

SCALAR-TENSOR AND $f(R)$ MODELS
OF DARK ENERGY IN THE UNIVERSE

A. A. STAROBINSKY

L.D. LANDAU MEMORIAL CONFERENCE

CHERNOGOLOVKA, 23.06.2008

Dark matter and dark energy -
two entities in the Universe seen
by their gravitational interaction
only

DM - non-relativistic
gravitationally clustered

DE - relativistic
unclustered

Definition through equations

I. DM - through the (generalized)
Poisson equation

$$\frac{\Delta \Phi}{a^2} = 4\pi G (\rho - \rho_0(t))$$

$\Phi(\vec{r})$ is measured using the motion
of 'test particles' in it

- a) Stars in galaxies → rotation curves
- b) Galaxies → peculiar velocities
- c) Hot gas in clusters → X-ray profiles
- d) Photons → gravitational
lensing
(strong and weak)

Result: DM is non-relativistic ($p \ll E$), collisionless, and has the same spatial distribution as visible matter for $L \gtrsim 1 \text{ Mpc}$

$$b \approx 1, \quad \Omega_{m, \text{tot}} = 0.28 \pm 0.03 \quad (26)$$

$$\Omega_{\text{bar}} = 0.046 \pm 0.003$$

II. DE - through relativistic gravitational field equations in the Einsteinian form

$$\frac{1}{8\pi G} \left(R_{\mu}^{\nu} - \frac{1}{2} \delta_{\mu}^{\nu} R \right) = \underbrace{T_{\mu}^{\nu}(\text{vis}) + T_{\mu}^{\nu}(\text{DM})}_{\text{approximately dust-like}} + T_{\mu}^{\nu}(\text{DE})$$

$G = G_0 = \text{const}$ - the Newton gravitational constant measured in laboratory

$$T_{\mu}^{\nu}(\text{DE})_{; \nu} = 0$$

(in the absence of decay of DM particles into DE)

Applications of the definition of $T_{\mu}^{\nu}(\Delta E)$

1. To the FRW background

$$\epsilon_{\Delta E} \equiv T_{00}^0(\Delta E) = \epsilon(\bar{z})$$

$$\bar{z} = \bar{z}(t) = \frac{a_0}{a(t)} - 1$$

$$p_{\Delta E} = -T_{\perp\perp}^{\perp}(\Delta E) = p_{\Delta E}(\bar{z})$$

(no summation on \perp)

$$\dot{\epsilon}_{\Delta E} + 3H(\epsilon_{\Delta E} + p_{\Delta E}) = 0$$

$$H \equiv \frac{\dot{a}}{a}$$

1 function

2. To the evolution of density
perturbations in the matter component
(baryons + DM) at small scales

$\left(\frac{\delta\rho}{\rho}\right)_m(\bar{z})$ - second observational function

Main up-to-date result

In the zero approximation ($\sim 10\%$ accuracy)

$$\epsilon_{\Delta E}(\bar{z}) = \epsilon_0 = \text{const}$$

$$p_{\Delta E} = -\epsilon_0, \quad T_{\mu}^{\nu}(\Delta E) = \epsilon_0 \delta_{\mu}^{\nu}$$

$$\Omega_{\Delta E} = 1 - \Omega_m$$

$$\rho_0 = \frac{\epsilon_0}{c^2} = 6.44 \cdot 10^{-30} \frac{\Omega_{\Delta E}}{0.7} \cdot \left(\frac{H_0}{70}\right)^2 \text{ g cm}^{-3}$$

$$\frac{G^2 k \epsilon_0}{c^7} = 1.25 \cdot 10^{-123} \cdot (\dots \downarrow \dots)$$

Investigation of dark energy

I. From observations to theory

Reconstruction program (1998)

- 1) $H(z), \epsilon_{DE}(z)$
- 2) $q(z), P_{DE}(z), w_{DE}(z)$
- 3) $r(z), \frac{dw_{DE}(z)}{dz}$

1. Inversion of classical cosmological tests $\mathcal{D}_L(z) \rightarrow H(z)$
2. CMB (acoustic peaks spacing, ISW), BAO
3. $\left(\frac{\delta\rho}{\rho}\right)_m(z), \Phi(z)$ from gravitational lensing, correlation of $\frac{\delta\rho}{\rho}$ with CMB

II. From theory to observations

Models (many of them!)
(qualitatively — the same as for inflation)

1. Fundamental constant Λ
2. Scalar field (with $m \sim 10^{-33} \text{ eV}$
 $w_{DE} \geq -1$)
3. Geometrical DE = modified gravity
(e.g., scalar-tensor and $f(R)$ DE models)

