The problem of initial conditions in cosmology

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Abstract. The creation of a quantum Universe is described by a density matrix which yields an ensemble of universes with the cosmological constant limited to a bounded range $\Lambda_{\rm min} \leq \Lambda \leq \Lambda_{\rm max}$. The domain $\Lambda < \Lambda_{\rm min}$ is ruled out by a cosmological bootstrap requirement (the self-consistent back reaction of hot matter). The upper cutoff results from the quantum effects of vacuum energy and the conformal anomaly mediated by a special ghost-avoidance renormalization. The cutoff $\Lambda_{\rm max}$ establishes a new quantum scale – the accumulation point of an infinite sequence of garland-type instantons. The Euclidean path integral formalism used for the construction of the fundamental density matrix for a mixed state of the Universe is justified by proving its correspondence to the microcanonical ensemble in quantum cosmology. The cosmological evolution starting with these initial conditions also have some new features: the stage of cosmic acceleration can be followed by a big boost singularity – a rapid growth up to infinity of the scale factor acceleration parameter. From the developed approach it follows that the notion of the density matrix plays a more fundamental role than that was traditionally prescribed to it.

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INTRODUCTION

Many years ago when speaking to his students L.D. Landau used to say, according to I.M. Khalatnikov's reminiscences, that the future physical theory should incorporate not only equations of motion but also initial conditions for them [1]. It is difficult to imagine how preferred initial conditions could be prescribed to an equation of motion in the framework of classical physics. The same could be said about the non-relativistic quantum mechanics or about quantum field theory in the Minkowski spacetime. The only theory where the idea of natural initial conditions was realized is quantum cosmology, the branch of quantum theory treating the Universe as a unique quantum object described by its quantum state. The basic equation of quantum cosmology - the Wheeler-DeWitt equation was formulated in the sixties [2]. However, certain prescriptions for the wave function of the universe, satisfying this equation were suggested only in the early eighties in papers [3, 4, 5, 6, 7]. In papers mentioned above the two related approaches were used: the analogy with the tunneling processes in quantum mechanics [4] and the apparatus of the Euclidean field theory [3]. In both cases the phenomenon of the so called "quantum birth of the universe from nothing" was employed. Both approaches used the instanton solutions of the Euclidean Einstein equations, however their physical predictions were different because the Euclidean action entered with different signs the exponential of the wave function of the universe, calculated in the semiclassical approximation. Namely, the Hartle-Hawking or "no-boundary" wave function of the universe [3] which behaves in the lowest order of the WKB approximation as $\psi_{NB} \sim \exp(-\Gamma)$, where Γ is the Euclidean action on the underlying instanton, predicts the quantum birth of a universe with a very large (infinite) initial radius, which looks quite counter-intuitive. The tunneling or Vilenkin wave function of the universe [4] behaves as $\psi_T \sim \exp(+\Gamma)$ and predicts the birth of a universe with an infinitely small radius. Besides, both of these functions are non-normalizable and it is hardly possible to prescribe to them the traditional quantum-mechanical probabilistic interpretation.

Considering solutions of the Wheeler-DeWitte equation in the one-loop approximation, one can achieve (imposing some constraints on the particle content of the theory) the normalizability of the wave function of the universe in both the tunneling and no-boundary prescriptions [8]. Moreover, for the tunneling wave function of the universe one can predict a peak of the probability of the quantum birth of the universe with reasonable initial parameters [9].

However, the traditional approach to quantum cosmology limited to the consideration of only pure quantum states and associated with them instantons looks too restrictive. It appears that relaxing the requirement of the "purity" of possible quantum states of the universe and taking into account the possibility of existence of the gravitational instantons with more complicated geometries than those considered in the above works on quantum cosmology, one can obtain some, at first glance, unexpected results. In our papers [10, 11] we have generalized the traditional scheme of quantum cosmology. The main goals of our approach were the following:

1. Description of the birth of the universe from nothing in a mixed state and the use of the density matrix instead of the wave function of the universe.
2. Predicition of initial conditions for the cosmological evolution, which we call "cosmological landscape" in analogy with a very popular string landscape [12].
3. Elimination of "infrared catastrophe" (an infinitely large probablity of the birth of the universe of an infinitely large size) in the Hartle-Hawking prescription.
4. Establishing connections with string theory.

