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SUPERFLUIDTY OF TRAPPED ATOMIC GASES

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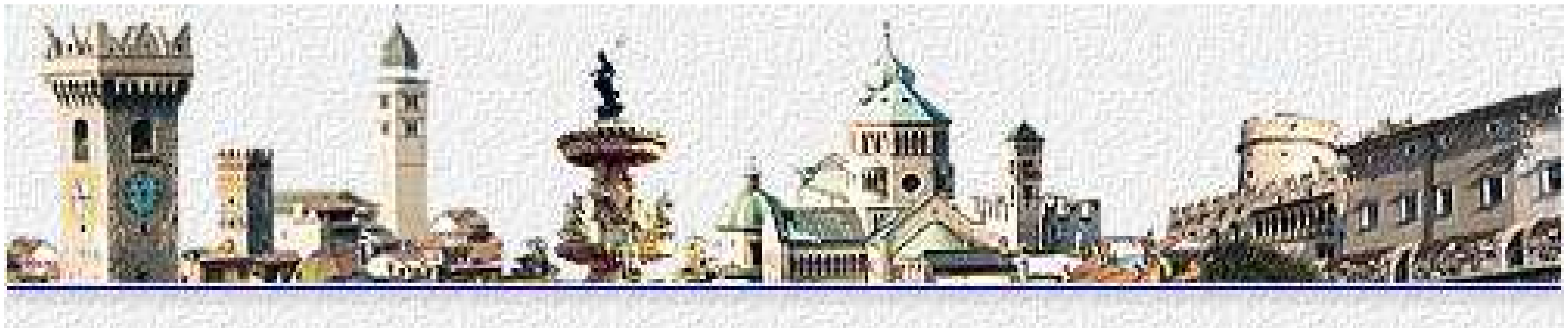
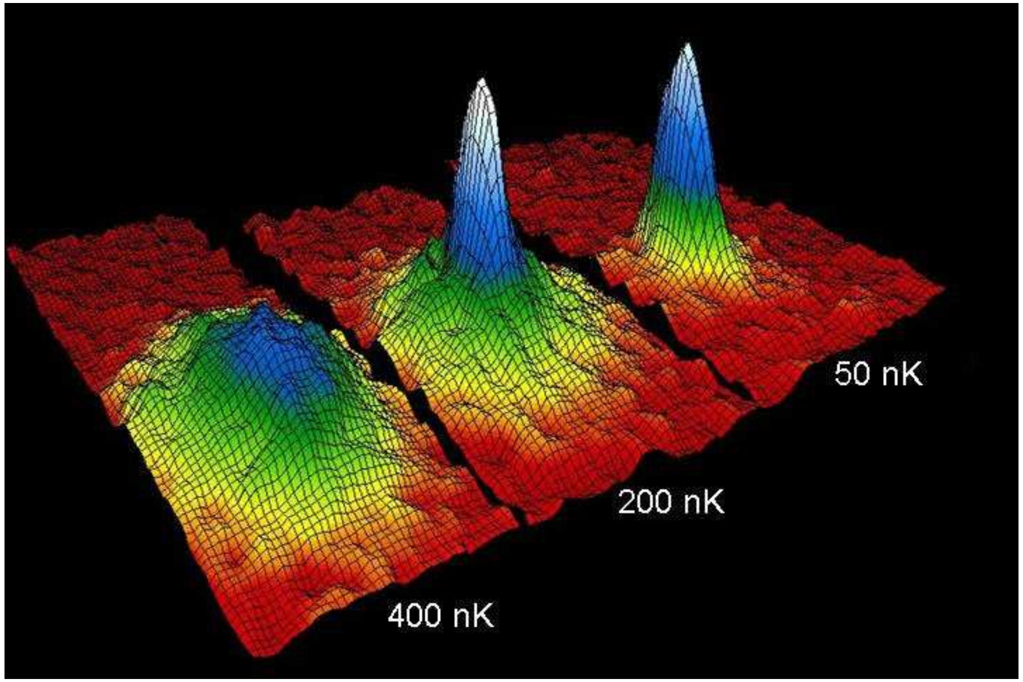
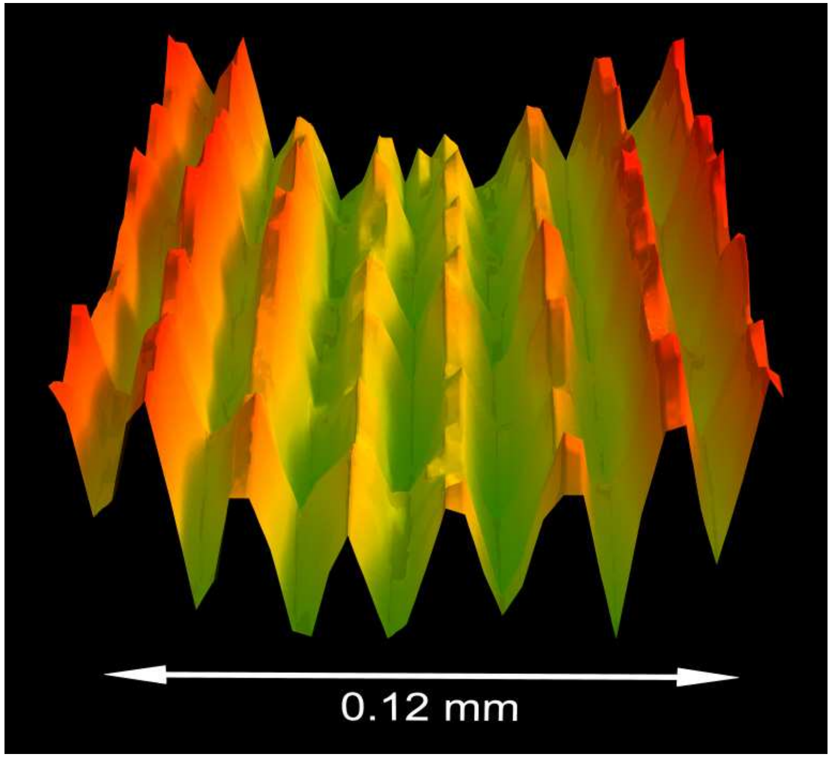


Bose-Einstein condensation

Bimodal distribution
(Jila 1995)



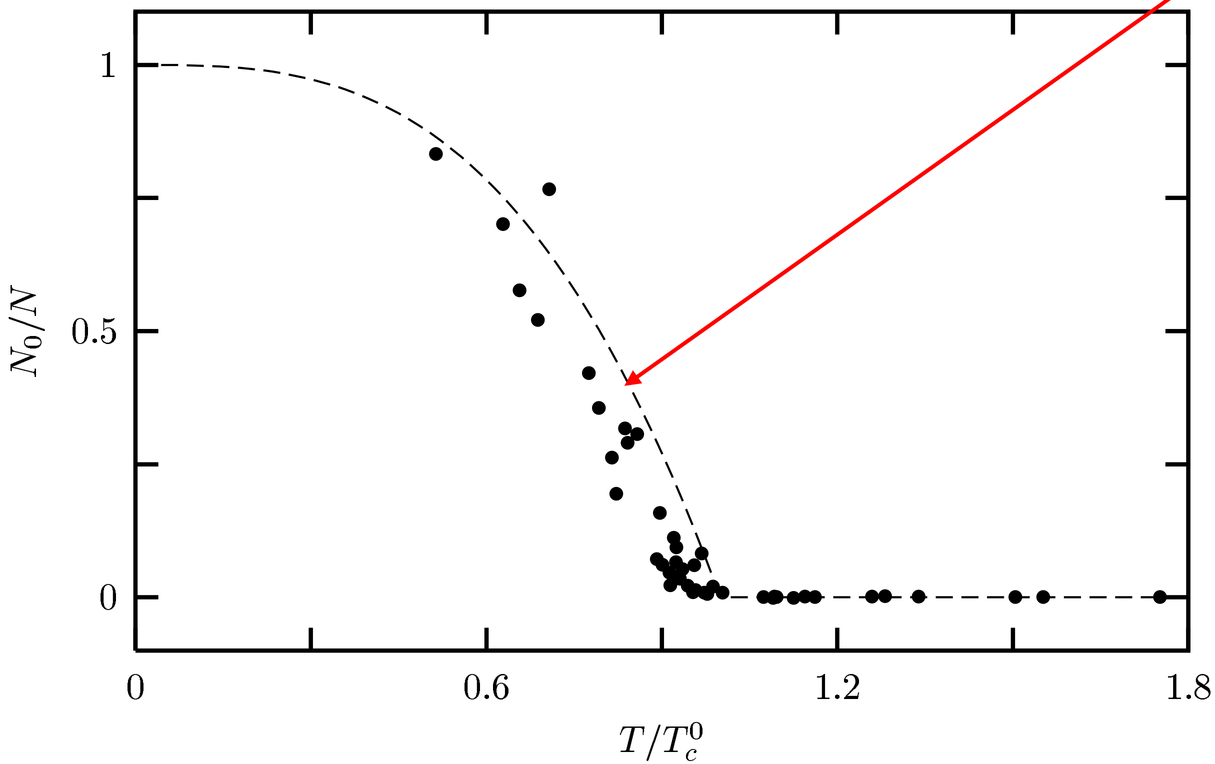
Interference (Mit 1996)



Condensate fraction and phase transition

$$k_B T_c = 0.94\hbar\omega_{ho} N^{1/3}$$

$$N_0 / N = 1 - (T / T_C)^3$$



Exp data: Jila 1996

ideal gas predictions in harmonic trap:
Tc = fraction of microKelvin

$$V_{ext} = \frac{1}{2} m\omega_{ho} r^2$$

What is new with BEC in trapped atomic gases ?

- Bose-Einstein condensation
in both momentum and coordinate space
- Diluteness (Gross-Pitaevskii eq. for order parameter)

New important knobs available
(in addition to temperature, density, angular velocity)

- tuning of scattering length
  (BEC-BCS crossover in Fermi superfluids)
- flexible trapping conditions (anisotropy of harmonic trapping, optical lattices, 1D and 2D configurations)

SUPERFLUIDITY in trapped atomic gases

- **Hydrodynamic** behavior at **T=0** (irrotationality)
- Quenching of **moment of inertia**
- **Quantized vortices**
- **Josephson** oscillations
- Absence of **viscosity** (Landau's critical velocity)

……..