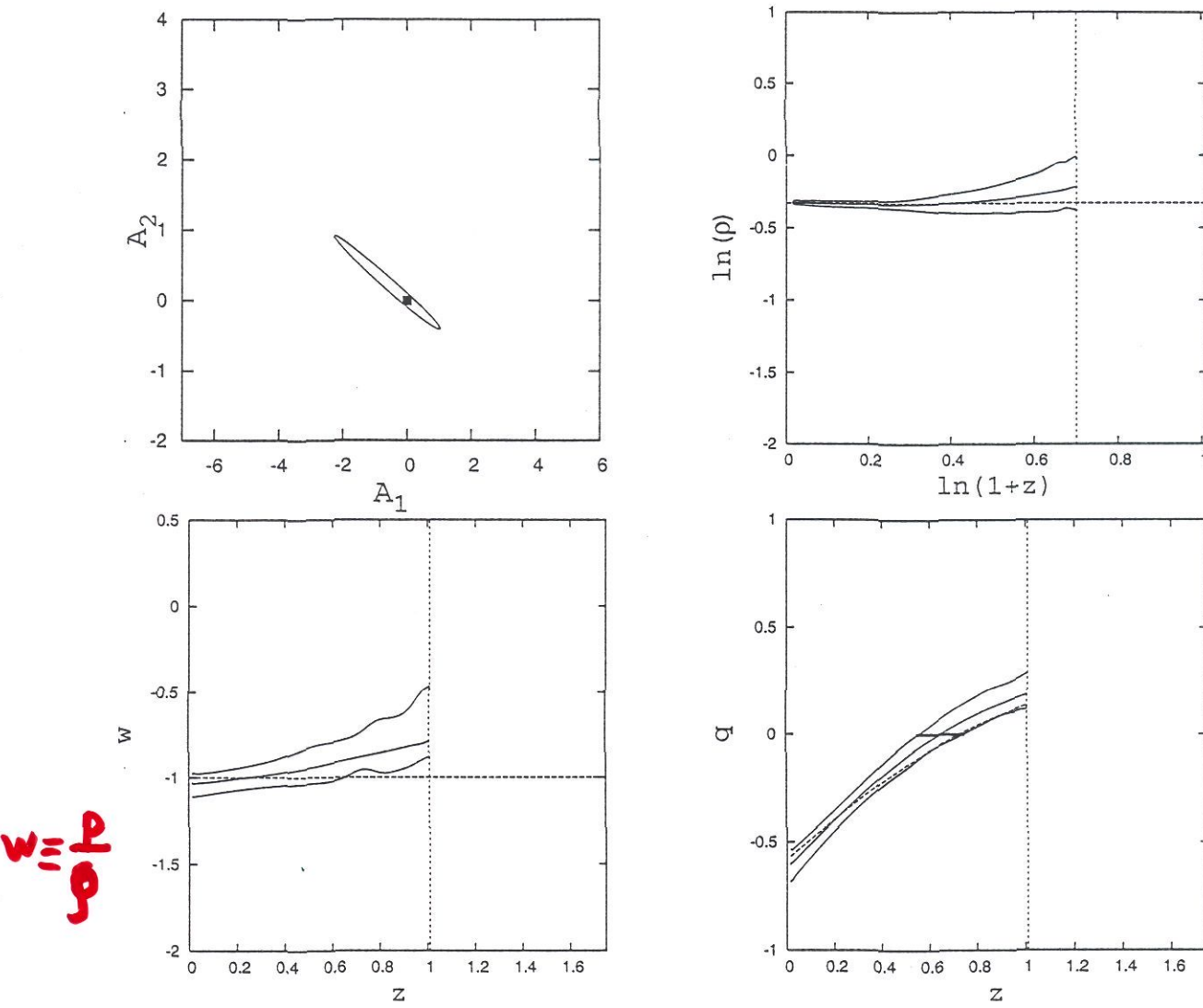


FIG. 7: 2σ confidence levels for the SNLS+CMB+BAO dataset using $\Omega_{\text{om}} = 0.28 \pm 0.03$. The upper left hand panel shows the confidence levels in $A_1 - A_2$, with the black dot representing Λ CDM. The upper right hand panel shows the logarithmic 2σ variation of the DE density in terms of redshift. The dashed line represents Λ CDM. The lower left and right hand panels represent the variation of the equation of state and deceleration parameter respectively. The dashed lines in both panels represent Λ CDM. The thick solid line in the lower right hand panel shows the acceleration epoch, i.e. the redshift at which the universe started accelerating. Results are shown upto redshift $z = 1.01$.