The tools which we have used were

1. Quantum theory of tunneling : Euclidean quantum gravity.
2. Quantum field theory: renormalization in curved spacetime; effective action formalism.
3. The account of non-local effects due to back reaction of the conformal anomaly of quantum fields and their radiation.

Our main results can be formulated as follows:

1. The closed system of equations describing the quantum birth of the universe is derived: the generalized Friedmann equation with the quantum radiation source and the "bootstrap" equation for the latter.
2. The solution of these equations gives the families of acceptable parameters, characterizing initial conditions for cosmological evolution - "cosmological landscape".
3. The problem of "infrared catastrophe" in the Hartle-Hawking prescription is resolved.
4. The scenarios of the future evolution of the Universe are studied.

DENSITY MATRIX, RADIATION AND INSTANTONS

The idea to consider the density matrix of the universe instead of the wave function of the universe was put forward in [13] where it was also noticed that such a density matrix is based on an instanton with two disjoint boundaries (see Fig.1). The density matrix describes a mixed state which might account for the presence of radiation [14]. For the pure quantum state [3] the instanton bridge between Σ and Σ' breaks down (see Fig.2). However, the radiation stress tensor prevents these half instantons from

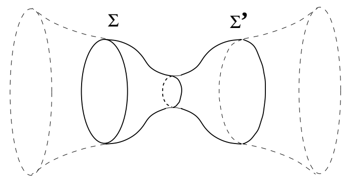

FIGURE 1. Picture of instanton representing the density matrix. Dashed lines depict the Lorentzian Universe nucleating from the instanton at the minimal surfaces Σ and Σ'.

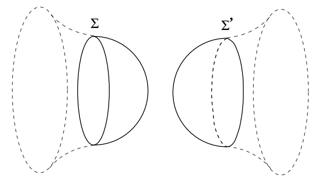

FIGURE 2. Density matrix of the pure Hartle-Hawking state represented by the union of two vacuum instantons.

closure. Indeed, the Euclidean Friedmann equation for a closed universe with the metric

$$ds^2 = N^2(\tau)\,d\tau^2 + a^2(\tau)\,d^2\Omega^{(3)} \quad (1)$$

in the presence of a cosmological constant $\Lambda = 3H^2$ and radiation characterized by a constant C

$$\frac{\dot{a}^2}{a^2} = \frac{1}{a^2} - H^2 - \frac{C}{a^4} \quad (2)$$

has the solution $a = \frac{1}{\sqrt{2}H}\sqrt{1-(1-4CH^2)^{1/2}\cos 2H\tau}$ with two turning points, neither of them vanishing, $a_\pm = \frac{1}{\sqrt{2}H}\sqrt{1\pm(1-4CH^2)^{1/2}},\ 4H^2C \leq 1.$

The relevant density matrix is the path integral

$$\rho[\varphi,\varphi'] = e^{\Gamma} \int\limits_{g,\phi\,|_{\Sigma,\Sigma'} = (\varphi,\varphi')} D[g,\phi]\,\exp\big(-S_{\rm E}[g,\phi]\big). \quad (3)$$

with the partition function $e^{-\Gamma}$ which follows from integrating out the field φ in the coincidence $\varphi' = \varphi$ corresponding to the identification of Σ' and Σ, the underlying instanton acquiring the toroidal topology (see Fig.3).

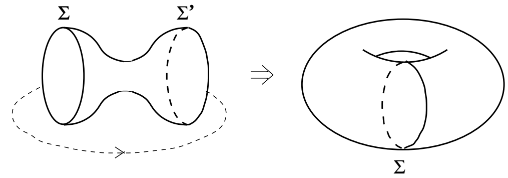

FIGURE 3. Calculation of the partition function represented by compactification of the instanton to a torus with periodically identified Euclidean time.

CONFORMAL ANOMALY AND GHOSTS

The metric of the instanton introduced above

$$ds^2 = a^2(\eta)(d\eta^2 + d^2\Omega^{(3)}), \tag{4}$$

is conformally equivalent to the metric of the Einstein static universe:

$$d\bar{s}^2 = d\eta^2 + d^2\Omega^{(3)}, \tag{5}$$

where η is the conformal time parameter. We shall consider conformally invariant fields. As is well known, the quantum effective action for such fields has a conformal anomaly first studied in cosmology in [15, 16]. It has the form

$$g_{\mu\nu}\frac{\delta\Gamma_{1-\mathrm{loop}}}{\delta g_{\mu\nu}} = \frac{1}{4(4\pi)^2} g^{1/2}\left(\alpha\Box R + \beta E + \gamma C^2_{\mu\nu\alpha\beta}\right), \tag{6}$$

where $E = R^2_{\mu\nu\alpha\gamma} - 4R^2_{\mu\nu} + R^2$ and $\Box$ is the four-dimensional Laplacian. This anomaly, when integrated functionally along the orbit of the conformal group, gives the relation between the actions on conformally related backgrounds [17].