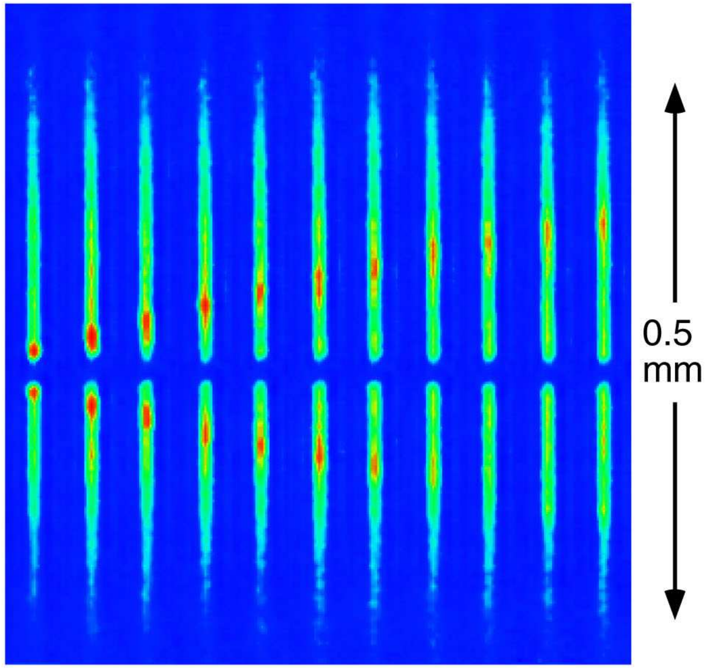
Furthermore, in **Fermi** gases

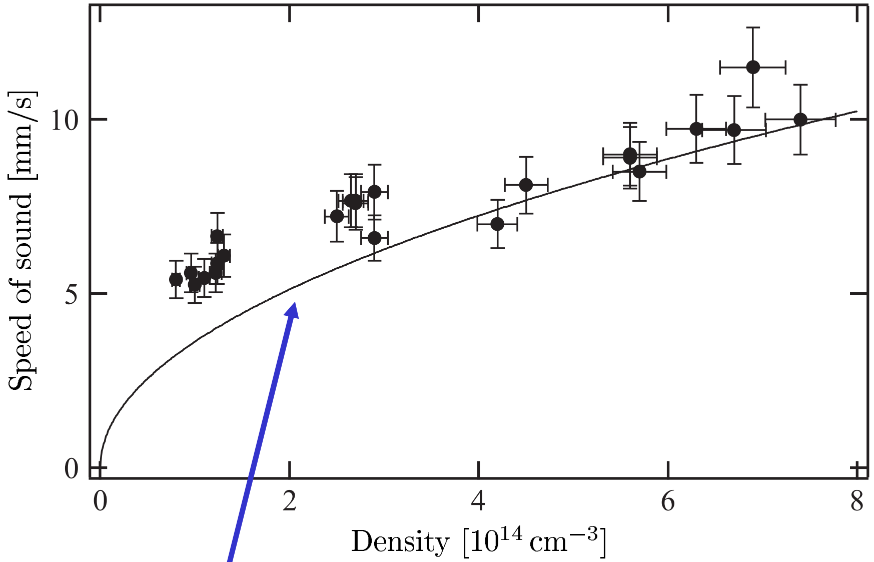
- **Pairing gap** (single particle excitations)
- **Phase separation** in the presence of polarization

T=0 HYDRODYNAMICS

- BOGOLIUBOV **SOUND**
- COLLECTIVE **OSCILLATIONS**
- ANISOTROPIC **EXPANSION**

Bogoliubov sound
(wave packet propagating in a BEC, Mit 97)



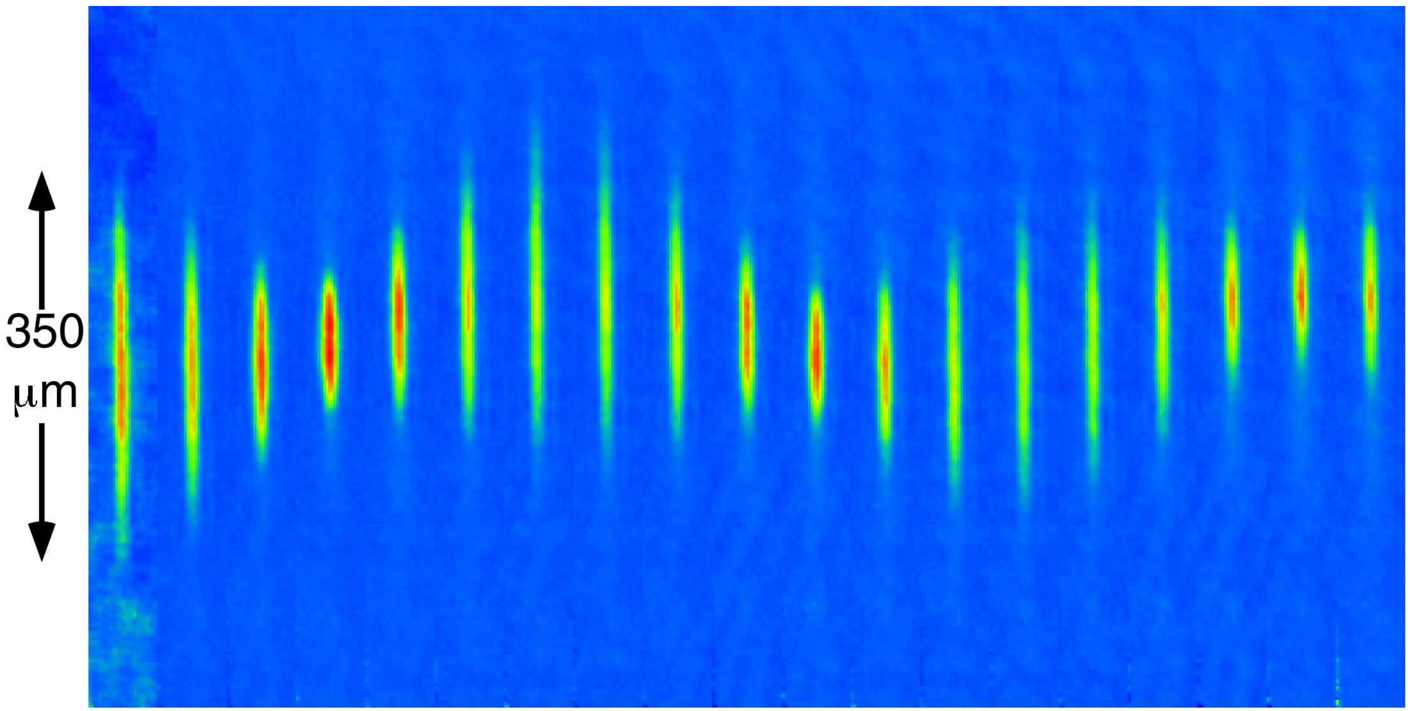


sound velocity as a function of central density

Collective oscillations at T=0
(axial compression mode in BEC)

Exp (Mit, 1997) $\omega = 1.57\omega_z$

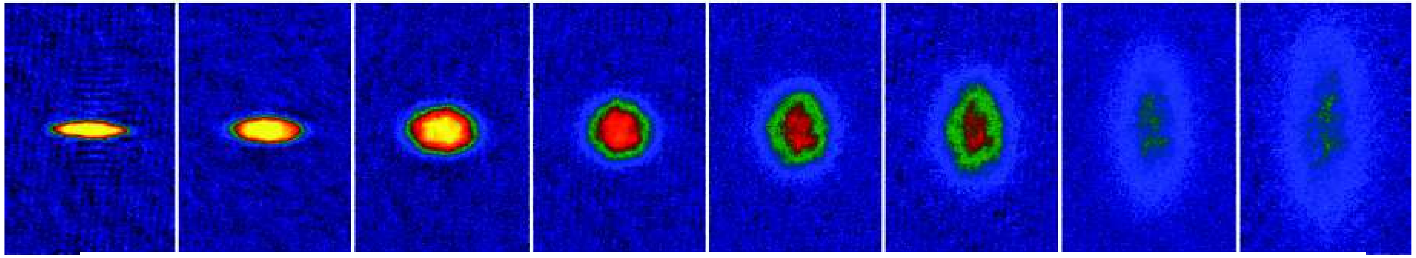
HD Theory (Trento, 1996): $\omega = \sqrt{5/2}\,\omega_z = 1.58\omega_z$

