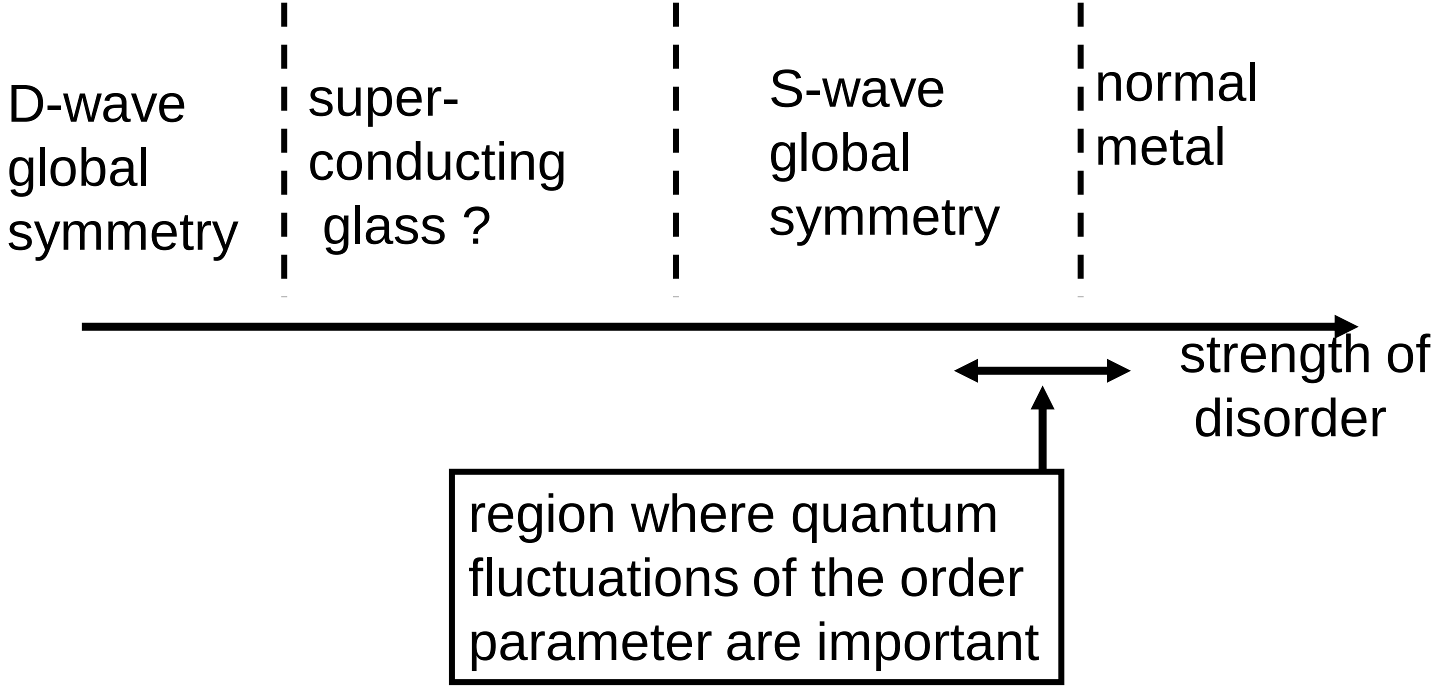
distribution function of the mean free path

$$P(l) = \frac{1}{\sqrt{2\pi}} \exp[-\frac{(l-\bar{l})^3}{2\sigma_l^2 \bar{l}^2}]$$

the correlation length is Λ

$$l_0 - l_c \approx G_\zeta \sigma_l^2 \left(\frac{\Lambda}{\xi_0} \right)^2$$

phase diagram of D-(or P) superconductors as a function of disorder (T=0)



D-wave
global
symmetry

super-
conducting
glass ?