$$\Gamma_{1-\mathrm{loop}}[g] = \Gamma_{1-\mathrm{loop}}[\bar{g}] + \delta\Gamma[g,\bar{g}], \tag{7}$$

$$g_{\mu\nu}(x) = e^{\sigma(x)}\bar{g}_{\mu\nu}(x), \tag{8}$$

where

$$\begin{aligned}
\delta\Gamma[g,\bar{g}] &= \frac{1}{2(4\pi)^2}\int d^4x\bar{g}^{1/2}\left\{\frac{1}{2}\left[\gamma\bar{C}^2_{\mu\nu\alpha\beta}\right.\right.\\
&\left.+\beta\left(\bar{E} - \frac{2}{3}\bar{\Box}\bar{R}\right)\right]\sigma\\
&\left.+\frac{\beta}{2}\left[(\bar{\Box}\sigma)^2 + \frac{2}{3}\bar{R}(\bar{\nabla}_\mu\sigma)^2\right]\right\}\\
&-\frac{1}{2(4\pi)^2}\left(\frac{\alpha}{12} + \frac{\beta}{18}\right)\\
&\times\int d^4x\left(g^{1/2}R^2(g) - \bar{g}^{1/2}R^2(\bar{g})\right).
\end{aligned} \tag{9}$$

One can show that the higher-derivative in σ terms are all proportional to the coefficient α. The α-term can be arbitrarily changed by adding a local counterterm $\sim g^{1/2}R^2$. We fix this local renormalization ambiguity by an additional criterion of the absence of ghosts. The conformal contribution to the renormalized action on the minisuperspace background equals

$$\begin{aligned}
\delta\Gamma[g,\bar{g}] &\equiv \Gamma_R[g] - \Gamma_R[\bar{g}]\\
&= m_P^2 B\int d\tau\left(\frac{\dot{a}^2}{a} - \frac{1}{6}\frac{\dot{a}^4}{a}\right),
\end{aligned} \tag{10}$$

$$m_P^2 B = \frac{3}{4}\beta, \tag{11}$$

with the constant $m_P^2 B$ which for scalars, two-component spinors and vectors equals respectively $1/240$, $11/480$ and $31/120$.

EFFECTIVE ACTION ON A STATIC EINSTEIN INSTANTON

For a conformal scalar field

$$S[\bar{g},\phi] = \frac{1}{2}\sum_\omega\int_0^\eta d\eta'\left(\left(\frac{d\phi_\omega}{d\eta'}\right)^2 + \omega^2\phi_\omega^2\right), \tag{12}$$

where $\omega = n$, $n = 0,1,2,...$, labels a set of eigenmodes and eigenvalues of the Laplacian on a unit 3-sphere. Thus

$$\begin{aligned}
&e^{-\Gamma_{1-\mathrm{loop}}[\bar{g}]}\\
&= \int\prod_\omega d\varphi_\omega \int_{\phi_\omega(\eta)=\phi_\omega(0)=\varphi_\omega} D[\phi]\exp\left(-S[\bar{g},\phi]\right)\\
&= \mathrm{const}\prod_\omega\left(\sinh\frac{\omega\eta}{2}\right)^{-1},
\end{aligned} \tag{13}$$

and the effective action equals the sum of contributions of the vacuum energy E_0 and free energy $F(\eta)$ with the inverse temperature played by η — the circumference of the toroidal instanton in units of a conformal time,

$$\begin{aligned}
\Gamma_{1-\mathrm{loop}}[\bar{g}] &= \sum_\omega\left[\eta\frac{\omega}{2} + \ln\left(1 - e^{-\omega\eta}\right)\right]\\
&= m_P^2 E_0\eta + F(\eta),
\end{aligned} \tag{14}$$

$$m_P^2 E_0 = \sum_\omega\frac{\omega}{2} = \sum_{n=1}^\infty\frac{n^3}{2}, \tag{15}$$

$$F(\eta) = \sum_\omega\ln\left(1 - e^{-\omega\eta}\right) \tag{16}$$

$$= \sum_{n=1}^\infty n^2\ln\left(1 - e^{-n\eta}\right). \tag{17}$$

Similar expressions hold for other conformally invariant fields of higher spins. In particular, the vacuum energy (an analog of the Casimir energy) on Einstein static spacetime is

$$m_P^2 E_0 = \frac{1}{960}\times\left\{\begin{array}{c}4\\17\\88\end{array}\right. \tag{18}$$

respectively for scalar, spinor and vector fields.