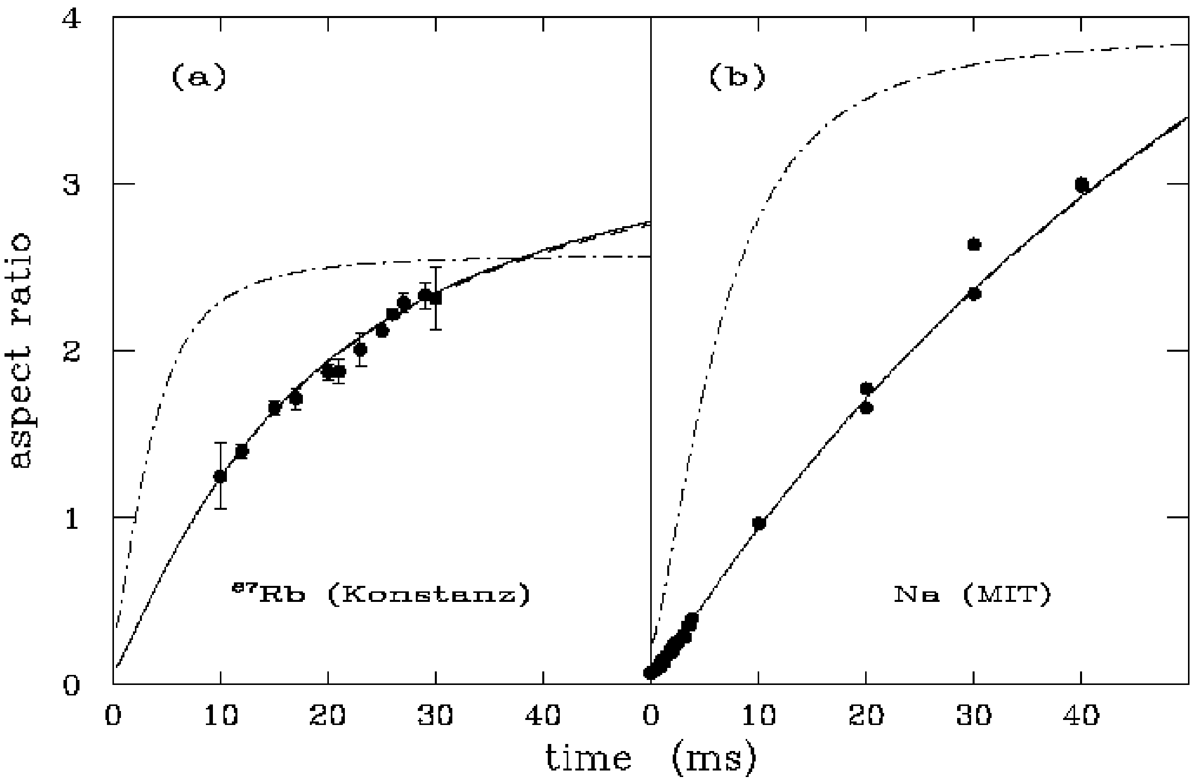


5 milliseconds per frame

Hydrodynamics predicts anisotropic expansion of the superfluid

(Kagan, Surkov, Shlyapnikov 1996; Castin, Dum 1996,





Rotational effects

Superfluids rotate differently from classical fluids
(due to irrotationality constraint)

At low angular velocities moment of inertia is quenched
(consequences on **scissors** mode)

Quantized vortices are formed at higher angular velocity

Scissors mode

Direct measurement of moment of inertia difficult because images of atomic cloud probe **density** distribution (**not velocity** distribution)

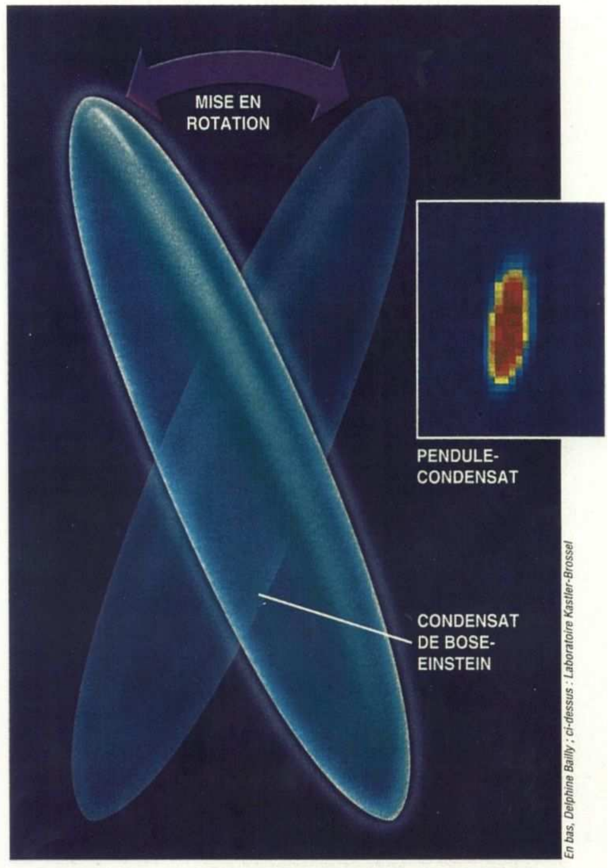
In deformed traps **rotation** is however **coupled** to **density** oscillations.
Exact relation, holding also in the presence of 2-body forces:

$$[H, L_z] = im(\omega_y^2 - \omega_x^2)\sum_i x_i y_i$$

angular momentum **quadrupole operator**

Response to **transverse** probe measurable thorugh **density** response function !!

Example of coupling is provided by **scissor mode**.
If confining (deformed) trap is suddenly rotated by angle θ the gas is no longer in equilibrium.
Behaviour of resulting oscillation depends crucially on value of moment of inertia (**irrotational** vs **rigid**)



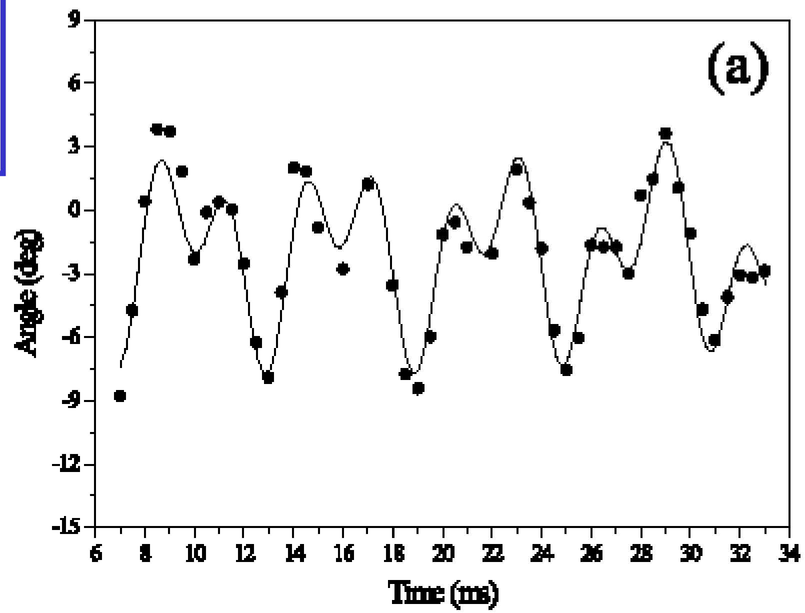
Theory of scissors mode
(Guery-Odelin and S.S., PRL 83 4452 (1999))

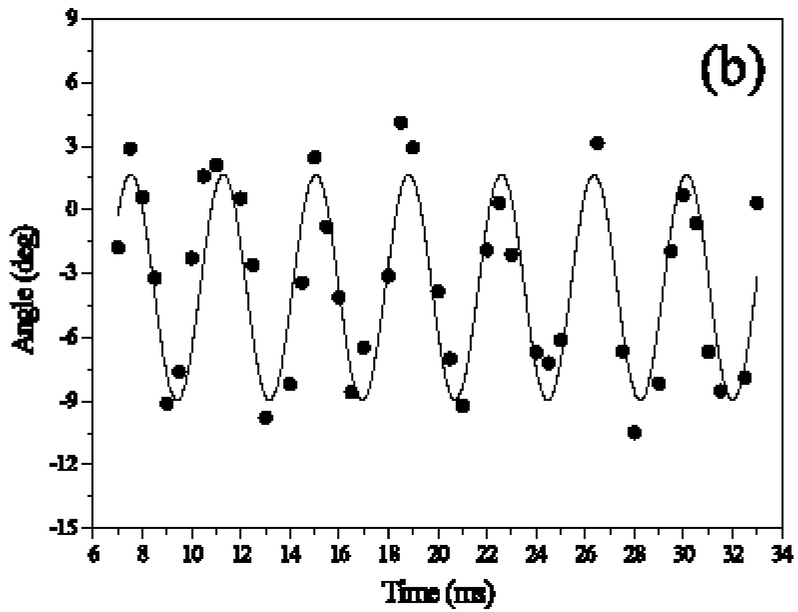
Scissors measured at Oxford
(Marago'et al, PRL 84, 2056 (2000))

Above T_C (normal)

2 modes: $\omega_{\pm} = |\omega_x \pm \omega_y|$

Below T_C (superfluid) :
single mode: $\omega = \sqrt{\omega_x^2 + \omega_y^2}$





Quantized vortices

Superfluids carry angular momentum through quantized vortices

Quantized vortices are **macroscopic** configurations described by order parameter

$$\Psi = \Psi_V e^{i\varphi}$$

azimuthal angle

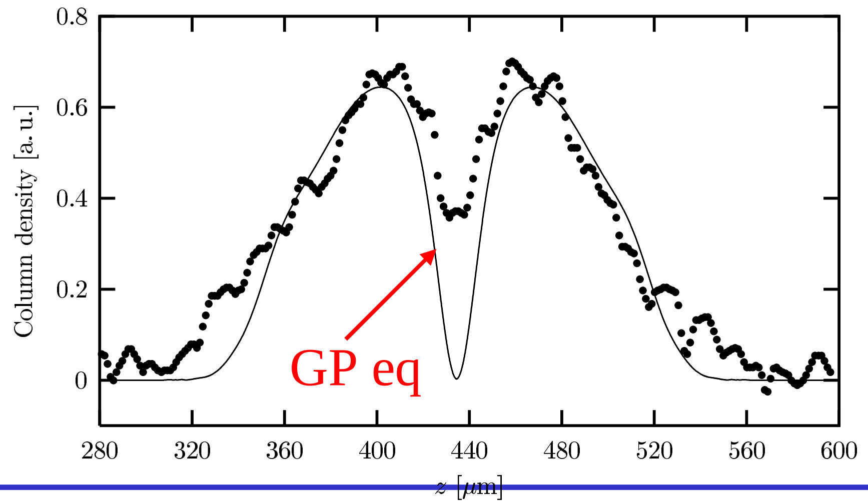
Circulation of velocity is **quantized**. Quantum of circulation: **h/m**