S-wave
global
symmetry

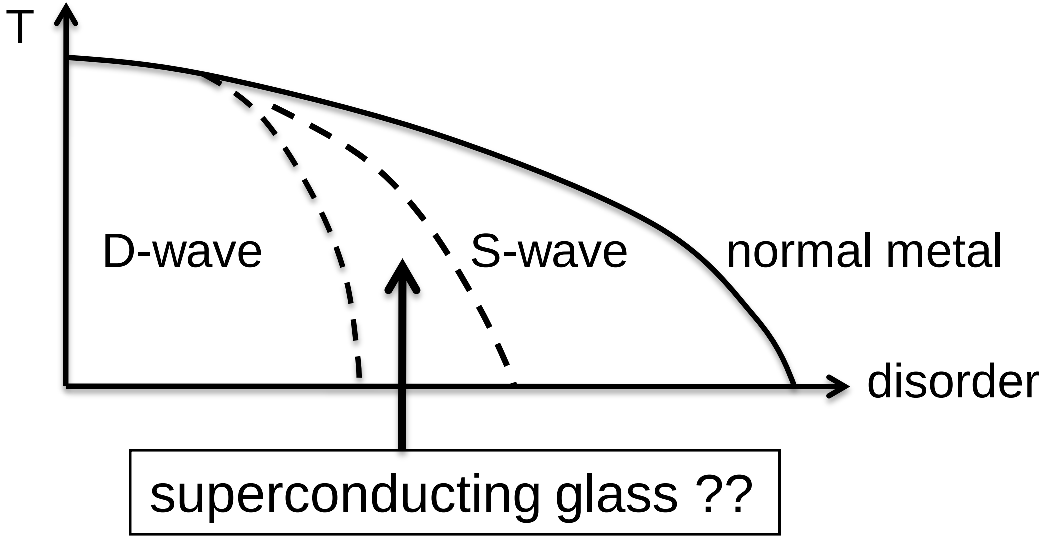
normal
metal

strength of
disorder

region where quantum fluctuations of the order parameter are important

it is likely that this sequence of phases take place on over-doped site of HTC.

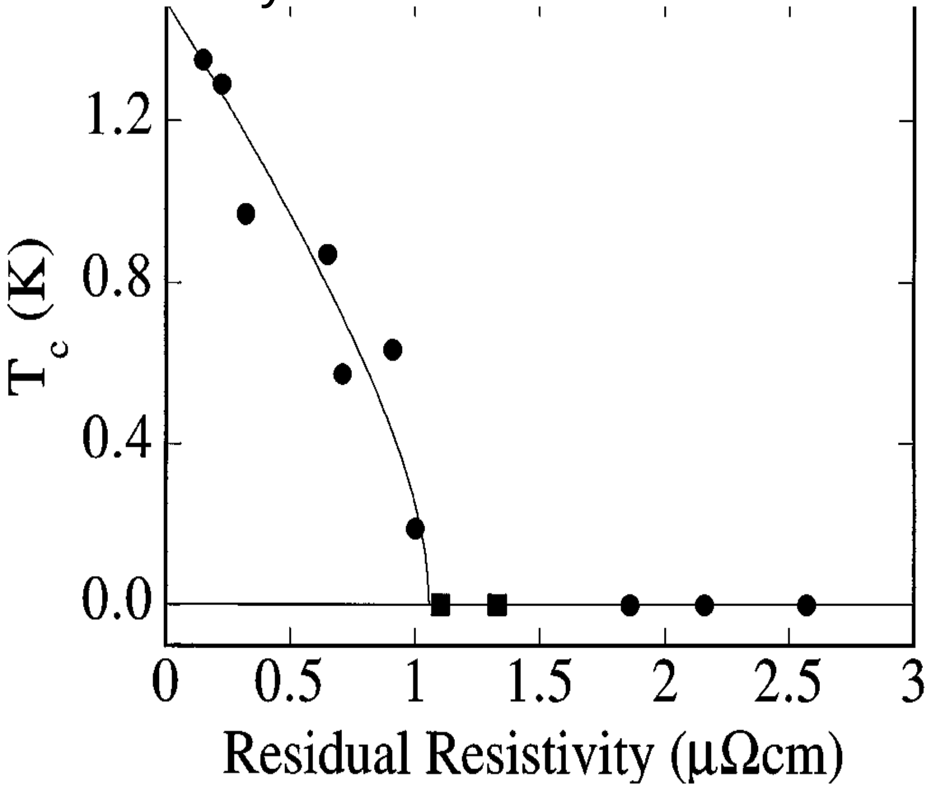
The phase diagram of disordered D-wave superconductors



superconducting glass ??

near the point of the transition the order parameter has global S-wave symmetry regardless of its symmetry in pure state

can we say the same thing about superconducting ruthanates which are suspects for P-wave superconductivity?



Sr_2RuO_4

A. P. Mackenzie at all

properties of the exotic metal near the quantum superconductor-metal transition:

conductivity of the “metal” is enhanced

Hall coefficient is suppressed

magnetic susceptibility is enhanced

in which sense such a metal is Fermi liquid?
For example, what is the size of quasi-particles ?
Is electron focusing at work in such metals ?