We should take into account the effect of the finite ghost-avoidance renormalization denoted below by a subscript R, which results in the replacement of E_0 above by a new parameter C_0:

$$\Gamma_R[\bar{g}] = m_P^2 C_0\eta_0 + F(\eta), \tag{19}$$

$$m_P^2 C_0 = m_P^2 E_0 + \frac{3}{16}\alpha. \tag{20}$$

A direct observation indicates the following universality relation for all conformal fields of low spins

$$m_P^2 C_0 = \frac{1}{2}m_P^2 B. \tag{21}$$

EFFECTIVE FRIEDMANN AND BOOTSTRAP EQUATIONS

Now we can write down the effective Friedmann equation governing the Euclidean evolution of the universe. First of all, the full conformal time on the instanton is

$$\eta = 2\int_{\tau_-}^{\tau_+} \frac{d\tau\, N(\tau)}{a(\tau)}, \tag{22}$$

where $\tau_\pm$ label the turning points for $a(\tau)$ – its minimal and maximal values.

The effective action is ($m_P^2 \equiv 3/4\pi G$)

$$\begin{aligned}
&\Gamma[a(\tau), N(\tau)] \\
&= 2m_P^2 \int_{\tau_-}^{\tau_+} d\tau \left(-\frac{a\dot{a}^2}{N} - Na + NH^2a^3\right) \\
&+2Bm_P^2 \int_{\tau_-}^{\tau_+} d\tau \left(\frac{\dot{a}^2}{Na} - \frac{1}{6}\frac{\dot{a}^4}{N^3 a}\right) \\
&+F\left(2\int_{\tau_-}^{\tau_+} \frac{d\tau\, N}{a}\right) + Bm_P^2 \int_{\tau_-}^{\tau_+} \frac{d\tau\, N}{a},
\end{aligned} \tag{23}$$

and the effective Friedmann equation reads

$$\begin{aligned}
\frac{\delta\Gamma}{\delta N} &= 2m_P^2\left(\frac{a\dot{a}^2}{N^2} - a + H^2a^3\right) \\
&+2Bm_P^2\left(-\frac{\dot{a}^2}{N^2 a} + \frac{1}{2}\frac{\dot{a}^4}{N^4 a}\right) \\
&+\frac{2}{a}\left(\frac{dF(\eta)}{d\eta} + \frac{B}{2}m_P^2\right) = 0.
\end{aligned} \tag{24}$$

In the gauge $N = 1$ this equation takes form

$$\frac{\dot{a}^2}{a^2} + B\left(\frac{1}{2}\frac{\dot{a}^4}{a^4} - \frac{\dot{a}^2}{a^4}\right) = \frac{1}{a^2} - H^2 - \frac{C}{a^4}, \tag{25}$$

where the amount of radiation constant C is given by the bootstrap equation

$$m_P^2 C = m_P^2\frac{B}{2} + \frac{dF(\eta)}{d\eta} \equiv \frac{B}{2}m_P^2 + \sum_\omega \frac{\omega}{e^{\omega\eta}-1}. \tag{26}$$

The Friedmann equation can be rewritten as

$$\begin{aligned}
\dot{a}^2 &= \sqrt{\frac{(a^2-B)^2}{B^2} + \frac{2H^2}{B}(a_+^2 - a^2)(a^2 - a_-^2)} \\
&-\frac{(a^2-B)}{B}
\end{aligned} \tag{27}$$

and has the same two turning points $a_\pm$ as in the classical case provided

$$a_-^2 \geq B. \tag{28}$$

This requirement is equivalent to

$$C \geq B - B^2H^2,\ BH^2 \leq \frac{1}{2}. \tag{29}$$

Together with

$$CH^2 \leq \frac{1}{4},$$

the admissible domain for instantons reduces to the curvilinear wedge below the hyperbola and above the straight line to the left of the critical point (see Figure 4)

$$C = \frac{B}{2},\ H^2 = \frac{1}{2B}.$$

For a scalar field the numerical analysis of the Friedmann and bootstrap equations shows that the one-parameter family of instantons interpolates between the point on the lower line boundary with the parameters

$$H^2 \approx 2.997\,m_P^2,\ C \approx 0.004\,m_P^{-2},\ \Gamma_0 \approx -0.1559, \tag{30}$$

and the point on the upper hyperbolic boundary

$$H^2 \approx 12.968\,m_P^2,\ C \approx 0.0193\,m_P^{-2},\ \Gamma_0 \approx -0.0883. \tag{31}$$