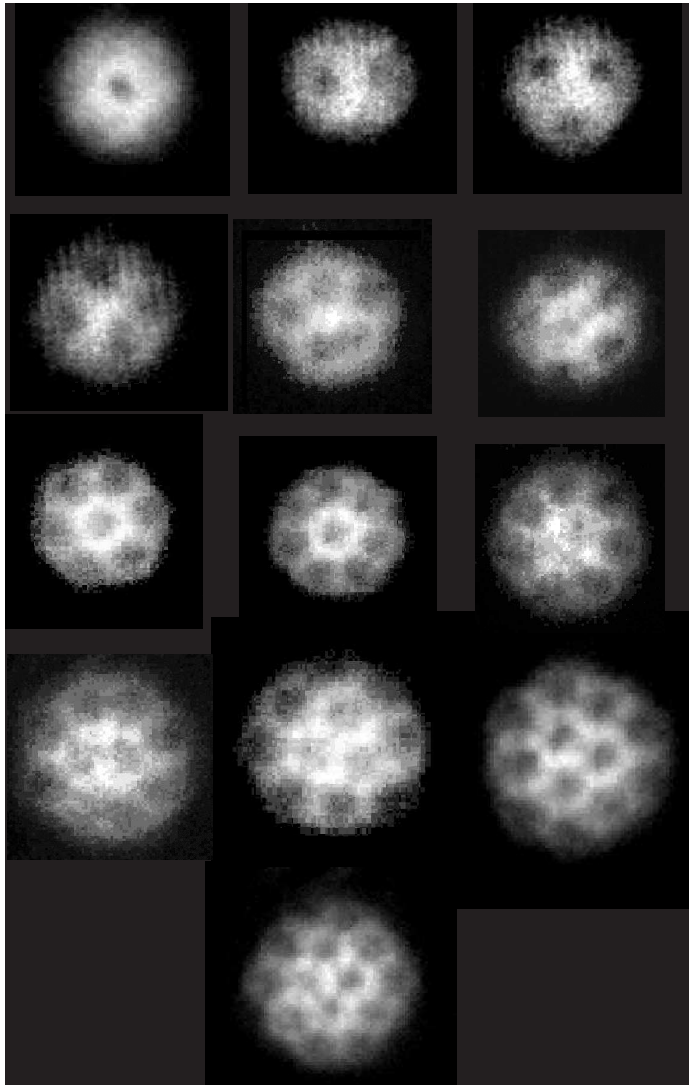
Equation for the order parameter: **Gross-Pitaevskii** (1961).
Describes the condensate at the scale of the **healing length**

Vortex detection in Bose-Einstein condensed gases

Size of vortices is of order of healing length (< 1 micron),
Cannot be resolved in situ.
Visibility emerges **after expansion**

Good **agreement** between measured (Madison et al. 2000) and calculated (Dalfovo & Modugno, 2000) density profiles **after expansion**





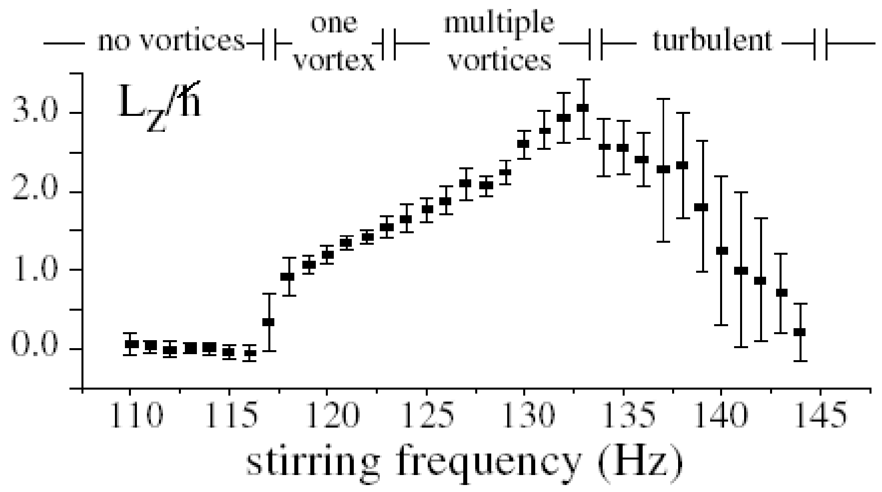
Vortices at ENS
Chevy, 2001

Spectroscopic measurement of angular momentum

Splitting between m=+2 and m=-2 quadrupole frequencies proportional to angular momenetum
(Zambelli and Stringari,1999)

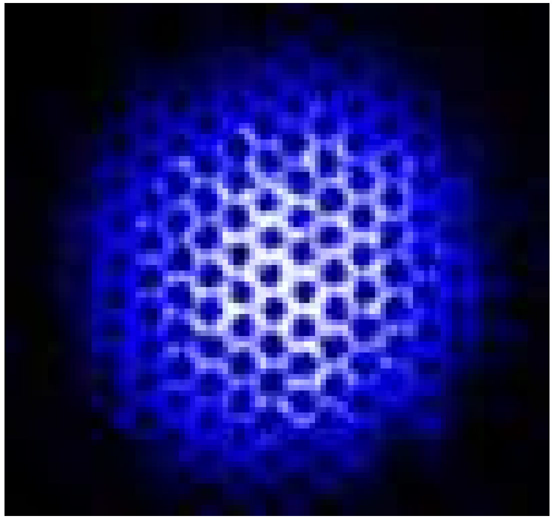
$$\omega_+ - \omega_- = \frac{2}{M}\frac{\langle l_z \rangle}{\langle r_\perp^2 \rangle}$$

Measurement of angular momentum in BEC's
(Chevy et al., 2000)



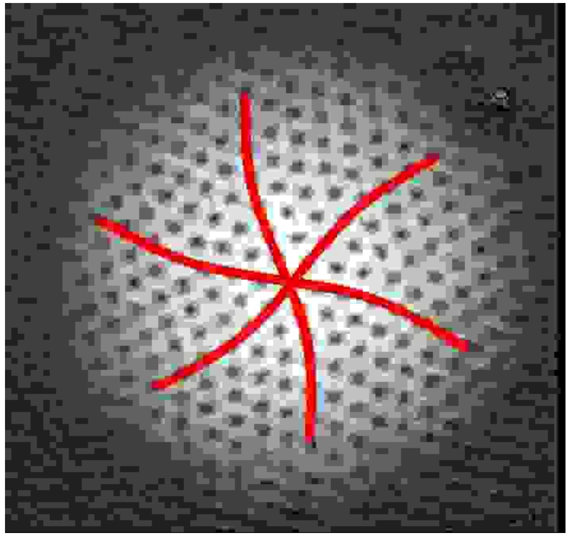
Vortex lattices

By increasing angular velocity one can nucleate more vortices (vortex lattice)



(Jila 2002)

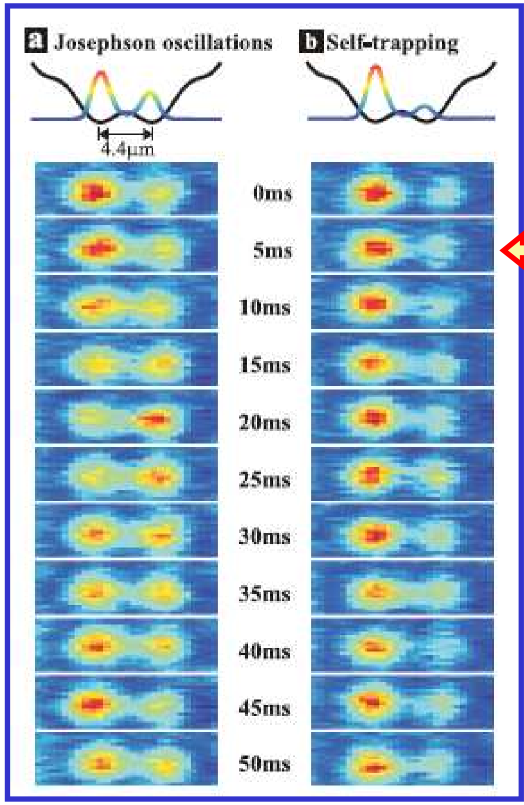
Vortices form a regular
triangular lattice
(cfr Abrikosov lattice
In superconductors)



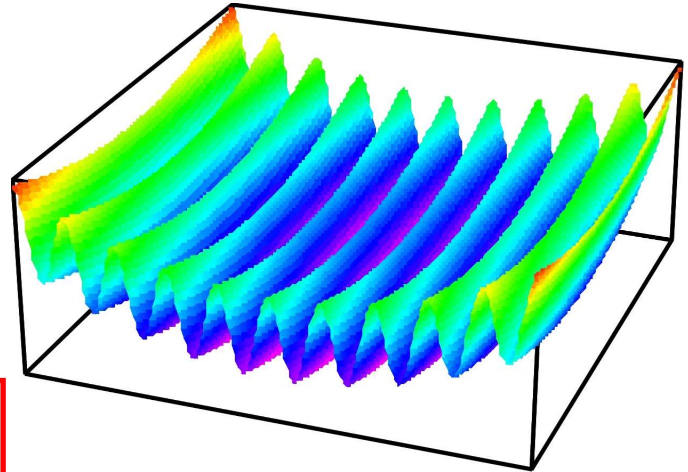
(Jila 2003)

Tkachencko (elastic) waves
In a BEC vortex lattice

JOSEPHSON OSCILLATIONS

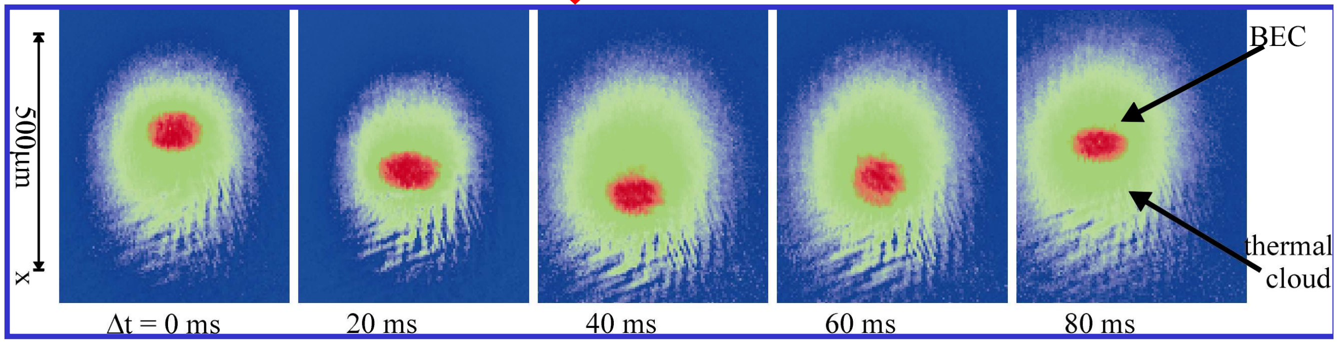


Double well
(Heidelberg 2004)