Conclusion:

in between of D-wave and normal metal phases there is a superconducting phase with S-wave “global” symmetry

Other theoretical possibilities:

a. Quantum S-wave superconductor-metal transition in an external magnetic field

b. If the electron-electron interaction constant has random sign the system may exhibit quantum superconductor-metal transition

If R_i <Rc and the variance σ_R is small, than

$N_c \sim 1/R^D$

this result can be obtained on the level of mean field

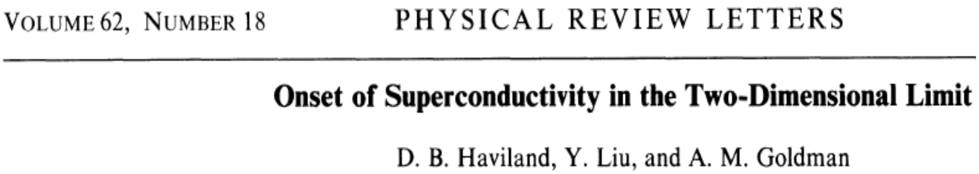
$$\Delta(\vec{r}) = \lambda(\vec{r}) \int d\vec{r}\,' K(\vec{r}, \vec{r}\,')\Delta(\vec{r}\,') + a\,|\,\Delta\,|^2\,\Delta$$

$$K(\vec{r}, \vec{r}\,') \propto \frac{1}{|\,r - r'|^D}$$

interaction constants $\lambda_N < 0$, λ_S **>0.**

Examples of experimental data

Experiments suggesting existence of quantum superconductor-insulator transition



VOLUME 62, NUMBER 18 PHYSICAL REVIEW LETTERS

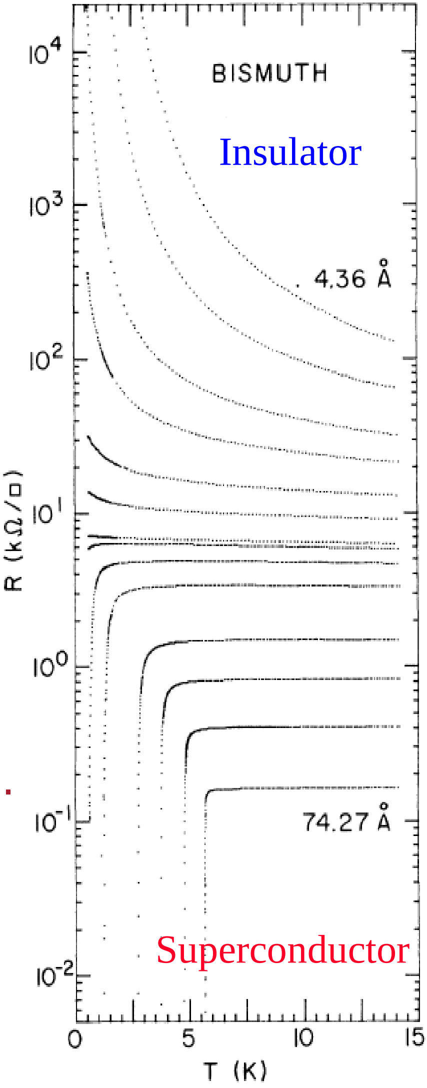
Onset of Superconductivity in the Two-Dimensional Limit

D. B. Haviland, Y. Liu, and A. M. Goldman

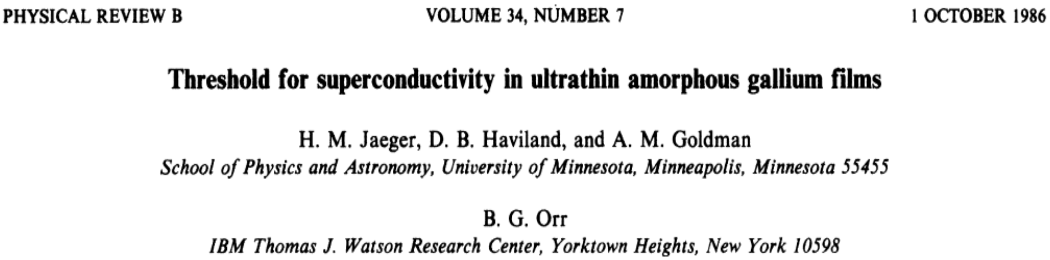
$R_c = 4\ h/e$

Bi layer on amorphous Ge.