The last instanton describes the creation of a static Einstein Universe of the constant size

$$a = a_+ = a_- = 1/(\sqrt{2}H)$$

with the hot gas of a conformally-invariant scalar field particles in the equilibrium state with the temperature

$$T = \frac{1}{a\eta} = \frac{H}{\pi\sqrt{1-2BH^2}}. \tag{32}$$

INFRARED CATASTROPHE IS ELIMINATED.

The suggested approach allows to resolve the problem of the so-called infrared catastrophe for the no-boundary state of the Universe based on the Hartle-Hawking instanton. This problem is related to the fact that the Euclidean action on this instanton is negative and inverse proportional to the value of the effective cosmological constant. This means that the probability of the universe creation with an infinitely big size is infinitely high. We shall show now that the conformal anomaly effect allows one to avoid this counter-intuitive conclusion.

Indeed, outside of the admissible domain for the instantons with two turning points, obtained above, one can also construct instantons with one turning point which smoothly close at $a_- = 0$ with $\dot{a}(\tau_-) = 1$. Such instantons correspond to the Hartle-Hawking pure quantum state. However, in this case the on-shell effective action, which reads for the set of solutions obtained above as

$$\begin{aligned}
\Gamma_0 &= F(\eta) - \eta\frac{dF(\eta)}{d\eta} \\
&+4m_P^2\int_{a_-}^{a_+}\frac{da\dot{a}}{a}\left(B - a^2 - \frac{B\dot{a}^2}{3}\right),
\end{aligned} \tag{33}$$

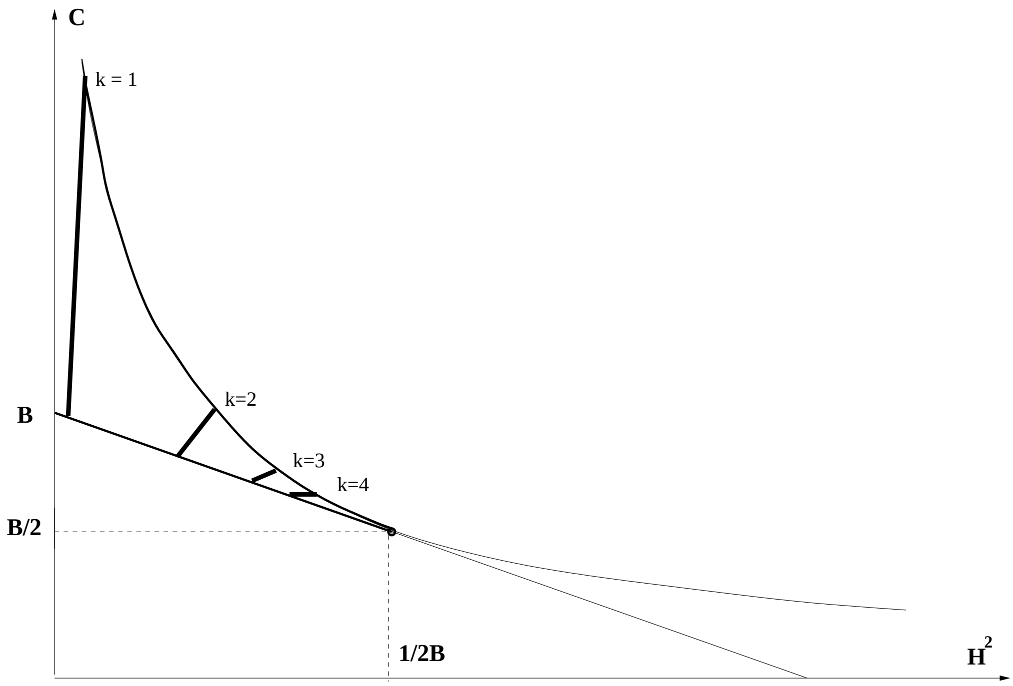

FIGURE 4. The instanton domain in the (H^2, C)-plane is located between bold segments of the upper hyperbolic boundary and lower straight line boundary. The first one-parameter family of instantons is labeled by $k = 1$. Families of garlands are qualitatively shown for $k = 2, 3, 4$. $(1/2B, B/2)$ is the critical point of accumulation of the infinite sequence of garland families.