Fit:

$$\frac{H^2(z)}{H_0^2} = A_0 + A_1(1+z) + A_2(1+z)^2 + \Omega_m(1+z)^3$$

$$A_0 + A_1 + A_2 + \Omega_m = 1$$

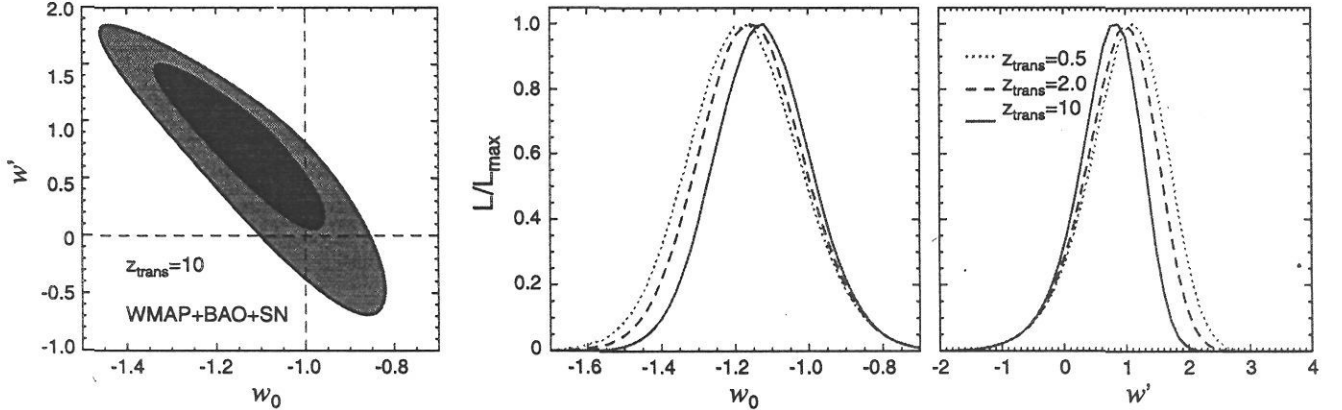


FIG. 14.— Constraint on models of time-dependent dark energy equation of state, $w(z)$ (Eq. [70]), derived from the WMAP distance priors (l_A , R , and z_*) combined with the BAO and SN distance data (§ 5.4.2). There are three parameters: w_0 is the value of w at the present epoch, $w_0 \equiv w(z=0)$, w' is the first derivative of w with respect to z at $z=0$, $w' \equiv dw/dz|_{z=0}$, and z_{trans} is the transition redshift above which $w(z)$ approaches to -1 . Here, we assume flatness of the universe, $\Omega_k = 0$. (Left) Joint two-dimensional marginalized distribution of w_0 and w' for $z_{\text{trans}} = 10$. The contours show the $\Delta\chi^2_{\text{total}} = 2.30$ (68.3% CL) and $\Delta\chi^2_{\text{total}} = 6.17$ (95.4% CL). (Middle) One-dimensional marginalized distribution of w_0 for $z_{\text{trans}} = 0.5$ (dotted), 2 (dashed), and 10 (solid). (Right) One-dimensional marginalized distribution of w' for $z_{\text{trans}} = 0.5$ (dotted), 2 (dashed), and 10 (solid). The constraints are similar for all z_{trans} . We do not find evidence for the evolution of dark energy.