Disorder is varied by changing film thickness.



Experiments suggesting existence of quantum superconductor-metal transition



PHYSICAL REVIEW B VOLUME 34, NUMBER 7 1 OCTOBER 1986

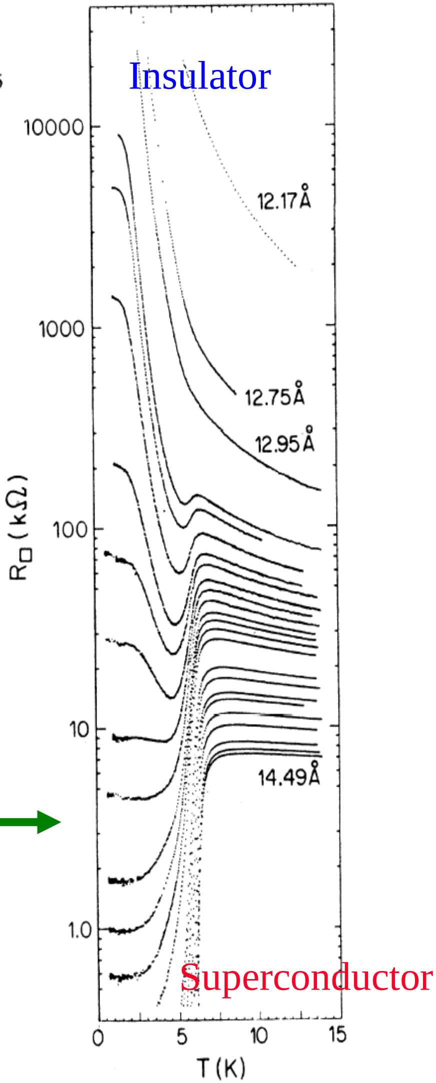
Threshold for superconductivity in ultrathin amorphous gallium films

H. M. Jaeger, D. B. Haviland, and A. M. Goldman
School of Physics and Astronomy, University of Minnesota, Minneapolis, Minnesota 55455

B. G. Orr
IBM Thomas J. Watson Research Center, Yorktown Heights, New York 10598

Ga layer grown on amorphous Ge

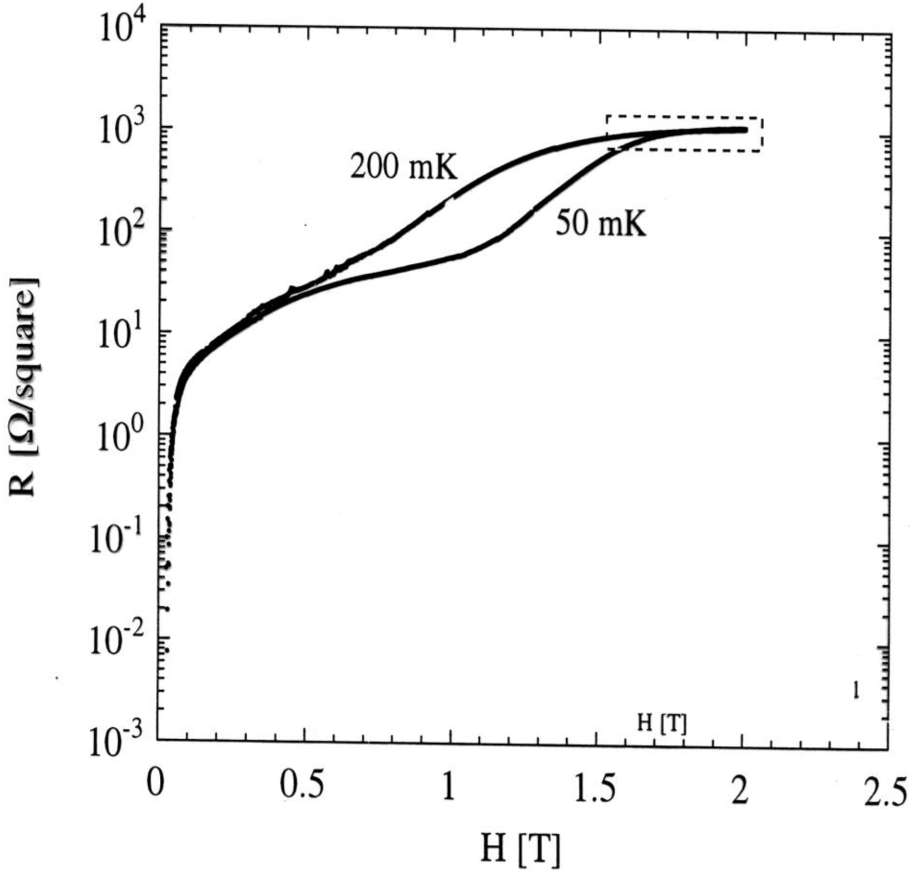
A metal?