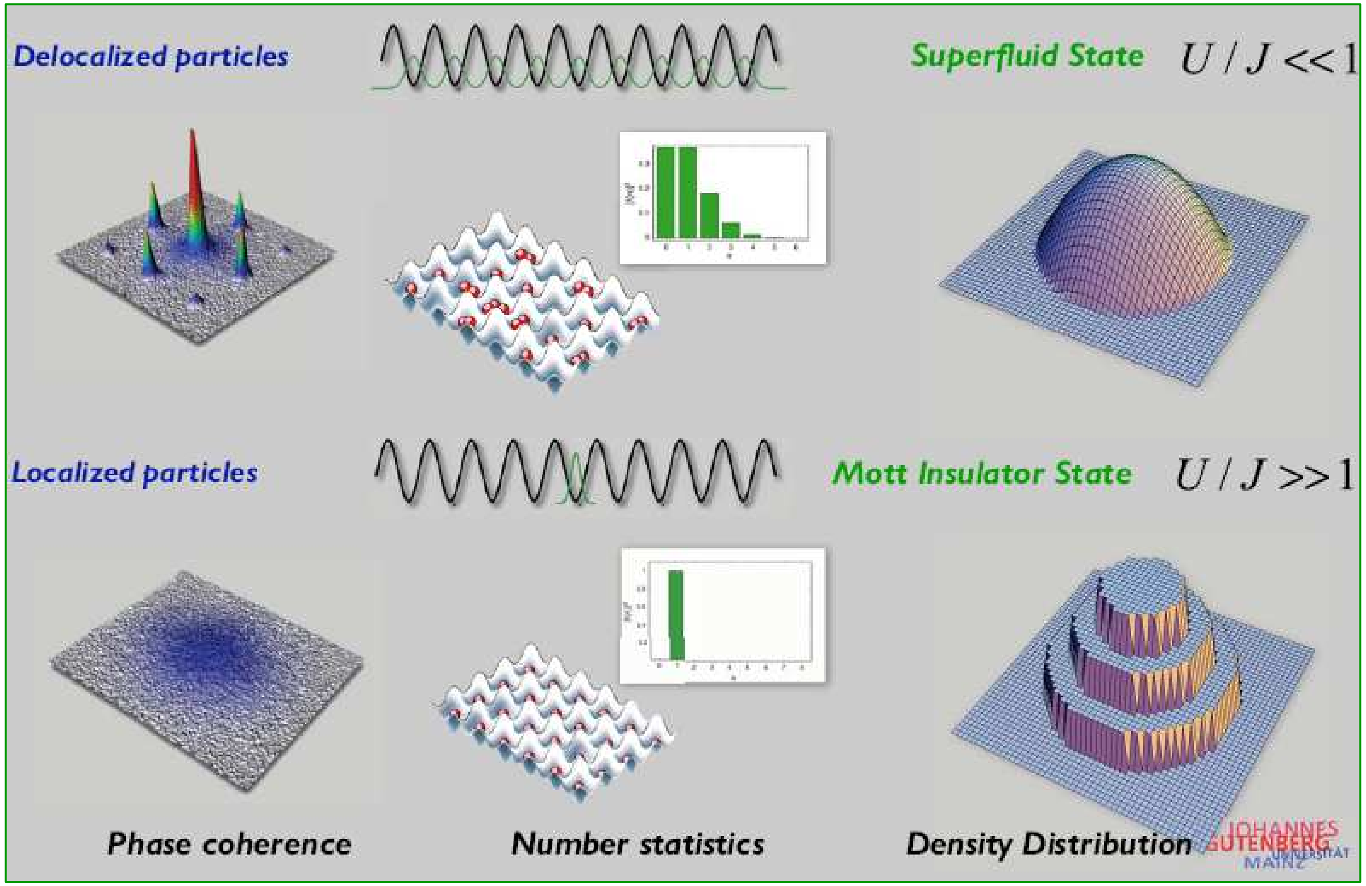
Periodic potential
(Firenze 2001)

Only superfluid coherently tunnels through the barrier



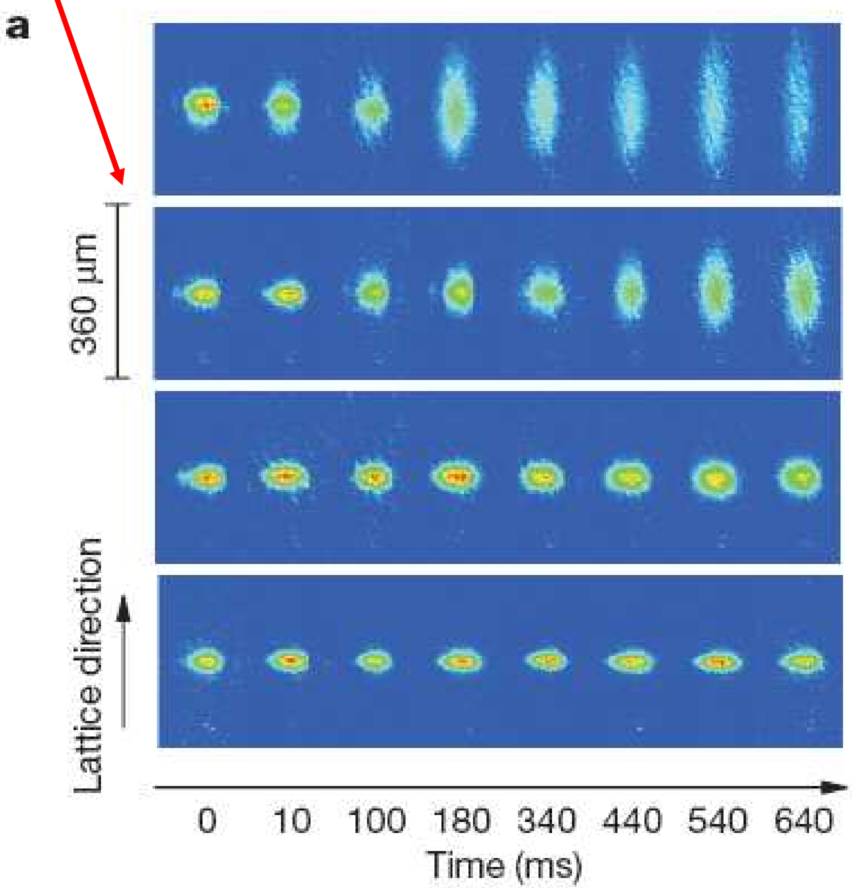
LOCALIZATION: enemy of BEC and superfluidity:
- deep optical lattice (superfluid-Mott transition)
- disorder (Anderson localization)

Superfluid to Mott Insulator transition
(Greiner et al. Nature 2002)



Anderson localization in almost non interacting Bose gas produced by **laser speckles** (Instute d'Optique, Palaiesau) and **quasi-periodic** optical potentials (Lens, Firenze)

(Nature 2008, June 12 !)



no quasi-periodicity

1D expansion

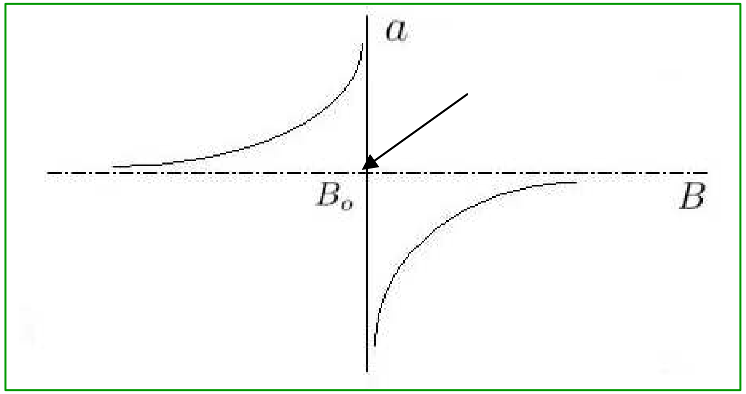
high quasi-periodicity

s-wave FERMI SUPERFLUIDITY and **BCS-BEC** CROSSOVER

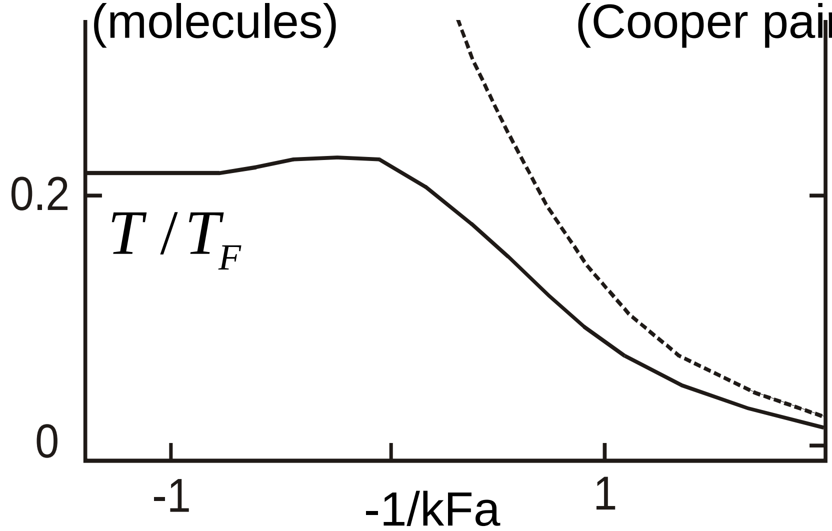
Interaction between fermions in two different hyperfine states described by **s-wave scattering length**.