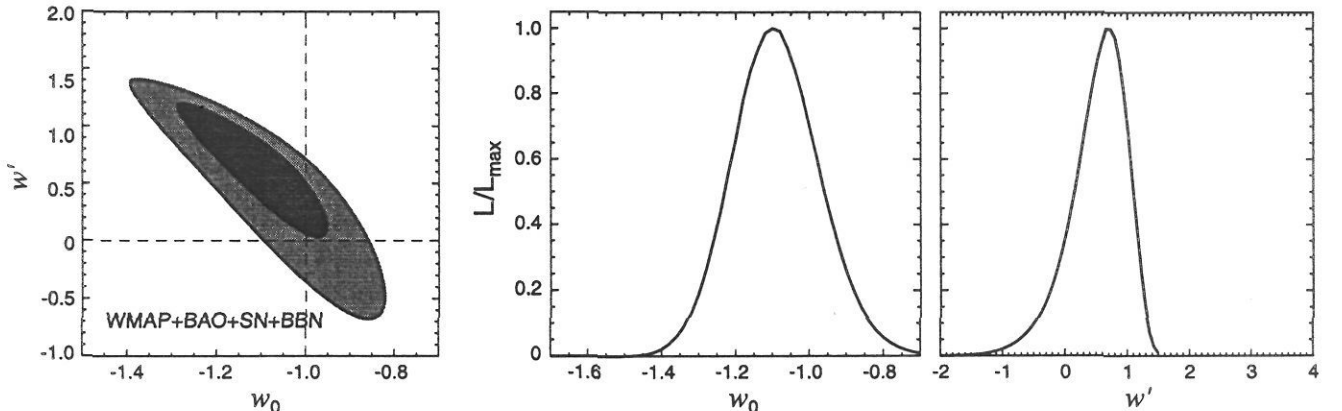


FIG. 15.— Constraint on the linear evolution model of dark energy equation of state, $w(z) = w_0 + w'z/(1+z)$, derived from the WMAP distance priors (l_A , R , and z_*) combined with the BAO and SN distance data as well as the Big Bang Nucleosynthesis (BBN) prior (Eq. [71]). Here, we assume flatness of the universe, $\Omega_k = 0$. (Left) Joint two-dimensional marginalized distribution of w_0 and w' . The contours show the $\Delta\chi^2_{\text{total}} = 2.30$ (68.3% CL) and $\Delta\chi^2_{\text{total}} = 6.17$ (95.4% CL). (Middle) One-dimensional marginalized distribution of w_0 . (Right) One-dimensional marginalized distribution of w' . We do not find evidence for the evolution of dark energy. Note that Linder (2003) defines w' as the derivative of w at $z=1$, whereas we define it as the derivative at $z=0$. They are related by $w'_{\text{Linder}} = 0.5w'_{\text{WMAP}}$.

The 95% limit on w_0 for $z_{\text{trans}} = 10$ is $-0.38 < 1 + w_0 < 0.14$ ⁴⁶, whose upper limit is surprisingly close to the limit for a constant w model in a flat universe, $-0.097 < 1 + w < 0.142$ (95% CL). Therefore, we have a fairly robust upper limit on the present-day value of the equation of state, $1 + w_0 < 0.14$. (This statement is, however, true only for a flat universe.) On the other hand, the lower limit has weakened significantly – by a factor of about four. Our results are consistent with the previous work using the WMAP 3-year data (see Wang & Mukherjee 2007; Wright 2007, for recent work and references therein). We find that our upper limit on w_0 and lower limit on w' are better than the previous work by a factor of ~ 2 .

Alternatively, one may take the linear form, $w(a) = w_0 + (1-a)w_a$, literally and extend it to an arbitrarily high redshift. This can result in an undesirable situation

⁴⁶ The 68% intervals are $w_0 = -1.12 \pm 0.13$ and $w' = 0.70 \pm 0.53$ (WMAP+BAO+SN; $\Omega_k = 0$).

in which the dark energy is as important as the radiation density at the epoch of the Big Bang Nucleosynthesis (BBN); however, one can constrain such a scenario severely using the limit on the expansion rate from BBN (Steigman 2007). We follow Wright (2007) to adopt a Gaussian prior on

$$\sqrt{1 + \frac{\Omega_\Lambda (1 + z_{\text{BBN}})^{3[1+w_{\text{eff}}(z_{\text{BBN})}]}{\Omega_m (1 + z_{\text{BBN}})^3 + \Omega_r (1 + z_{\text{BBN}})^4 + \Omega_k (1 + z_{\text{BBN}})^2}} = 0.942 \pm 0.030, \quad (71)$$

where we have kept Ω_m and Ω_k for definiteness, but they are entirely negligible compared to the radiation density at the redshift of BBN, $z_{\text{BBN}} = 10^9$. Figure 15 shows the constraint on w_0 and w' for the linear evolution model derived from the WMAP distance priors, the BAO and SN data, and the BBN prior. The 95% limit on w_0 is

PRESENT STAGE OF DARK ENERGY RECONSTRUCTION

1. In the first approximation, DE is well described by a cosmological constant

$$w_{DE} \approx -1$$

2. $w_{DE} = -1$ is inside 2 σ error bars for all data

3. If $w_{DE} = \text{const} \neq -1$, then

$$|w_{DE} + 1| \leq 0.1$$

E.g. W.J. Percival et al., arXiv: 0705.3323

$$w_{DE} = -1.004 \pm 0.089$$

$$\text{WMAP5 + BAO + SN: } w_{DE} = -0.97 \pm 0.06$$

4. No evidence for permanent phantom DE.

No evidence for the Big Rip in future

($\Delta T > 50$ by l. y.)

$$\uparrow a(t) \propto (t_i - t)^{-p}$$

$p > 0$

However

5. SNe: small discrepancy between Gold and SNLS samples

BAO: comparison of $D_V(0.2)$ with $D_V(0.35)$ slightly favors $w_{DE} < -1$ for $z < 0.35$

6. If the assumption $w_{DE} = \text{const}$ is omitted, some place for 'temporary phantom' DE still exists for $z \lesssim 0.3$

But $\overline{w_{DE}}(0 < z \lesssim 0.5) \simeq -1$

WMAP5 + BAO + SN: $-0.38 < 1 + w(0) < 0.14$
 $-0.7 < w'(0) < 1.5$

7. Place for dynamical dark energy (especially, a geometric one) still exists!

Recent review on the reconstruction approach:

V. Sahni, A.A. Starobinsky,
IJMPD 15, 2105 (2006) [astro-ph/0610026]

What if recent phantom behaviour ($w < -1$) of dark energy will be confirmed by observations?