diverges to plus infinity. Indeed, for $a_- = 0$ and $\dot{a}_- = 1$

$$\eta = \int_0^{a_+} \frac{da}{\dot{a}a} = \infty,\ F(\infty) = F'(\infty) = 0, \quad (34)$$

and hence the effective Euclidean action diverges at the lower limit to $+\infty$. Thus,

$$\Gamma_0 = +\infty,\ \ \exp(-\Gamma_0) = 0,$$

and this fact completely rules out all pure-state instantons, and only mixed quantum states of the universe, described by the cosmological density matrix appear to be admissible.

INSTANTON GARLANDS

One should consider also the multiple instanton configurations, which could be called "Instanton garlands" (see Figure 5). The total conformal time for such an instanton garland is

$$\eta_0^{(k)} = 2k \int_{\tau_-}^{\tau_+} \frac{d\tau}{a} = 2k \int_{a_-}^{a_+} \frac{da}{a\dot{a}}, \quad (35)$$

where k is the number of simple instanton folds in a garland.

Numerical analysis for $k = 2$ shows the existence of the one-parameter family of instantons similar to the case of $k = 1$. It interpolates between the point on the lower boundary of (C, H^2)-plane

$$H_{(2)}^2 \approx 45.89\, m_P^2,\ C_{(2)} \approx 0.0034\, m_P^{-2},\ \Gamma_0^{(2)} \approx -0.0113, \quad (36)$$

and the point on the upper (hyperbolic) boundary

$$H_{(2)}^2 \approx 61.12\, m_P^2,\ C_{(2)} \approx 0.0041\, m_P^{-2},\ \Gamma_0^{(2)} \approx -0.0145. \quad (37)$$

Such families exist for all $k, 1 \leq k \leq \infty$, and their infinite sequence is saturated at the critical point,

$$\eta_0^{(k)} \simeq \ln k^2 \quad (38)$$

$$H_{(k)}^2 \simeq \frac{1}{2B}\left(1 - \frac{\ln^2 k^2}{2k^2\pi^2}\right), \quad (39)$$

$$C_{(k)} \simeq \frac{B}{2}\left(1 + \frac{\ln^2 k^2}{2k^2\pi^2}\right), \quad (40)$$

$$\Gamma_0^{(k)} \simeq -m_P^2 B\, \frac{\ln^3 k^2}{4k^2\pi^2}. \quad (41)$$

The length of instanton families decreases as $1/k^4$. Infinite garlands ($k \to \infty$) do not dominate the instanton distribution because their action grows with k rather than decreases to $-\infty$.

A growing spin of a conformal particle decreases the instanton size and makes its probability weight higher.

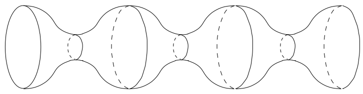

FIGURE 5. Segment of the garland consisting of three folds of a simple instanton glued at surfaces of a maximal scale factor.

For N fields

$$C \to NC, \tag{42}$$

$$B \to NB, \tag{43}$$

$$\eta_0 \to \eta_0, \tag{44}$$

$$F(\eta_0) \to NF(\eta_0), \tag{45}$$

$$H^2 \to \frac{H^2}{N}. \tag{46}$$

The initial size of the universe grows with the growing spin and number of fields.

WHERE EUCLIDEAN QUANTUM GRAVITY AND COSMOLOGY COMES FROM ?

In the preceding sections we have described a new approach to the problem of initial conditions in cosmology based on the use of the combination of two ideas: the density matrix formalism and Euclidean quantum gravity. A natural question arises: where Euclidean quantum gravity comes from? The answer can be formulated briefly as follows: from the Lorentzian quantum gravity (LQG) [18]. Namely, the density matrix of the Universe for the microcanonical ensemble in Lorentzian quantum cosmology of spatially closed universes describes an equipartition in the physical phase space of the theory, but in terms of the observable spacetime geometry this ensemble is peaked about a set of cosmological instantons (solutions of the Euclidean quantum cosmology) limited to a bounded range of the cosmological constant. These instantons obtained above as fundamental in Euclidean quantum gravity framework, in fact, turn out to be the saddle points of the LQG path integral, belonging to the imaginary axis in the complex plane of the Lorentzian signature lapse function [18].