- Availability of **Feshbach** resonances permits to reach favourable conditions for superfluidity

- **BCS-BEC crossover** (Eagles, Leggett, Nozieres-Schmitt Rink, Randeria,)

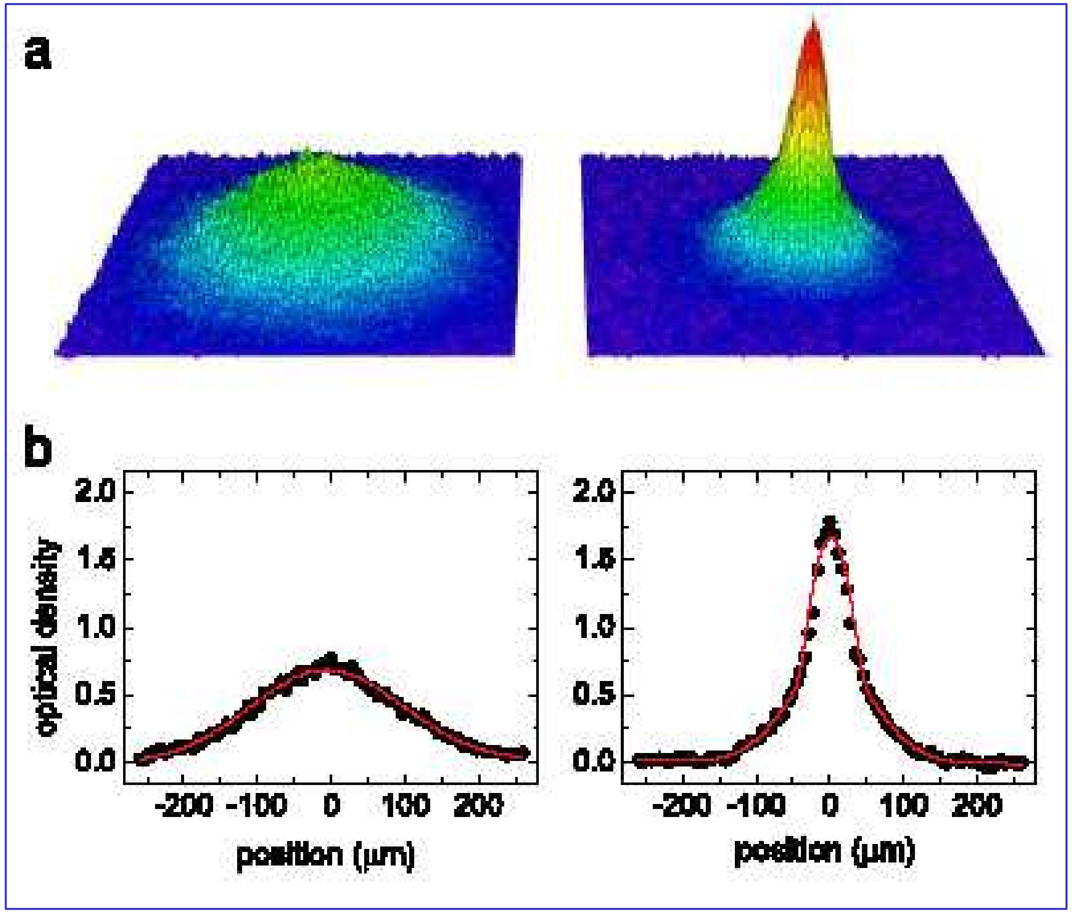


BEC (molecules) unitarity BCS (Cooper pairs)



Pairs of fermions forms dimers and dimers give rise to BEC

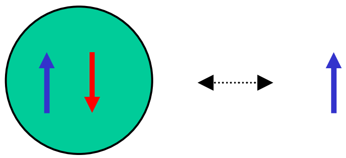
Bose-Einstein condensation
emerging from the Fermi sea



Jila 2003:
(see also Mit, Innsbruck, ENS, Rice)

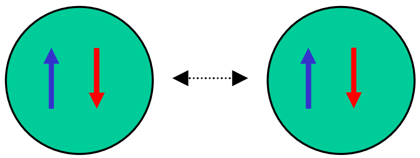
How do dimers interact with dimers ?
How do dimers interact with atoms ?

Theory:



dimer-atom $a_{bf} = 1.2a$

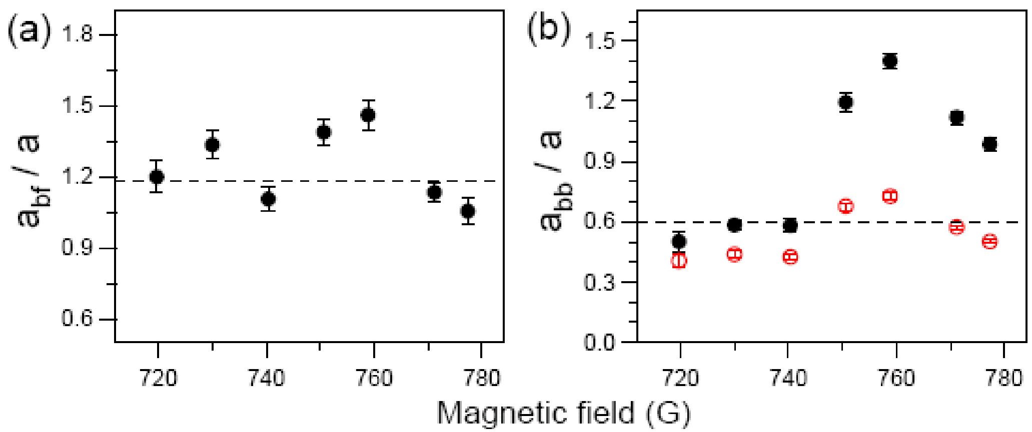
(Skorniakov and Terr-Martirosian1957)



dimer-dimer $a_{bf} = 0.6a$

(Petrov, Salomon and Shlyapnikov 2004)

Exp determination from density profile of polarized Fermi gases



Shin et al.(Mit 2008)

Unitary Fermi gas (1/a=0): main features

- diluteness
(interparticle distance >> range of inetraction)

- strong interactions
(scattering length >> interparticle distance)

- universality
(no dependence on interaction parameters)

- robust superfluidity (high critical velocity)

- **high Tc**
(of the order of
Fermi temperature

Conventional superconductors	10(-5)-10(-4)
Superfluid He3	10(-3)
High-temperature superconductors	10(-2)
Fermi gases with resonant interactions	0.2

The quest for superfluidity in Fermi gases

Some examples:

- collective **oscillations** and eq. of state
- Landau's **critical velocity**
- **spin polarization**

COLLECTIVE OSCILLATIONS IN SUPERFLUID PHASE (T=0)

- **Surface** modes: unaffected by equation of state

- **Compression** modes sensitive to **equation of state.**
  Hydrodynamic theory of superfluids predicts
  universal value $\omega_{rad} = \sqrt{10/3}\,\omega_r$ **at unitarity** (1/a=0)
  for radial compression mode in elongated trap
  (Stringari 2004)

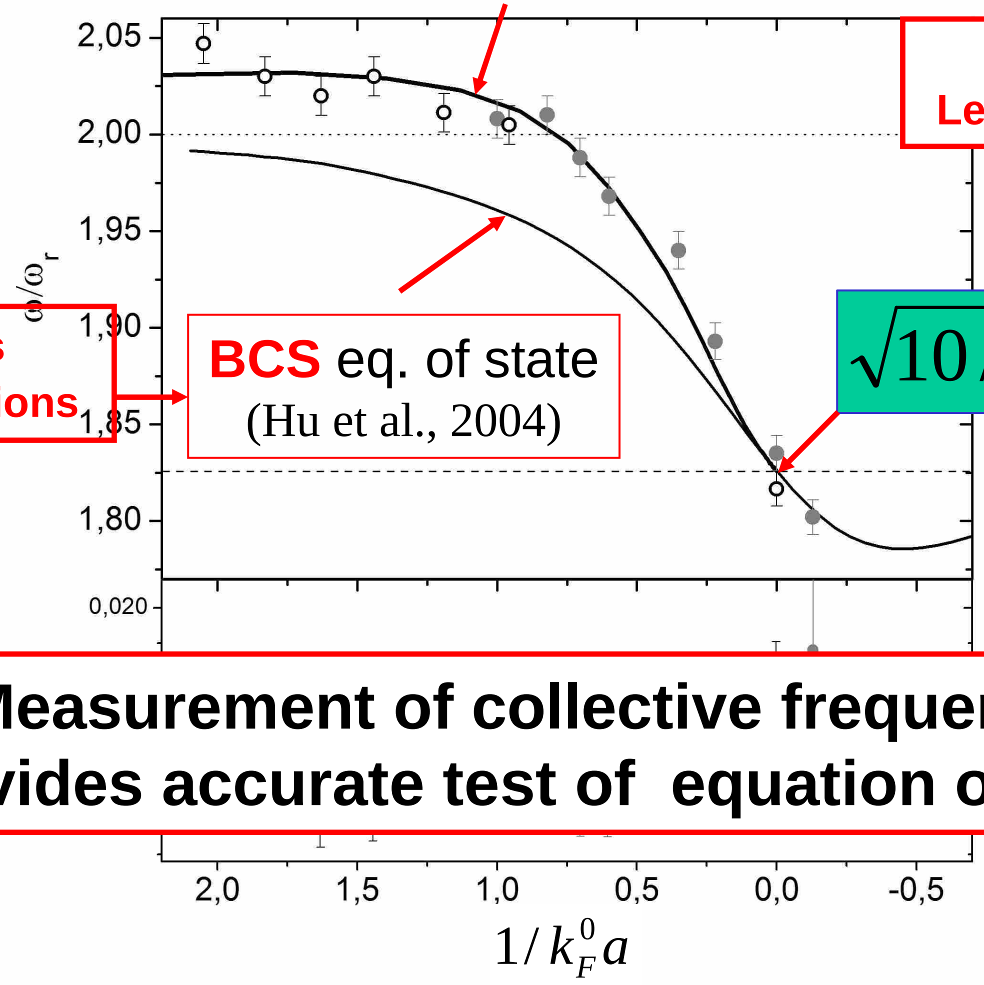
Breathing mode in elongated Fermi superfluids

Exp: Altmeyer et al. (Innsbruck)

Theory: Hydrodynamics with Monte Carlo eq. of state

MC equation of state (Astrakharchick et al., 2005)

Includes Lee-Huang-Yang correlations



does not includes quantum correlations

BCS eq. of state (Hu et al., 2004)

$$\sqrt{10/3}$$

universality !

Measurement of collective frequencies provides accurate test of equation of state !!