Ghost phantom models of dark energy are bad.

1. Quantum instability



2. At the classical level:

does not explain homogeneity and isotropy of the Universe

E.g.: for a given $\bar{H} = \frac{1}{3} \frac{d}{dt} \ln abc$, it is much more probable to have very different $\frac{\dot{a}}{a}$, $\frac{\dot{b}}{b}$, $\frac{\dot{c}}{c}$ compensated by the negative energy density of the ghost field.

Scalar-tensor models of dark energy do not have this problem.

Reconstruction of dark energy in scalar-tensor gravity

B. Boisseau, G. Esposito-Farese,

D. Polarski, A.S.

Phys. Rev. Lett. 85, 2236 (2000)

$\epsilon_{DE} + p_{DE} < 0$ is permitted

$$\mathcal{I} = \frac{1}{2} (F(\varphi) R + Z(\varphi) \varphi_{,\mu} \varphi^{,\mu}) - V(\varphi) + \mathcal{I}_m$$

Includes $R + f(R)$ theory for $Z(\varphi) = 0$.

$$Z(\varphi) = 1$$

$$\omega^{-2}(\varphi) = F^{-2} \left(\frac{dF}{d\varphi} \right)^2$$

Two independent observable
cosmological functions are
required for reconstruction
of $F(\varphi)$ and $V(\varphi)$

$$D_L(z), \quad \delta(z)$$



$$H(z) \longrightarrow F(z) \begin{matrix} \longrightarrow V(z) \\ \longrightarrow \varphi(z) \end{matrix}$$

Background equations

$$3FH^2 = \rho_m + \frac{\dot{\phi}^2}{2} + V - 3H\dot{F}$$

$$-2F\dot{H} = \rho_m + \dot{\phi}^2 + \ddot{F} - H\dot{F} \quad \rho_m \propto a^{-3}$$

Their consequence:

$$\ddot{\phi} + 3H\dot{\phi} + \frac{dV}{d\phi} - 3(\dot{H} + 2H^2) \frac{dF}{d\phi} = 0$$

In terms of redshift:

$$F'' + \left[(\ln H)' - \frac{4}{1+z} \right] F' + \left[\frac{6}{(1+z)^2} - \frac{2(\ln H)'}{1+z} \right] F \\ = \frac{2V}{(1+z)^2 H^2} + 3(1+z) \left(\frac{H_0}{H} \right)^2 F_0 \Omega_{m,0}$$

$$\phi'^2 = -F'' - \left[(\ln H)' + \frac{2}{1+z} \right] F' + \frac{2(\ln H)'}{1+z} F \\ - 3(1+z) \frac{H_0^2}{H^2} F_0 \Omega_{m,0}$$

I. First step \rightarrow as in GR

$$H(z) = \left[\frac{d}{dz} \left(\frac{D_L(z)}{1+z} \right) \right]^{-1}$$

II. Equation for sufficiently small-scale inhomogeneities

$$\ddot{\delta} + 2H\dot{\delta} - 4\pi G_{\text{eff}} \rho_m \delta = 0$$

where $\delta \equiv \left(\frac{\delta \rho}{\rho} \right)_{\text{CDM+baryon}}$ at a fixed comoving scale $\lambda = a(t)/k$ and

$$\frac{k^2}{a^2} \gg \max \left(\frac{d^2 V}{dy^2}, H^2 \cdot \max \left(1, \frac{d^2 F}{dy^2} \right) \right);$$

$$G_{\text{eff}} = \frac{1}{8\pi F} \cdot \frac{F + 2 \left(\frac{dF}{dy} \right)^2}{F + \frac{3}{2} \left(\frac{dF}{dy} \right)^2}$$

From this, excluding dy :

a second-order differential

equation for $F(z)$.

Properties of scalar-tensor models of dark energy

R. Garrouri, D. Polarski, A. Ranquet, A. S.
JCAP 09, 016 (2006) [astro-ph/0606287]

1. Temporary phantom behaviour and crossing of the phantom boundary $w = -1$ are possible for an open set of $F(\varphi)$ and non-zero and non-constant $V(\varphi)$.