COSMOLOGICAL EVOLUTION AND BIG BOOST SINGULARITY

Now let us consider the cosmological evolution of the unverse starting from the initial conditions described above. Making the transition from the Euclidean time to the Lorentzian one, $\tau = it$, we can write the modified Lorentzian Friedmann equation as [19]

$$\frac{\dot{a}^2}{a^2} + \frac{1}{a^2} = \frac{1}{B}\left\{1 - \sqrt{1 - \frac{16\pi G}{3}B\varepsilon}\right\}, \tag{47}$$

$$\varepsilon = \frac{3}{8\pi G}\left(H^2 + \frac{\mathscr{C}}{a^4}\right), \tag{48}$$

$$\mathscr{C} \equiv C - \frac{B}{2}, \tag{49}$$

where ε is a total gravitating matter density in the model (including at later stages also the contribution of particles created during inflationary expansion and thermalized at the inflation exit). A remarkable feature of this equation is that the Casimir energy is totally screened here and only the thermal radiation characterized by $\mathscr{C}$ weighs.

If one wants to compare the evolution described by Eq. (49) with the real evoltuion of the universe, first of all it is necessary to have a realistic value for an effective cosmological constant $\Lambda = 3H^2$. The only way to achieve this goal is to increase the number of conformal fields and the corresponding parameter B, (11), of the conformal anomaly (6). The mechanisms for growing number of the conformal fields exist in some string inspired cosmological models with extra dimensions [18]. If some of these mechanisms work we can encounter an interesting phenomenon: if the B grows with a faster than the rate of decrease of the energy density ε one encounters a new type of the cosmological singularity - Big Boost. This singularity is characterized by finite values of the cosmological radius a_{BB} and of its time derivative $\dot{a}_{BB}$, while the second time variable $\ddot{a}$ has an infinite positive value. The universe reaches this singularity at some finite moment of cosmic time t_{BB}:

$$a(t_{BB}) = a_{BB} < \infty, \tag{50}$$

$$\dot{a}(t_{BB}) = \dot{a}_{BB} < \infty, \tag{51}$$

$$\lim_{t \to t_{BB}} \ddot{a}(t) = \infty. \tag{52}$$

It is interesting to compare this singularity with other types of cosmological singularities arising in isotropic and homogeneous Friedmann cosmological models. The most known and well studied one is the Big Bang (or Big Crunch) singularity which is characterized by a vanishing scale factor at the initial or final moments of the cosmological evolution:

$$a(t_{In,Fin}) = 0. \tag{53}$$

Such a singularity arises, for example, in Friedmann universes filled by dust, radiation and other types of "standard" matter with the equation of state parameter $w = p/\rho > -1/3$.

During the last decade another type of singularity has acquired some popularity. This is the so called Big Rip singularity [20, 21], when the radius of the universe, its first time derivative and the Hubble variable tend to infinity at some finite moment of time:

$$\lim_{t \to t_{BR}} a(t) = \infty, \tag{54}$$

$$\lim_{t \to t_{BR}} \dot{a}(t) = \infty, \tag{55}$$

$$\lim_{t \to t_{BR}} \frac{\dot{a}(t)}{a(t)} = \infty. \tag{56}$$

Such a singularity arises in the models where the phantom dark energy (i.e. dark energy for which the equation of state parameter $w < -1$) is present. Some modern cosmological observations give certain indications in favour of models, including phantom dark energy.

Another type of cosmological singularity was found in some cosmological models based on tachyon (Born-Infeld-type) field [22]. Such a singularity is a result of a decelerating evolution of the universe which culminates at some finite moment of time t_{BBr} when the cosmological radius has some finite value, its first time derivative and the Hubble variable are equal to zero, while the second time derivative of the cosmological radius tends to $-\infty$:

$$a(t_{BBr}) = a_{BBr} < \infty, \tag{57}$$

$$\dot{a}(t_{BBr}) = 0, \tag{58}$$

$$\lim_{t \to t_{BBr}} \ddot{a}(t) = -\infty. \tag{59}$$

This type of cosmological singularity arises, for example in a simple model, representing a flat Friedmann universe filled with the “anti-Chaplygin gas” [22, 23], whose equation of state is $p = A/\rho$ (where A ia a positive constant) in analogy with the Chaplygin gas cosmological model [24], whose equation of state is $p = -A/\rho$ and which describes a unified model of dark energy and dark matter. However, when one considers a richer tachyon model [22] the situation looks more interesting. Indeed, the cosmic deceleration era in this model can follow a long period of accelerated expansion. Thus, one cannot exclude, that after the period of the accelerated quasi-de Sitter expansion which we experience now, a quite new phase of cosmological evolution will come.