Landau's critical velocity

$$v_{cr} = \min_p \frac{\varepsilon(p)}{p}$$

Dispersion law of elementary excitations

- **Landau's criterion for superfluidity** (**metastability**):
  fluid moving with velocity **smaller** than critical velocity cannot decay (**persistent current**)

- Ideal Bose gas and ideal Fermi gas one has $v_{cr} = 0$

- In interacting Fermi gas one predicts two limiting cases:

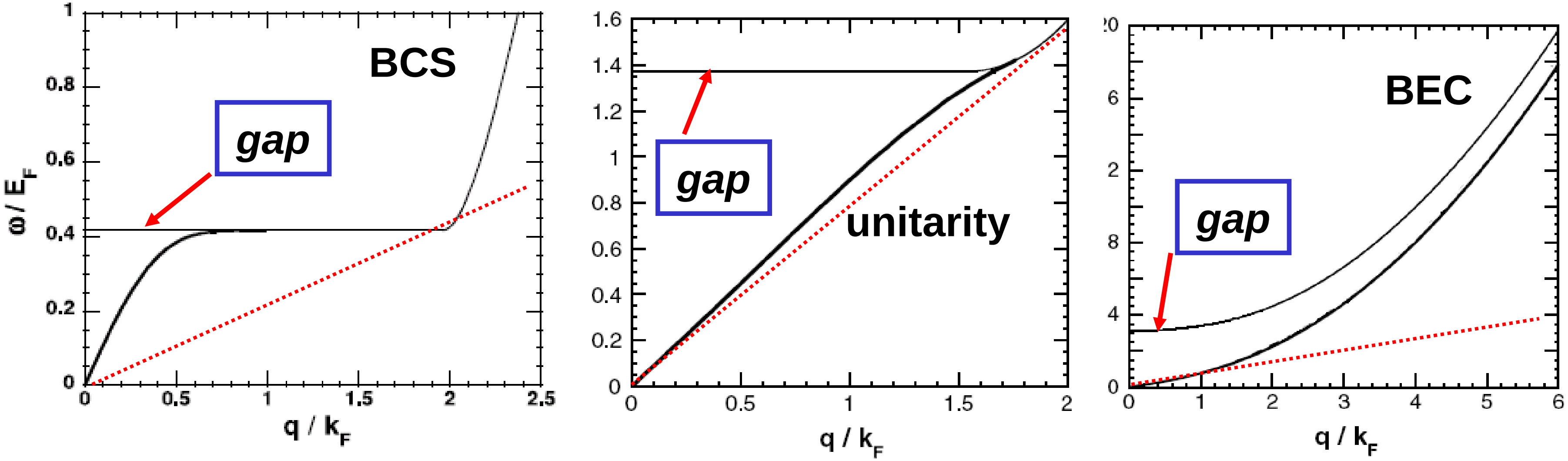
BEC (Bogoliubov dispersion)

$$v_{cr} = c \propto \sqrt{a}$$ **(sound velocity)**

BCS (role of the gap)

$$v_{cr} = \Delta / p_F \propto \exp(\pi / 2k_F a)$$

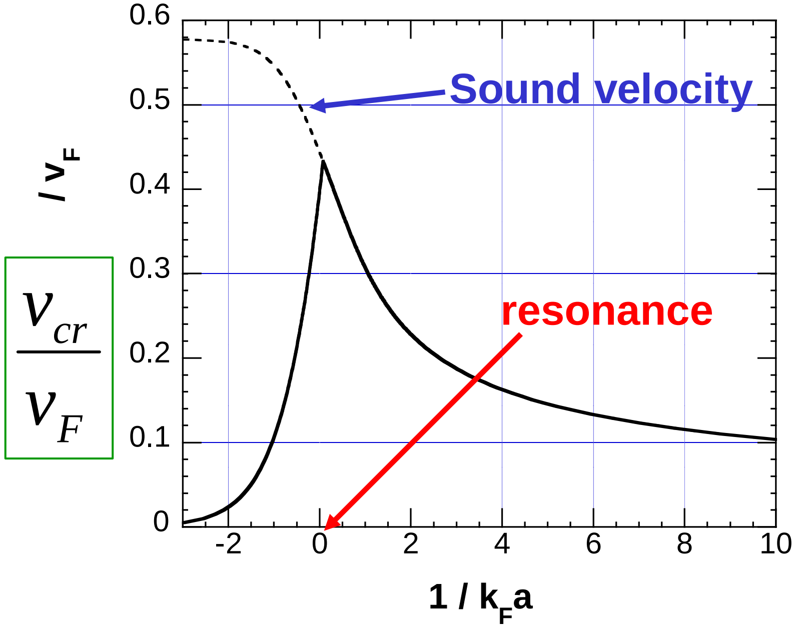
Dispersion law along BCS-BEC crossover



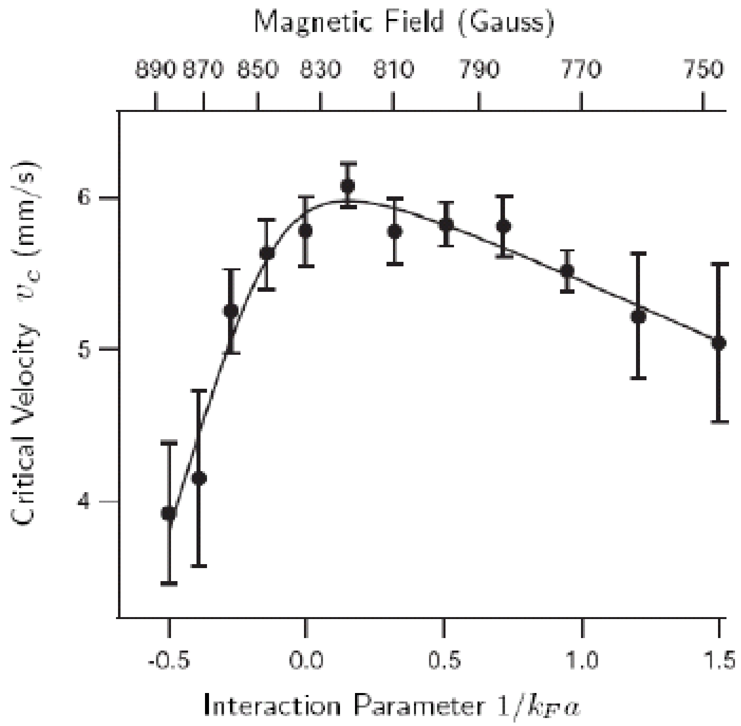
(R. Combescot, M. Kagan and S. Stringari 2006)

Landau's critical velocity

theory

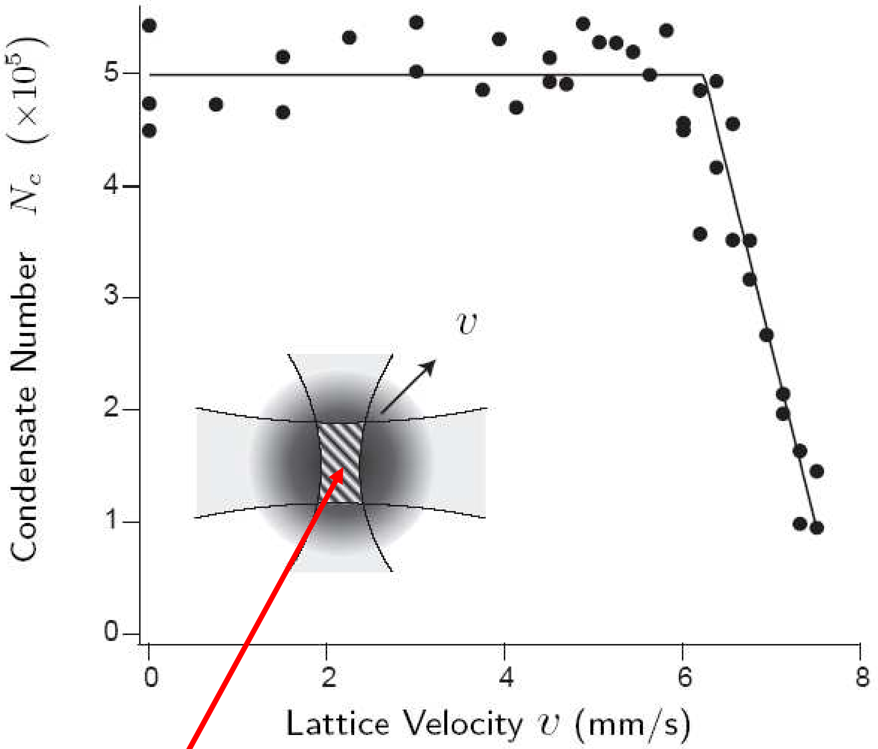


Experiment (Miller et al. 2007)



Landau's critical velocity
is
highest near unitarity !!

Measurement of Landau's critical velocity



Above critical velocity dissipative effect produced by moving optical lattice is observed

(Mit, Miller et al, 2007)