"Curvature induced phantomness"

2. In the absence of dust-like matter ($\Omega_m = 0$), power-law solutions leading to the Big Rip singularity in future and to $w < -1$ exist if

$$F = \alpha \varphi^2, \quad \varphi \rightarrow \infty$$

$$V = V_0 |\varphi|^n, \quad 2 < n < 4$$

(Barrow & Maeda
1990)

Then $a(t) \propto (t_0 - t)^q$

$$\varphi(t) \propto (t_0 - t)^2$$

$$q = \frac{2(n+2+\frac{1}{\alpha})}{(n-2)(n-4)} < 0$$

$$r = \frac{2}{2-n} < 0$$

However, for these solutions $|w+1| \leq \frac{q}{3} \sim \frac{1}{\omega_{BS}}$
and very small.

Present observational bounds:

$$\gamma_{PN} - 1 = (2.1 \pm 2.3) \cdot 10^{-5} \quad \text{Bertotti et al., 2003} \rightarrow \text{Cassini mission}$$

$$\beta_{PN} - 1 = (0 \pm 1) \cdot 10^{-4} \quad \text{Pitjeva, 2005} \rightarrow \text{ephemerides}$$

$$\frac{\dot{G}_{eff,0}}{G_{eff,0}} = (-0.2 \pm 0.5) \cdot 10^{-13} \text{ y}^{-1} \text{ of planets}$$

$$\beta_{PN} - 1 = (1.2 \pm 1.1) \cdot 10^{-4} \quad \text{Williams et al., 2005} \rightarrow \text{lunar laser ranging}$$

$$\omega_{BD,0} \equiv \left(\frac{F}{\left(\frac{dF}{dy} \right)^2} \right)_0 > 4 \cdot 10^4$$

3. Small z expansion.

$$\frac{F(z)}{F_0} = 1 + F_1 z + F_2 z^2 + \dots$$

$$\frac{V(z)}{3F_0 H_0^2} = \Omega_{V,0} + u_1 z + u_2 z^2 + \dots$$

$$\frac{H^2(z)}{H_0^2} = 1 + k_1 z + k_2 z^2 + \dots$$

$$F_0^{-1/2} \psi'(z) = y_0' + y_1' z + y_2' z^2 + \dots$$

$$\omega_{DE}(z) = w_0 + w_1 z + w_2 z^2 + \dots$$

$$|F_1| < 10^{-2}$$

$$y_0'^2 = 6(1 - \Omega_{m,0} - \Omega_{V,0} - F_1) \geq 0$$

What is required to get significant phantomness ($|w+1| \gg \frac{1}{\omega_{\text{ph},0}}$)?

$$F_2 < 0, \quad |F_2| \sim 1 \gg |F_2| \quad (|F_2| < 10^{-2})$$

$$|F_2| > 3 (\Omega_{\text{DE},0} - \Omega_{\nu,0}) > 0$$

$\hookrightarrow 1 - \Omega_{m,0}$

$$w_0 + 1 = \frac{2F_2 + 6(\Omega_{\text{DE},0} - \Omega_{\nu,0})}{3\Omega_{\text{DE},0}} < 0$$

k_2 can be negative, too

4. Connection with post-Newtonian parameters in the significantly phantom case.

$$\gamma_{\text{PN}} - 1 = - \frac{F_1^2}{6(\Omega_{\text{DE},0} - \Omega_{\nu,0})} < 0$$

$$\beta_{\text{PN}} - 1 = - \frac{F_1^2 F_2}{72(\Omega_{\text{DE},0} - \Omega_{\nu,0})} > 0$$

$$-4 < \frac{\gamma_{\text{PN}} - 1}{\beta_{\text{PN}} - 1} = \frac{12(\Omega_{\text{DE},0} - \Omega_{\nu,0})}{F_2} < 0$$

However, $|\gamma_{\text{PN}} - 1|$ and $|\beta_{\text{PN}} - 1|$ may be much smaller than $|1+w|$ if F_2 is very small

$$\frac{\dot{G}_{\text{eff},0}}{G_{\text{eff},0}} = H_0 F_2 \left(1 - \frac{F_2}{3(\Omega_{\text{DE},0} - \Omega_{\nu,0})} \right)$$

Positive detection of $\gamma_{PN} < 1$, $\beta_{PN} > 1$

may be a strong argument for significant phantomness of present DE.

Negative detection tells nothing.