Insulator

Superconductor

T=0 superconductor-metal transition in a perpendicular magnetic field



**N. Masson,
A. Kapitulnik**

**There are conductors whose T=0 conductance
is four order of magnitude larger than the Drude value.**

superconductor – glass transition in a magnetic field parallel to the film

W. Wu, P. Adams,

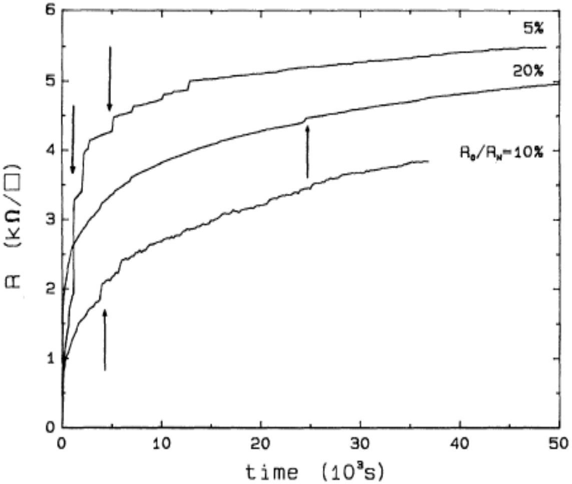
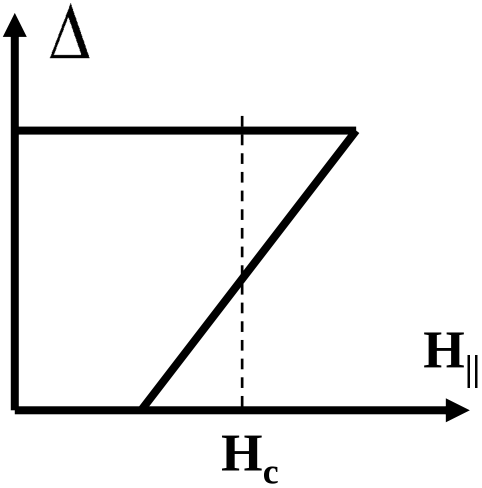


FIG. 2. R versus time after $H_{\|}$ was held constant when R_0/R_N reached desired values during field-up sweeps. Arrows indicate some of the avalanches. Note that the $R_0/R_N = 5\%$ curve actually jumped above the $R_0/R_N = 20\%$ curve.



The mean field theory:
The phase transition is of first order.

There are long time (hours) relaxation processes reflected in the time dependence of the resistance.

Is it a superconductor-glass transition in a parallel magnetic field?

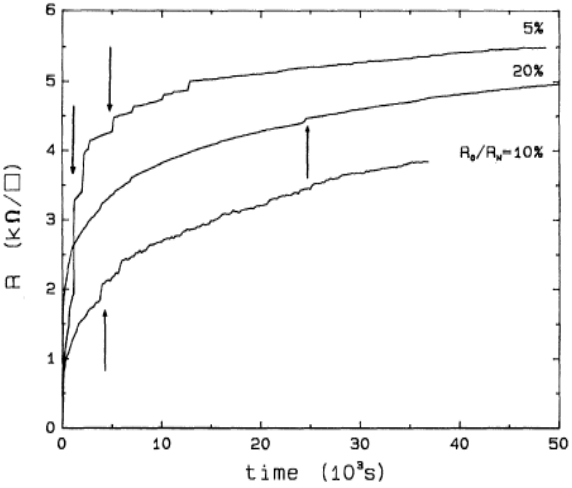


FIG. 2. R versus time after $H_{\parallel}$ was held constant when R_0/R_N reached desired values during field-up sweeps. Arrows indicate some of the avalanches. Note that the $R_0/R_N = 5\%$ curve actually jumped above the $R_0/R_N = 20\%$ curve.

W. Wo, P. Adams, 1995