CONCLUSION: INITIAL CONDITIONS, SINGULARITIES, DENSITY MATRIX AND LANDAU'S LEGACY

In conclusion we would like to make some remarks concerning connections between topics touched in the reported series of works [10, 11, 18, 19] and the scientific legacy of L.D. Landau.

First, let us remember that the main goal of works in quantum cosmology is the construction of the quantum state of the universe, which can predict initial conditions for its subsequent classical evolution. As we have already noticed in the Introduction, it was L.D. Landau who pointed out that a consistent physical theory should not only present the equations of motion for the system under consideration, but also be able to predict initial conditions for these equations [1].

Second, considering some string-inspired cosmological models, we have found a new type of the cosmological singularity - the Big Boost singularity, characterized by an infinite value of the cosmic acceleration [19]. The importance of the problem of cosmological singularity was also underlined by Landau in the fifties on equal footing with such topics as the theory of phase transitions and superconductivity [1]. Development of both the theoretical and observational cosmology has confirmed the correctness of his opinion. Indeed, the theoretical study of the anisotropic universe in the vicinity of the Big Bang (Crunch) moment has resulted in the discovery of the phenomenon of the oscillatory approach to the cosmological singularity [25, 26, 27], while the recent observations of the cosmic acceleration phenomenon have stimulated study of other types of cosmological singularities (for a recent review see e.g. [28]).

Finally, the results of the reported works open a new insight into the role of a density matrix in quantum theory. Let us recollect the main features of the density matrix of the universe, which we advocate:

1. It was shown that the density matrix, corresponding to mixed quantum states is a fundamental object, not less fundamental than the wave function, describing pure quantum states.
2. Appearance of the density matrix is not necessarily a result of our ignorance or an artifact of tracing out a part of the degrees of freedom.
3. The universe in the framework of quantum cosmology can be born in a mixed and not in a pure quantum state.

This means that the density matrix plays a more fundamental role than that which was prescribed to it until now.

Here we would like to say that it was L.D. Landau who has introduced the notion of the density matrix in 1927 in the paper “*Das Dämpfungsproblem in der Wellenmechanics*”, Z. Physik, 45 (1927) 430 in parallel with F. Bloch and J. von Neumann [30]. This happened at the dawn of quantum mechanics and all three of them were very young. Landau was the youngest of them – he was 19 year old.

The existence of mixed quantum states described by the density matrix was and is considered as a result of lack of information, which is not connected with the basic laws of quantum theory. The probability weights present in a density matrix are the numbers reflecting traditional statistic probabilities which exist already in the framework of the classical theory. These probability weights which are given by eigenvalues of the density matrix are the relative weights of different pure quantum states constituting a mixed quantum state. Thus, these probability weights play the role similar to one played by the probability distribution function on the phase space in the classical statistical mechanics. Considering pure classical states, where all the coordinates and momenta have determined values, one does not encounter the probabilities. In the case of quantum theory considering pure quantum states, one eliminates the statistical probabilities, while quantum mechanical probabilities are always present. Hence, the statistical probabilities are usually considered to be less fundamental than the quantum mechanical ones. Moreover, one normally thinks that the universe is born and, hence, always exists in the pure quantum state, while its subsystems are usually represented by density matrices due to quantum entanglement effects. The results of the presented series of works show that, at least in some cosmological models, the universe as a whole always exists in the mixed quantum state represented by the cosmological density matrix, while the probability of its birth in the pure quantum state (Hartle-Hawking no-boundary state) is equal to zero. That means that the statistical or "thermodynamical" probability is in a way no less fundamental than the quantum mechanical probability, because, even in principle, we cannot get rid of it, choosing to work only with pure states. Thus, the density matrix introduced by L.D. Landau, F. Bloch and J. von Neumann in 1927, is a more fundamental object than these authors could have imagined at that time.

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