5. Correct asymptotic behaviour

for large z ($\dot{\varphi}^2 \gg 0$, $w_{DE} \leq 0$)

requires non-zero and non-constant $V(\varphi)$

E.g. $F(\varphi) \rightarrow F_\infty < F_0$

$$V(\varphi) \propto \exp\left(\sqrt{\frac{3}{2F_0\Omega_{u,\infty}}} \varphi\right)$$

$$z, \varphi \rightarrow \infty$$

6. In the stable case $F > 0$, $w_{DE} > -\frac{2}{3}$,

no possibility to construct a stable wormhole (even with an electromagnetic field)

(K. A. Bronnikov & A.S., JETP Lett.

85, 1 (2007) [gr-qc/0612032])

Geometrical $f(R)$ model of \mathcal{DE}

$$S = \frac{1}{16\pi G} \int f(R) \sqrt{-g} d^4x + S_m$$

$$f(R) = R + F(R) \quad R \equiv R_{\mu}^{\mu}$$

$$R_{\mu}^{\nu} - \frac{1}{2} \delta_{\mu}^{\nu} R = -8\pi G (T_{\mu}^{\nu}(m) + T_{\mu}^{\nu}(\mathcal{DE}))$$

$$8\pi G T_{\mu}^{\nu}(\mathcal{DE}) \equiv F'(R) R_{\mu}^{\nu} - \frac{1}{2} F(R) \delta_{\mu}^{\nu} \\ + (\nabla_{\mu} \nabla^{\nu} - \delta_{\mu}^{\nu} \nabla_{\rho} \nabla^{\rho}) F'(R)$$

Particle content: graviton +
massive scalar particle ($M^2 = \frac{1}{3f''(R)}$)
(dubbed "scalaron" in A.S., 1980)

Stability conditions:

- ① $f' > 0$ graviton is not a ghost
- ② $f'' > 0$ scalaron is not a tachyon

imposed for $R \geq R_{\text{now}}$ at least

(i.e. during the whole evolution of the Universe)

Violation of these conditions is undesirable from the classical point of view, too!

$f'(R_0) = 0$ - instant loss of homogeneity and isotropy

$f''(R_0) = 0$ - weak singularity

$$R(t) = R_0 + O(\sqrt{t})$$

$$a(t) = a_0 + a_1 t + a_2 t^2 + O(t^{5/2})$$

③ Existence of the Newtonian regime
($\Delta\varphi = 4\pi G\rho$)

$$|F| \ll R, |F'(R)| \ll 1, R|F''(R)| \ll 1$$

for $R_{\text{now}} \ll R$ (at up to some very large R)



De Sitter regime

$$Rf' = 2f$$

Stable if

$$f'(R_1) > R_1 f''(R_1)$$



Equivalent to $\omega_{BD} = 0$ scalar-tensor gravity

Use for inflation

$$f(R) = R + \frac{R^2}{6M^2} \quad (+ \text{small non-local terms})$$

AS, 1980

Internally self-consistent inflationary model with slow-roll decay, a graceful exit to the subsequent RD FRW stage (through an intermediate matter-dominated stage) and sufficiently effective reheating

$$\tau \sim M_{\text{Pl}}^2 / M^3 \quad N \sim 50$$

Remains viable

$$M = 3.0 \times 10^{-6} (N/50)^{-1} M_{\text{Pl}}$$

$$n_s = 1 - \frac{2}{N} = 0.96 \quad \text{for } N=50$$

$$r = \frac{12}{N^2} = 4.8 \times 10^{-3} (N/50)^2$$

$$\text{Exp. : } \bar{n}_s = 0.96 \pm 0.014, \quad r < 0.20$$

Use for DE

$$F(R) \propto R^{-n} \text{ for } R \rightarrow 0$$

Does not work for many reasons

Viable model - regular at $R=0$

$$f(R) = R + \lambda R_0 \left(\frac{1}{\left(1 + \frac{R^2}{R_0^2}\right)^n} - 1 \right)$$

AS, JETP Lett. 86, 157 (2007)

arXiv: 0706.2041 [astro-ph]

or even

$$f(R) = R - \lambda R_0 \tanh^2 \frac{R}{R_0}$$

$f(0) = 0$ - 'disappearing' cosmological constant in flat space-time

Induced Λ at high curvatures:

$$\Lambda_\infty \equiv -\frac{1}{2} F(\infty) = \frac{\lambda R_0}{2}$$

Observational restrictions

1. Cosmology

Anomalous growth of non-relativistic matter perturbations in the regime

$$k \gg M(R) a$$

$$G_{\text{eff}} = 4G/3f'(R) \approx \frac{4G}{3}$$

$$\left(\frac{\delta\rho}{\rho}\right)_m \propto t^{\frac{\sqrt{33}-1}{6}} \quad (\text{instead of } \propto t^{2/3})$$