Spin polarized Fermi superfluids

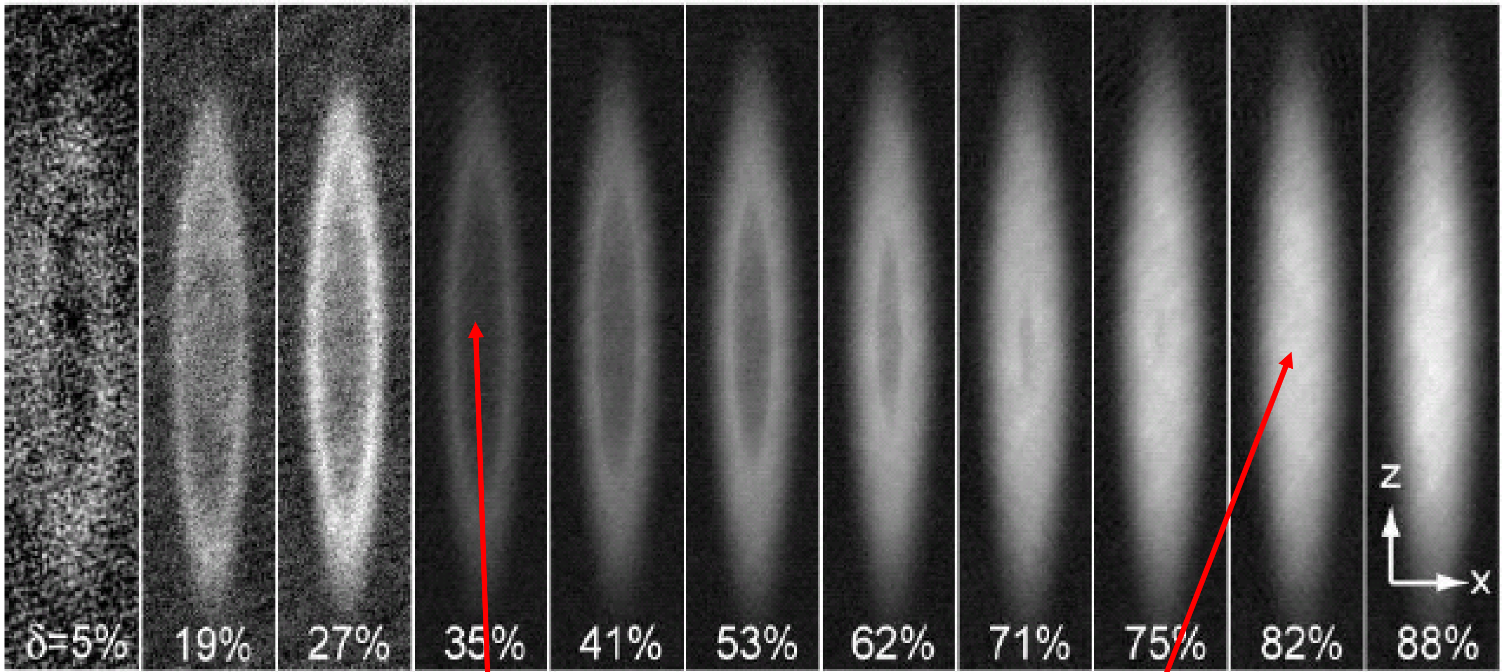
$$\boxed{N_\uparrow \neq N_\downarrow}$$

Differently from BEC's
phase separation is not easily observed
by imaging density profiles of Fermi gas
(bimodal distribution is absent
at unitarity as well as in BCS

Phase separation can be nevertheless
observed in **spin polarized** samples

Occurrence of phase separation in spin polarized Fermi gas observed experimentally at unitarity (see also Rice exp)

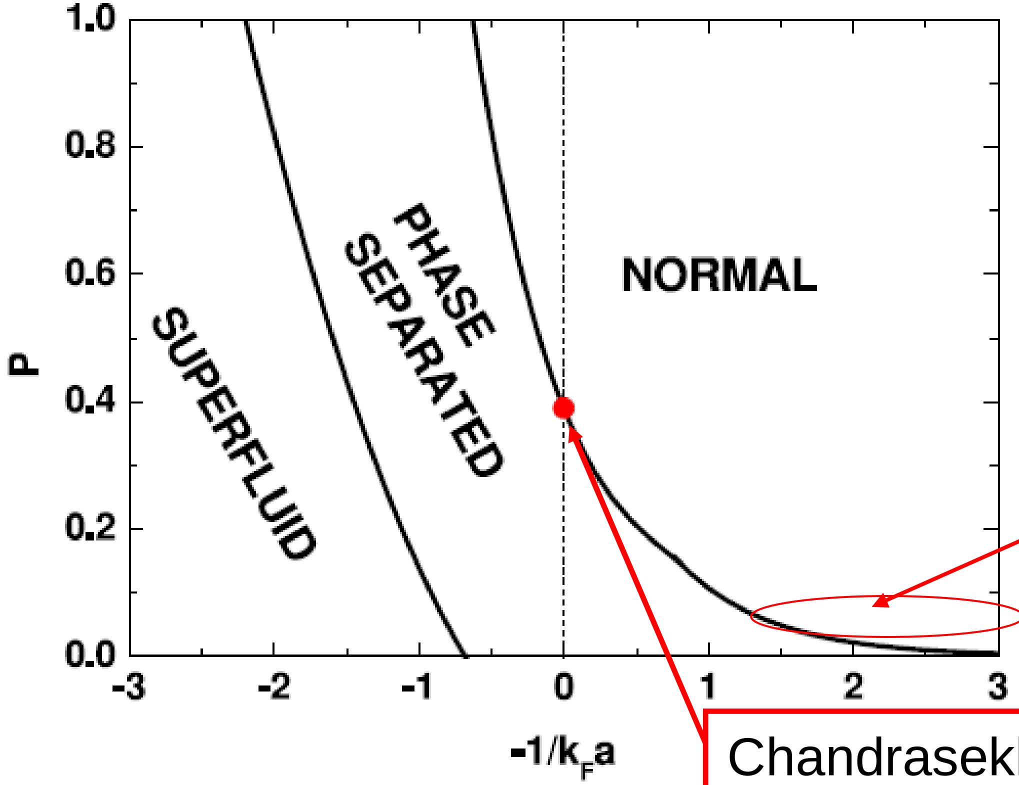
Density difference $n_\uparrow - n_\downarrow$
(phase contrast imaging, MIT 2006)



In superfluid phase $n_\uparrow = n_\downarrow$

In polarized normal phase $n_\uparrow > n_\downarrow$

Phase diagram of uniform matter at T=0



$$P = \frac{N_\uparrow - N_\downarrow}{N_\uparrow + N_\downarrow}$$

FFLO phase

Chandrasekher-Clogston limit at unitarity $P_C = 0.39$

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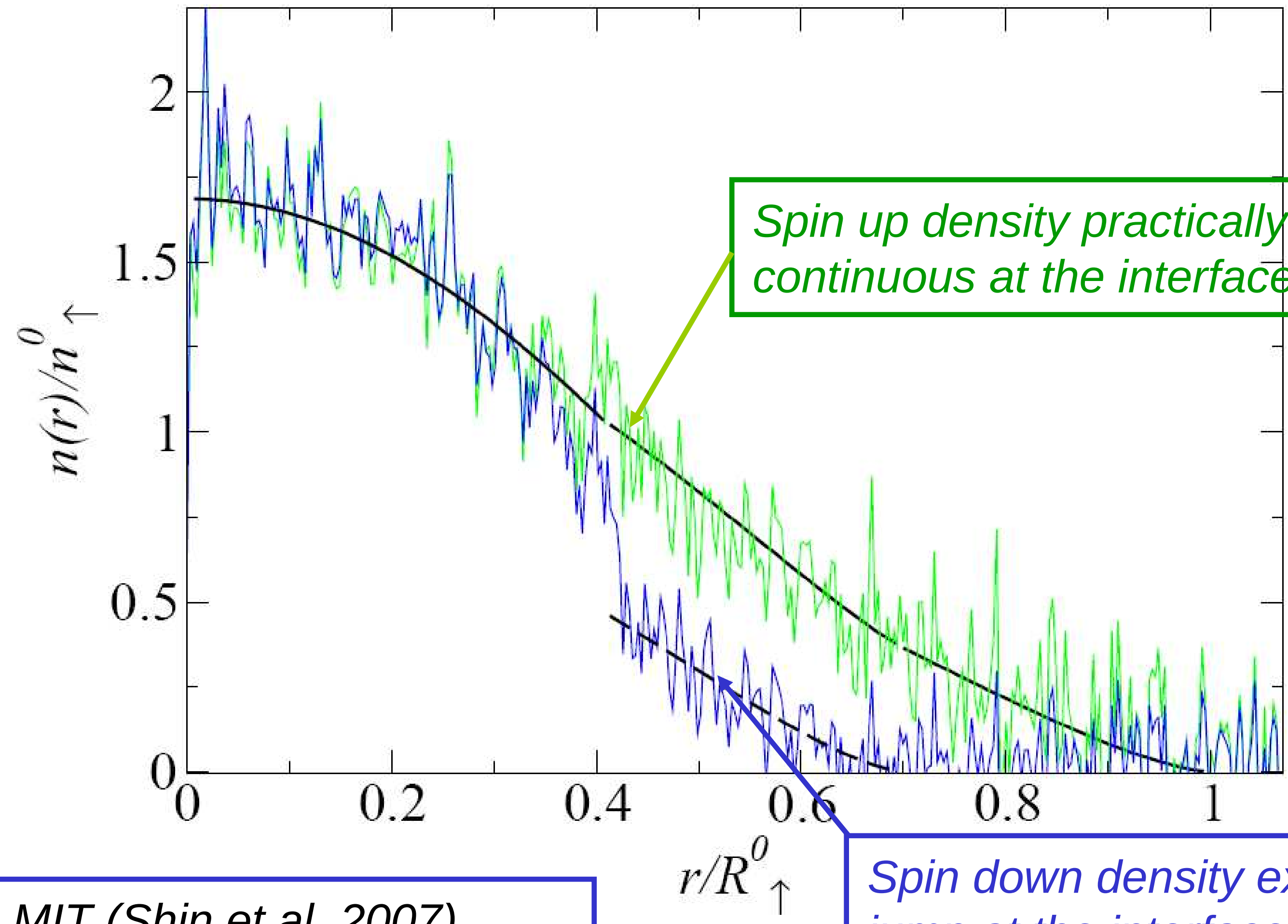
Interactions in normal phase play a crucial role in determining critical polarization.
Example: neglecting interactions in normal phase yields $P_C \approx 1$

Density jump at the interface (1/a=0)



Spin up density practically continuous at the interface

Spin down density exhibits jump at the interface

Exp: MIT (Shin et al. 2007)
Theory: Trento (Lobo et al.2006)

Based on MC equations of state for superfluid and polarized normal phase
Theory predicts critical polarization in excellent agreement with exps

MAIN CONCLUSION

- TRAPPED ATOMIC GASES: WELL SUITED TO EXPLORE THE EFFECTS OF **SUPERFLUIDITY**

- IMPORTANT RECENT ADVANCES IN **FERMI** SUPERFLUIDITY

Main collaborators in Trento

Franco Dalfovo

Stefano Giorgini

Lev Pitaevskii

Theory of Bose-Einstein condensation in trapped gases
F. Dalfovo et al.
Rev.Mod.Phys, 71, 463 (1999)

Theory of ultracold atomic Fermi gases
S.Giorgini et al.
Rev.Mod.Phys, in press (2008)
cond-mat/0706.3360