Results in the apparent mismatch

$$\Delta n_s = n_s^{(\text{gal})} - n_s^{(\text{CMB})} = \frac{\sqrt{33}-5}{2(3n+2)}$$

$$\Delta n_s < 0.05 \rightarrow n \geq 2$$

$$n=2 \quad (F(R) \propto R^{-4}) \Rightarrow \text{increase of } G_0 \text{ from } 0.75 \text{ to } 0.9$$

2. Laboratory and Solar system tests

$$M(R) L \gg 1 \quad \text{with } R = 8\pi G T_m = 8\pi G \rho_m$$

Otherwise, $\gamma_{\text{PN}} = \frac{1}{2}$ and the 'fifth'

force appears

$$M(R/\rho_m) \propto \rho_m^{n+1} \quad (R = 8\pi G \rho_m)$$

$n \geq 2$ is sufficient for all tests

Structure of corrections
at the matter-dominated and earlier
stages

$$R = R^{(0)} + \delta R_{ind} + \delta R_{osc}$$

$$R^{(0)} = 2\pi G T_m \propto a^{-3}$$

$$\delta R_{ind} = \left(R F'(R) - 2F(R) - 3 \nabla_\mu \nabla^\mu F'(R) \right)_{R=R^{(0)}}$$

$$R \gg R_0 : \delta R_{ind} \approx \text{const} = -2F(\infty) = 4\Lambda(\infty)$$

No Dolgov-Kawasaki instability

$$\delta R_{osc} \propto \begin{cases} t^{-3n-4} \sin(\text{const} \cdot t^{-2n-1}) & \text{MD} \\ t^{-\frac{3n}{4}-3} \sin(\text{const} \cdot t^{-(3n+2)/2}) & \text{RD} \end{cases}$$

$\frac{\delta a}{a}$ is small but $\frac{\delta R_{osc}}{R^{(0)}}$ diverges for $t \rightarrow 0$

δR_{osc} should be very small just from
beginning - problem for those $F(R)$
models which do not let R become
negative (due to the crossing of the
 $F''(R) = 0$ point)

"Scalaron overproduction" problem

"Big Boost" SINGULARITY WITH $R \rightarrow \infty$

AND ITS ELIMINATION

If $F(R) \rightarrow 0$ at $R \rightarrow \infty$, then a new generic "Big Boost" singularity can arise:

$$F(R) \propto R^{-2n}; \quad F''(\infty) = 0$$

$$a = a_0 + a_1(t-t_0) + a_2|t-t_0|^k, \quad 1 < k = \frac{2n+1}{n+1} < 2$$

$$R \propto |t-t_0|^{k-2} > 0$$

Elimination:

$$\text{add } \frac{R^2}{6M^2} \text{ to } F(R) \quad M^2(\infty) = M$$

Additional advantages:

1. No unlimited growth of $M(R)$
2. A toy "UV-completion" - further radiative corrections are logarithmic only
3. A possibility to unify inflation and present dark energy in one $F(R)$ model if $M = 3 \cdot 10^{-6} M_{Pl}$

$$\tau_{dec} \sim \frac{M_{Pl}^2}{M^3} \rightarrow M > 10^4 \text{ GeV for scalarons to decay before BBN}$$

CONCLUSIONS FOR $f(R)$ MODELS

1. With a regular $f(R)$ satisfying

$$f'(R) > 0, \quad f''(R) > 0 \quad \text{for all } R$$

$$|f - R| \ll R, \quad |f' - 1| \ll 1, \quad R/f'' \ll 1$$

$$\text{for } R_0 \ll R \ll M^2 \\ \text{with } M \gtrsim 10^4 \text{ GeV} \\ \text{for } R \rightarrow \infty,$$

$$f(R) \approx R^2$$

it is possible to construct viable models of DE, satisfying all existing cosmological, Solar system and laboratory data, and distinguishable from Λ CDM

2. Further unification of primordial DE (producing inflation) and present DE is possible for the specific choice of M : $M \approx 3 \cdot 10^{-6} M_{pl}$

3. The most critical test of these DE models: anomalous growth of scalar perturbations at recent time ($z \sim 1-3$ for $L = 8R^{-2} \text{ Mpc}$)

CONCLUSIONS

1. Deviation of dynamical DE from an exact cosmological constant is $\lesssim 10\%$, but still may exist
2. The simplest DE model which can accommodate its possible recent phantom behaviour and crossing of the "phantom boundary" $w_{DE} = -1$ is based on scalar-tensor gravity and does not have ghosts or instabilities
3. Viable models in $f(R)$ gravity, though more restricted, are possible, too
4. However close the present DE may be to Λ , simply by analogy with primordial DE, one should not expect it to be absolutely stable and eternal